УДК 517.98

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NOTE ON SEPARATELY SYMMETRIC POLYNOMIALS ON THE CARTESIAN PRODUCT OF ℓ_1

F. Jawad. *Note on separately symmetric polynomials on the Cartesian product of* ℓ_1, Mat. Stud. **50** (2018), 204–210.

In the paper, we describe algebraic bases in algebras of separately symmetric polynomials which are defined on Cartesian products of n copies of ℓ_1. Also, we describe spectra of algebras of entire functions, generated by these polynomials as Cartesian products of spectra of algebras of symmetric analytic functions of bounded type on ℓ_1. Finally, we consider algebras of separately symmetric analytic functions of bounded type on infinite direct sums of copies of ℓ_1. In particular, we show that there is a homomorphism from such algebra onto the algebra of all analytic functions of bounded type on a Banach space X with an unconditional basis.

1. Introduction and preliminaries. Let X be a complex Banach space. A function $f\colon X \to \mathbb{C}$ is called *G-analytic* if its restriction to any finite dimensional subspace of X is analytic. A continuous G-analytic function is called *analytic* on X or *entire*. If an entire function is bounded on all bounded subsets, then it is called a function *of bounded type*. An analytic function P_n is *n-homogeneous polynomial* if $P_n(\lambda x) = \lambda^n P_n(x)$, $x \in X$, $\lambda \in \mathbb{C}$. 0-homogeneous polynomial is just a constant. A finite sum of homogeneous polynomials is a *polynomial*. It is well known that any entire function f of bounded type can be represented by the series

$$f(x) = \sum_{n=0}^{\infty} P_n(x), \tag{1}$$

where P_n are n-homogeneous polynomials and the series converges uniformly on all bounded subsets. On the other hand, every series (1) defines an entire function of bounded type if and only if the radius $\varrho_0(f)$ of convergence at zero is equal to infinity, where

$$\varrho_0(f) = \Big(\limsup_{n\to\infty} \|P_n\|^{1/n}\Big)^{-1}. \tag{2}$$

For the general theory of analytic functions on Banach spaces we refer the reader to [9], [17].

Symmetric (invariant) polynomials and analytic functions with respect to actions of a group of operators on an infinite dimensional Banach space X were investigated in many papers (see e.g. [3], [4], [8]). Uniform algebras of symmetric analytic functions on ℓ_p and their

2010 *Mathematics Subject Classification:* 46J15, 46J20.
Keywords: polynomials and analytic functions on Banach spaces; symmetric analytic functions; block-symmetric polynomials; separately symmetric polynomials.
doi:10.15330/ms.50.2.204-210

spectra were investigated in [1]. Topological and algebraic structures on spectra of algebras of symmetric entire functions of bounded type on ℓ_p were studied in [5]–[7] and on L_∞ in [10], [11], [18]. Note that for different representations of a given group we can have different algebras of all symmetric polynomials.

Let S be the group of all bijections of natural numbers $\mathbb{N}$. For every $\sigma \in S$ we define an action of σ on ℓ_1 by

$$\sigma(x) = (x_{\sigma(1)}, x_{\sigma(2)}, \dots),$$

where $x = (x_1, x_2, \dots) \in \ell_1$. A polynomial $P\colon \ell_1 \to \mathbb{C}$ is said to be *symmetric* if $P(\sigma(x)) = P(x)$ for every $X \in \ell_1$ and permutation $\sigma \in S$. It is known [8] that polynomials

$$F_k(X) = \sum_{n=1}^{\infty} x_n^k, \quad k = 1, 2, \dots$$

form an algebraic basis in the algebra of all symmetric polynomials $\mathcal{P}_s(\ell_1)$.

In the paper, we consider algebras of separately symmetric polynomials on the Cartesian product of a finite number of copies of ℓ_1, $\underbrace{\ell_1 \times \cdots \times \ell_1}_{n}$ and describe algebraic bases of these algebras. Also, we consider algebras of separately symmetric analytic functions of bounded type on infinite direct sums of copies of ℓ_1.

Let us recall that a sequence of polynomials $\{P_1, P_2, \dots, P_n, \dots\}$ on a complex linear space X is *algebraically independent* if for every $m \in \mathbb{N}$, for every polynomial $q\colon \mathbb{C}^m \to \mathbb{C}$ and every positive integer $i_1 < \ldots < i_m$ the identity

$$q(P_{i_1}(x), \dots, P_{i_m}(x)) = 0 \qquad \forall x \in X$$

implies that $q(z) = 0$ $\forall z \in \mathbb{C}^n$. A sequence of polynomials on X forms an *algebraic basis* in an algebra $\mathcal{P}$ of polynomials on X if it is algebraically independent and generates this algebra.

