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INFINITE-MODAL APPROXIMATE SOLUTIONS OF THE BRYAN-PIDDUCK EQUATION[1]

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The nonlinear integro-differential Bryan-Pidduck equation for a model of rough spheres is considered. An approximate solution is constructed in the form of an infinite linear combination of some Maxwellian modes with coefficient functions that depend on time and spatial coordinate. Sufficient conditions for the infinitesimality of the uniformly-integral error between the parts of the Bryan-Pidduck equation are obtained.

1. Statement of the problem. This article describes a model of rough spheres [1] which was firstly introduced in 1894 by Bryan. The methods developed by Chapman and Enskog for general non-rotating spherical molecules were extended to Bryan's model by Pidduck in 1922. The model possesses an advantage over all other variably rotating models in that no variables are required to specify its orientation in the space.

These molecules are perfectly elastic and perfectly rough is to be interpreted as follows. When two molecules collide, the points which come into contact will not, in general, possess the same velocity. It is supposed that the two spheres grip each other without slipping; first each sphere is strained by the other, and then the strain energy is reconverted into kinetic energy of translation and rotation, no energy being lost; the effect is that the relative velocity of the spheres at their point of contact is reversed by the impact.

The model applies to monatomic molecules and, given the possibility of rotation, is more physical than the model of hard balls and interesting to explore.

The Boltzmann equation for the model of rough spheres (or the Bryan-Pidduck equation) has the form ([1–4])

$$D(f) = Q(f,f); \tag{1}$$

where the differential operator $D(f)$ is analytically expressed as follows:

$$D(f) \equiv \frac{\partial f}{\partial t} + \left(V, \frac{\partial f}{\partial x}\right), \tag{2}$$

and the collision integral $Q(f,f)$ has the form

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$$Q(f,f) \equiv \frac{d^2}{2}\int\limits_{\mathbb{R}^3} dV_1 \int\limits_{\mathbb{R}^3} d\omega_1 \int\limits_{\Sigma} d\alpha B(V-V_1,\alpha)\times$$
$$\times\Big[f(t,V_1^*,x,\omega_1^*)f(t,V^*,x,\omega^*) - f(t,V,x,\omega)f(t,V_1,x,\omega_1)\Big]. \qquad (3)$$

Here d is the diameter of the molecule, which is associated with the moment of inertia I by the following relation

$$I = \frac{bd^2}{4}, \qquad (4)$$

where b is the parameter, $b \in \left(0, \frac{2}{3}\right]$, characterizing the isotropic distribution of matter inside the gas particles; t is the time; $x = (x^1, x^2, x^3) \in \mathbb{R}^3$ is the spatial coordinate; $V = (V^1, V^2, V^3) \in \mathbb{R}^3$ and $w = (w^1, w^2, w^3) \in \mathbb{R}^3$ are the linear and angular velocity of the molecule, respectively; $\frac{\partial f}{\partial x}$ is the gradient of the function f of the variable x; Σ is the unit sphere in the space $\mathbb{R}^3$; α is the unit vector in $\mathbb{R}^3$, directed along the line connecting the centers of the colliding molecules;

$$B\,(V-V_1,\alpha) = |(V-V_1,\alpha)| - (V-V_1,\alpha) \qquad (5)$$

— the collision term.

The linear (V^*, V_1^*) and angular (w^*, w_1^*) molecular velocity after the collision can be expressed by the appropriate values before the collision as follows:

$$\begin{aligned}
V^* &= V - \frac{1}{b+1}\left(b(V_1-V) - \frac{bd}{2}\alpha\times(\omega+\omega_1) + \alpha(\alpha, V_1-V)\right),\\
V_1^* &= V_1 + \frac{1}{b+1}\left(b(V_1-V) - \frac{bd}{2}\alpha\times(\omega+\omega_1) + \alpha(\alpha, V_1-V)\right),\\
\omega^* &= \omega + \frac{2}{d(b+1)}\left\{\alpha\times(V-V_1) + \frac{d}{2}\left[\alpha(\omega+\omega_1,\alpha) - \omega - \omega_1\right]\right\},\\
\omega_1^* &= \omega_1 + \frac{2}{d(b+1)}\left\{\alpha\times(V-V_1) + \frac{d}{2}\left[\alpha(\omega+\omega_1,\alpha) - \omega - \omega_1\right]\right\},
\end{aligned} \qquad (6)$$

where the symbol $\times$ indicated the vector product. These formulas can be obtained using the laws of conservation of momentum, the total energy of translational and rotational motion (for the first time they are given in [1]).