2. Main results. Let us denote by

$$\ell_1^{(n)} = \underbrace{\ell_1 \times \cdots \times \ell_1}_{n}$$

the Cartesian product of n copies of ℓ_1 with

$$\|x\| = \|x^{(1)}\| + \cdots + \|x^{(n)}\|, \quad \text{where} \quad x = (x^{(1)}, \dots, x^{(n)}) \in \ell_1^{(n)}.$$

Definition 1. A polynomial $P\colon \ell_1^{(n)} \to \mathbb{C}$ is *separately symmetric* if for all permutations $\sigma_1, \dots, \sigma_n$ on $\mathbb{N}$

$$P(\sigma_1(x^{(1)}), \dots, \sigma_n(x^{(n)})) = P(x^{(1)}, \dots, x^{(n)}).$$

Definition 2. A polynomial $P\colon \ell_1^{(n)} \to \mathbb{C}$ is a *block-symmetric* or *MacMahon symmetric* polynomial if for every permutation σ on $\mathbb{N}$

$$P(\sigma(x^{(1)}), \dots, \sigma(x^{(n)})) = P(x^{(1)}, \dots, x^{(n)}).$$

We denote by $\mathcal{P}_{ss}(\ell_1^{(n)})$ the algebra of all separately symmetrical polynomials on $\ell_1^{(n)}$. Note that $\ell_1^{(n)}$ is isometrically isomorphic to ℓ_1. Indeed, the following map defined by

$$I^{(n)}(x^{(1)},\ldots,x^{(n)}) = (x_1^{(1)},\ldots,x_1^{(n)},x_2^{(1)},\ldots,x_2^{(n)},\ldots,x_k^{(1)},\ldots,x_k^{(n)},\ldots)$$

is an isomorphism from $\ell_1^{(n)}$ onto ℓ_1. If P is a symmetric polynomial on ℓ_1, then $P\circ I^{(n)}$ is separately symmetrical on $\ell_1^{(n)}$. So the map $P\mapsto P\circ I^{(n)}$ is a homomorphism of the algebra of all symmetric polynomials on ℓ_1, $\mathcal{P}_s(\ell_1)$ to $\mathcal{P}_{ss}(\ell_1^{(n)})$. Note that it is not onto. For example, $Q(x,y) = \sum_{i<j} x_ix_j + \sum_{i<j} y_iy_j$ is separately symmetric on $\ell_1^{(2)}$ but polynomial P on ℓ_1 defined by $P(x_1,y_1,\ldots,x_n,y_n,\ldots) = Q(x,y)$ is not symmetric.

Clearly, every separately symmetric polynomial is a block-symmetric polynomials. General algebraic properties of block-symmetric polynomials of finite number of variables can be found in [12]. Algebras generated by block-symmetric polynomials on ℓ_1 and their spectra were investigated in [13], [14], [15].

We will say that $x\sim y$ for some $x,y\in\ell_1^{(n)}$ if $P(x)=P(y)$ for every $P\in\mathcal{P}_{ss}(\ell_1^{(n)})$. For a given $x\in\ell_1^{(n)}$ and $1\le j\le n$ we denote the *support* of $x^{(j)}$ by

$$\operatorname{supp}_j(x) = \operatorname{supp}\bigl(x^{(j)}\bigr) = \{m\in\mathbb{N}\colon x_m^{(j)}\neq 0\}.$$

Lemma 1. *For every $x\in\ell_1^{(n)}$ there is $y\sim x$ such that $\operatorname{supp}_i(y)\cap\operatorname{supp}_j(y)=\varnothing$ for all $1\le i<j\le n$.*

Proof. We observe, first that if $z\in\ell_1^{(n)}$ such that for some $1\le l\le n$,

$$z^{(l)} = (0,x_1^{(l)},x_2^{(l)},\ldots,x_k^{(l)},\ldots)$$

and $z^{(j)}=x^{(j)}$ for all $j\neq l$, then $z\sim x$. Indeed, let $P\in\mathcal{P}_{ss}(\ell_1^{(n)})$. Let us denote by $P_l(z^{(l)})$ the polynomial $P(z)$ for fixed $z^{(j)}$, $j\neq l$. Then P_l is a symmetric polynomial on ℓ_1 and as it is well known for symmetric polynomials,

$$P_l(z^{(l)}) = P_l(0,x_1^{(l)},x_2^{(l)},\ldots,x_k^{(l)},\ldots) = P_l(x_1^{(l)},x_2^{(l)},\ldots,x_k^{(l)},\ldots) = P_l(x^{(l)}).$$

So $P(z)=P(x)$.

Now, it is enough to set

$$y^{(j)} = (\underbrace{0,\ldots,0}_{j-1},x_1^{(j)},\underbrace{0,\ldots,0}_{n-1},x_2^{(j)},\ldots,\underbrace{0,\ldots,0}_{n-1},x_k^{(j)},\ldots),\quad j=1,2,\ldots,n.$$

$\square$

Theorem 1. *Polynomials*

$$F_k^{(j)}(x) = F_k(x^{(j)}) = \sum_{m=1}^{\infty}\bigl(x_m^{(j)}\bigr)^k,\quad j=1,\ldots,n,\quad k=1,2,\ldots \tag{3}$$

form an algebraic basis in $\mathcal{P}_{ss}(\ell_1^{(n)})$.

Proof. As we observed, any separately symmetric polynomial P is a block-symmetric polynomial and so it can be represented by the following algebraic basis in the algebra of block-symmetric polynomials on ℓ_1 (see [15]):

$$F_{k_1\ldots k_n}(x) = \sum_{m=1}^{\infty} \left(x_m^{(1)}\right)^{k_1} \cdots \left(x_m^{(n)}\right)^{k_n}, \quad k_1, \ldots k_n \in \mathbb{Z}_+. \tag{4}$$

Note that polynomials in (4) are separately symmetric only if just one nonnegative integer $k_1, \ldots, k_n$ is greater than zero. Moreover, for every $x \in \ell_1^{(n)}$ we can find $y \sim x$ as in Lemma 1 and since $y^{(j)}$ have mutually disjoint support, $F_{k_1\ldots k_n}(y) = 0$ if for some two different numbers i and l, $k_i \neq 0$ and $k_l \neq 0$. So $P(y)$ can be represented as an algebraic span of $F_k^{(j)}(y)$, $1 \le j \le n$, $k \in \mathbb{N}$. But $P(x) = P(y)$, and $F_k^{(j)}(x) = F_k^{(j)}(y)$. Thus P is an algebraic span of separately symmetric polynomials $F_k^{(j)}(y)$, $1 \le j \le n$, $k \in \mathbb{N}$. Also, it is easy to check that $F_k^{(j)}$ are algebraically independent. So they form an algebraic basis in $\mathcal{P}_{ss}(\ell_1^{(n)})$. □

Let us denote by $H_{bss}(\ell_1^{(n)})$ the completion of $\mathcal{P}_{ss}(\ell_1^{(n)})$ with respect to the topology of uniform convergence on bounded subsets of $\ell_1^{(n)}$. Clearly, $H_{bss}(\ell_1^{(n)})$ is a uniform Fréchet algebra and $H_{bss}(\ell_1^{(n)})$ is a closed subalgebra of the algebra $H_b(\ell_1^{(n)})$ of entire functions of bounded type on $\ell_1^{(n)}$.

Let us recall that the *spectrum* of a Fréchet algebra is the set of all nonzero continuous complex homomorphisms. It is well known that the spectrum of algebra $H(\mathbb{C}^n)$ of entire functions on $\mathbb{C}^n$ consists of point-evaluation functionals δ_z, $z \in \mathbb{C}^n$, $\delta_z(f) = f(z)$. Spectra of algebras $H_b(X)$ of entire functions on Banach spaces X are much more complicated. In particular, in [2] it was shown that the spectra contain (but not limited to) point-evaluation functionals at elements of second dual spaces X'' of X. In [19] the spectra was described using second dual spaces to symmetric projective tensor products of X. Spectra of algebras of symmetric functions on ℓ_p were investigated in [1], [5], [6], [7]. It was shown that if $x, y \in \ell_p$ and $x = \sigma(y)$ for some permutation σ, then $\delta_x = \delta_y$. On the other hand, the spectrum of $H_{bs}(\ell_p)$ contains a family of elements ψ_λ, $\lambda \in \mathbb{C}$ which does not belong to the set of point-evaluation functionals. We denote by $M_{bss}^{(n)}$ the spectrum of $H_{bss}(\ell_1^{(n)})$.