In this work we need also the following generalization of formula (3):

$$Q(f,g) \equiv \frac{d^2}{2}\int\limits_{\mathbb{R}^3} dV_1 \int\limits_{\mathbb{R}^3} d\omega_1 \int\limits_{\Sigma} d\alpha B(V-V_1,\alpha)\times$$
$$\times\Big[f(t,V_1^*,x,\omega_1^*)g(t,V^*,x,\omega^*) - f(t,V,x,\omega)g(t,V_1,x,\omega_1)\Big],$$

which turns into (3) if $g = f$.

Maxwellians are the only known to the present moment the exact explicit solutions of the Boltzmann equation. The most general form of local Maxwellians (i.e. those that depend of t and x) was received at work [5].

Earlier ([3–5,11–14]), the authors constructed bimodal distributions in the form sums of two Maxwellians with coefficient functions that depends on t and x. In the present paper we will construct infinite-modal approximate solutions; i.e. such, that are presented in the form of a series

$$f(t,V,x,\omega)=\sum_{i=1}^{\infty}\varphi_i(t,x)M_i(V,\omega), \tag{7}$$

where $\varphi_i(t,x)$ (here and everywhere below $i\in\mathbb{N}$) are non-negative functions, smooth on $\mathbb{R}^4$, but in the role of Maxwellian M_i we firstly consider the global Maxwellians

$$M_i(V,\omega)=\rho_i I^{3/2}\left(\frac{\beta_i}{\pi}\right)^3 e^{-\beta_i\left(\left(V-\overline{V}_i\right)^2+I\omega^2\right)}, \tag{8}$$

ρ_i is a positive constant, denoting density of the i-th flow, β_i is the inverse temperature

$$\beta_i=\frac{1}{2T_i}; \tag{9}$$

the constant vector $\overline{V}_i\in\mathbb{R}^3$ denote mass velocity of i-th flow.

As a measure of the deviation between the parts of equation (1) we will consider a uniform-integral error of the form:

$$\Delta=\sup_{(t,x)\in\mathbb{R}^4}\int\limits_{\mathbb{R}^3}dV\int\limits_{\mathbb{R}^3}d\omega\left|D(f)-Q(f,f)\right|. \tag{10}$$

The aim of this paper is to find the form of the coefficient functions $\varphi_i(t,x)$ and determining the conditions for the hydrodynamic parameters of Maxwellian (8), such that error (10) could be made arbitrarily small.

2. Main results. Here we will formulate and prove some theorems that give sufficient conditions for infinitesimality of the error (10).

Theorem 1. *Let the coefficient functions $\varphi_i(t,x)$ in the distribution (7) be such that the functional series:*

$$\sum_{i=1}^{\infty}\varphi_i M_i,\quad \sum_{i=1}^{\infty}M_i\left|\frac{\partial\varphi_i}{\partial t}\right|,\quad \sum_{i=1}^{\infty}M_i\left|\frac{\partial\varphi_i}{\partial x}\right| \tag{11}$$

converge uniformly in the whole space of variables $(t,x,V)\in\mathbb{R}^7$.

Then there exists such a quantity Δ', that:

$$\Delta\leqslant\Delta' \tag{12}$$

and the following assertion holds

$$\lim_{\beta_i\to+\infty}\Delta'=\sum_{i=1}^{\infty}\rho_i\sup_{(t,x)\in\mathbb{R}^4}\left|\frac{\partial\varphi_i}{\partial t}+\left(\overline{V}_i,\frac{\partial\varphi_i}{\partial x}\right)\right|+2\pi d^2\sum_{\substack{i,j=1\\ i\neq j}}^{\infty}\rho_i\rho_j\left|\overline{V}_i-\overline{V}_j\right|\sup_{(t,x)\in\mathbb{R}^4}(\varphi_i\varphi_j). \tag{13}$$

Remark 1. Here and further a limiting procedure when $\beta_i\to+\infty$ it is necessary to understand in the following sense: for all $i\in\mathbb{N},\beta_i:=\theta_i+c$, where $\theta_i>0$, and $c>0$ is a fixed constant, and then $\sup\limits_{\theta_i,i\in\mathbb{N}}(\lim\limits_{c\to+\infty}\Delta')$ is equal to the right side (13).