Theorem 2. *The spectrum $M_{bss}^{(n)}$ of $H_{bss}(\ell_1^{(n)})$ is the Cartesian product of n copies of the spectrum $H_{bs}(\ell_1)$.*

Proof. Let φ be a continuous complex homomorphism of $H_{bss}(\ell_1^{(n)})$. Since $\{F_k^{(j)}\}$ forms an algebraic basis in $H_{bss}(\ell_1^{(n)})$, φ is completely defined by its values on the basis elements,

$$\xi_k^{(j)} = \varphi\left(F_k^{(j)}\right), \quad j = 1, \ldots, n, \quad k = 1, 2, \ldots. \tag{5}$$

For any fixed j, formula (5) defines a complex homomorphism $\varphi^{(j)}$ on $H_{bs}(\ell_1)$. So φ is defined by the collection $(\varphi^{(1)}, \ldots, \varphi^{(n)})$. Conversely, every such a collection uniquely defines an element in $M_{bss}^{(n)}$. □

Now let us consider the case of infinite direct sums. Recall that a sequence $\{e_n\}_{n=1}^{\infty}$ of a Banach space X is called an *unconditional (linear topological)* basis if it is linearly independent and every $a \in X$ can be represented by

$$a = \sum_{n=1}^{\infty} a_n e_n$$

such that the series converges unconditionally.

For a given Banach space X with an unconditional basis $\{e_n\}_{n=1}^{\infty}$ we consider a Banach space $\ell_1^{(X)}$ defined by the following way. If $x \in \ell_1^{(X)}$, then

$$x = (x^{(1)}, x^{(2)}, \dots, x^{(n)}, \dots),$$

where each $x^{(n)} = (x_1^{(n)}, \dots, x_k^{(n)}, \dots) \in \ell_1$ and

$$\sum_{n=1}^{\infty} \|x^{(n)}\|_{\ell_1} e_n \in X$$

with

$$\|x\| = \left\| \sum_{n=1}^{\infty} \|x^{(n)}\|_{\ell_1} e_n \right\|_X .$$

Note that if $X = \ell_1$, then $\ell_1^{(\ell_1)}$ is isometrically isomorphic to the complete projective tensor product $\ell_1 \widehat{\bigotimes} \ell_1$.

A polynomial P on $\ell_1^{(X)}$ is *separately symmetric* if for every sequence of permutations on $\mathbb{N}$, $\sigma = (\sigma_1, \dots, \sigma_n, \dots)$ we have $P(\sigma(x)) = P(\sigma_1(x^{(1)}), \dots, \sigma_n(x^{(n)}), \dots) = P(x)$ for all $x \in \ell_1^{(X)}$. Clearly, polynomials

$$F_m^{(j)}(x) = \sum_{k=1}^{\infty} (x_k^{(j)})^m, \quad j, m = 1, 2, \dots$$

are separately symmetric and algebraically independent. However, they does not form an algebraic basis in the algebra of all separately symmetric polynomials $\mathcal{P}_{ss}(\ell_1^{(X)})$ in the general case.

Lemma 2. *For every $a \in X$, $a = \sum_{j=1}^{\infty} a_j e_j$ there is a continuous complex homomorphism ψ_a on $H_{bss}(\ell_1^{(X)})$ such that $\psi_a(F_1^{(j)}) = a_j$ and $\psi_a(F_m^{(j)}) = 0$ for all $m > 1$ and $j \in \mathbb{N}$.*

Proof. Using an idea from [1], we define a sequence $v_n \in \ell_1^{(X)}$ by the following way

$$v_n = \sum_{j=1}^{\infty} (v^{(j)})_n e_j,$$

where $(v^{(j)})_n$ belongs to the jth copy of ℓ_1 and

$$(v^{(j)})_n = \Big(\underbrace{\frac{a_j}{n}, \dots, \frac{a_j}{n}}_{n \text{ times}}, 0, 0, \dots \Big).$$

Then

$$\|v_n\| = \left\| \sum_{j=1}^{\infty} \|(v^{(j)})_n\|_{\ell_1} e_j \right\|_X = \left\| \sum_{j=1}^{\infty} |a_j| e_j \right\|_X \le 2K \left\| \sum_{j=1}^{\infty} a_j e_j \right\|_X = 2K \|a\|_X,$$

where K is the unconditional constant of the basis $\{e_j\}_{j=1}^{\infty}$ (see [16, p. 19]). So the sequence $\{v_n\}_{n=1}^{\infty}$ is bounded. Since bounded subsets of spectra of Fréchet algebras are relatively compact, the corresponding sequence of point-evaluation complex homomorphisms $\{\delta_{v_n}\}_{n=1}^{\infty}$ must have a cluster point ψ_a. Since $F_1^{(j)}(v_n) = a_j$ for every $n \in \mathbb{N}$, $\psi_a(F_1^{(j)}) = a_j$. On the other hand, $F_m^{(j)}(v_n) = \frac{a_j^m}{n^{m-1}} \to 0$ as $n \to \infty$ and so $\psi_a(F_m^{(j)}) = 0$ for all $m > 1$ and $j \in \mathbb{N}$. $\square$

We denote by $M_{bss}^{(X)}$ the spectrum of $H_{bss}(\ell_1^{(X)})$. Let $\Psi\colon X \to H_{bss}(\ell_1^{(X)})$, $\Psi(a) = \psi_a$.

Theorem 3. *There is a homomorphism C_Ψ from $H_{bss}(\ell_1^{(X)})$ onto $H_b(X)$ defined by*

$$C_\Psi(f) = \hat{f} \circ \Psi,$$

where $\hat{f}$ is the Gelfand transform of $f \in H_{bss}(\ell_1^{(X)})$.

Proof. Let $g(a) = C_\Psi(f) = \hat{f} \circ \Psi(a) = \psi_a(f)$, $a \in X$. That is, g is well defined on X. If P_n is an n-homogeneous polynomial on X, then $C_\Psi(P_n)$ is an n-homogeneous polynomial on X and

$$|C_\Psi(P_n)(a)| \le \sup_m |P_n(v_m)| \le \|P_n\| \sup_m \|v_m\|^n = \|P_n\| \left\| \sum_{j=1}^{\infty} |a_j| e_j \right\|_X^n \le \|P_n\| (2K)^n \|a\|_X^n.$$

Thus $\|C_\Psi(P_n)\| \le (2K)^n \|P_n\|$, where K is the unconditional constant of the topological basis in X. So

$$g(a) = \sum_{n=0}^{\infty} C_\Psi(P_n)(a),$$

where P_n are like in (1) and according to (2), $\varrho_0(g) = \frac{1}{2K}\varrho_0(f) = \infty$. So $g \in H_b(X)$.

Conversely, let $g \in H_b(X)$ and

$$g(a) = \sum_{n=0}^{\infty} Q_n(a)$$

be the representation of g by the series of homogeneous polynomials. Since X has an unconditional basis, for $n > 0$

$$Q_n(a) = Q_n\Big(\sum_{m=1}^{\infty} a_m e_m\Big) = \sum_{i_1,\ldots,i_n=1}^{\infty} c_{i_1,\ldots,i_n} a_{i_1} \cdots a_{i_n},$$

for some constants $c_{i_1,\ldots,i_n}$. We set $P_0 = Q_0$ and

$$P_n(x) = \sum_{i_1,\ldots,i_n=1}^{\infty} c_{i_1,\ldots,i_n} F_1^{(i_1)}(x) \cdots F_1^{(i_n)}(x).$$

Since $\|F_1^{(j)}\| = 1$, $|F_1^{(j)}(x)| \le \|x^{(j)}\|$ and so $\|P_n\| \le \|Q_n\|$. Thus if we define

$$f(x) = \sum_{n=0}^{\infty} P_n(x), \qquad x \in \ell_1^{(X)},$$

then $\varrho_0(f) \ge \varrho_0(g) = \infty$ and so $f \in H_{bss}(\ell_1^{(X)})$. According to the definition of Ψ and Lemma 2, $C_\Psi(f) = g$ and so C_Ψ is onto. □

Note that in the general case $H_b(X)$ is nonseparable and so $H_{bss}(\ell_1^{(X)})$ is nonseparable and can not to be generated by a countable algebraic basis.

Corollary 1. *The spectrum $M_{bss}^{(X)}$ contains a copy of the spectrum of $H_b(X)$, in particular, it contains a point-wise copy of X''.*

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Received 30.10.2018
Revised 12.12.2018