Proof. Let us substitute of the distribution (7) in the left-hand side of (1), using the expression for it (2), which is possible due to the condition (11):

$$D(f)=\frac{\partial}{\partial t}\left(\sum_{i=1}^{\infty}\varphi_i M_i\right)+\left(V,\frac{\partial}{\partial x}\left(\sum_{i=1}^{\infty}\varphi_i M_i\right)\right)=$$

$$=\sum_{i=1}^{\infty}M_i\frac{\partial\varphi_i}{\partial t}+\sum_{i=1}^{\infty}\varphi_i\frac{\partial M_i}{\partial t}+\left(V,\sum_{i=1}^{\infty}M_i\frac{\partial\varphi_i}{\partial x}\right)+\left(V,\sum_{i=1}^{\infty}\varphi_i\frac{\partial M_i}{\partial t}\right)=$$

$$=\sum_{i=1}^{\infty}M_i\frac{\partial\varphi_i}{\partial t}+\sum_{i=1}^{\infty}\left(V,M_i\frac{\partial\varphi_i}{\partial x}\right)+\sum_{i=1}^{\infty}\varphi_i\frac{\partial M_i}{\partial t}+\sum_{i=1}^{\infty}\left(V,\varphi_i\frac{\partial M_i}{\partial t}\right)=$$

$$=\sum_{i=1}^{\infty}M_iD(\varphi_i)+\sum_{i=1}^{\infty}\varphi_iD(M_i).$$

Since M_i is an exact solution of equation (1), which draws both its parts into 0, we have

$$D(f)=\sum_{i=1}^{\infty}M_iD(\varphi_i). \tag{14}$$

Next we obtain the form of the collision integral $Q(f,f)$ in case of distribution (7), using the definition (3):

$$Q(f,f)=\frac{d^2}{2}\int_{\mathbb{R}^3}dV_1\int_{\mathbb{R}^3}d\omega_1\int_{\Sigma}d\alpha B(V-V_1,\alpha)\times$$

$$\times\left[\left(\sum_{i=1}^{\infty}\varphi_iM_i(V^*,\omega^*)\right)\cdot\left(\sum_{i=1}^{\infty}\varphi_iM_i(V_1^*,\omega_1^*)\right)-\left(\sum_{i=1}^{\infty}\varphi_iM_i(V,\omega)\right)\cdot\left(\sum_{i=1}^{\infty}\varphi_iM_i(V_1,\omega_1)\right)\right],$$

applying the well-known Cauchy theorem on the product of series, thanks to condition (11), we have:

$$Q(f,f)=\frac{d^2}{2}\int_{\mathbb{R}^3}dV_1\int_{\mathbb{R}^3}d\omega_1\int_{\Sigma}d\alpha B(V-V_1,\alpha)\times$$

$$\times\left[\sum_{i=1}^{\infty}\varphi_i^2M_i(V^*,\omega^*)M_i(V_1^*,\omega_1^*)+\sum_{ci,j=1,\ i\neq j}^{\infty}\varphi_i\varphi_jM_i(V^*,\omega^*)M_j(V_1^*,\omega_1^*)-\right.$$

$$\left.-\sum_{i=1}^{\infty}\varphi_i^2M_i(V,\omega)M_i(V_1,\omega_1)-\sum_{ci,j=1,\ i\neq j}^{\infty}\varphi_i\varphi_jM_i(V,\omega)M_j(V_1,\omega_1)\right]=$$

$$=\sum_{i=1}^{\infty}\varphi_i^2Q(M_i,M_i)+\sum_{ci,j=1,\ i\neq j}^{\infty}\varphi_i\varphi_jQ(M_i,M_j),$$

and again taking into account that $Q(M_i,M_i)=0$, finally we have

$$Q(f,f)=\sum_{i,j=1,\ i\neq j}^{\infty}\varphi_i\varphi_jQ(M_i,M_j). \tag{15}$$

Using the expressions (14), (15) for parts of the Boltzmann equation under the substitution of the distribution (7), we obtain the following estimate

$$\Big|D(f)-Q(f,f)\Big| = \left|\sum_{i=1}^{\infty} M_i\left(\frac{\partial\varphi_i}{\partial t}+\left(V,\frac{\partial\varphi_i}{\partial x}\right)\right)-\sum_{i,j=1,\ i\neq j}^{\infty}\varphi_i\varphi_j Q(M_i,M_j)\right| \leqslant$$
$$\leqslant \sum_{i=1}^{\infty} M_i\left|\frac{\partial\varphi_i}{\partial t}+\left(V,\frac{\partial\varphi_i}{\partial x}\right)\right|+\sum_{i,j=1,\ i\neq j}^{\infty}\varphi_i\varphi_j|Q(M_i,M_j)|. \tag{16}$$

We verify that the second series in the last inequality converges uniformly in the whole space $\mathbb{R}^7$, by the imposed conditions of the theorem. The following estimate

$$\varphi_i\varphi_j|Q(M_i,M_j)| =$$
$$=\frac{d^2}{2}\varphi_i\varphi_j\left|\int\limits_{\mathbb{R}^3}dV_1\int\limits_{\mathbb{R}^3}d\omega_1\int\limits_{\Sigma}d\alpha B(V-V_1,\alpha)\left[M_i(V^*,\omega^*)M_j(V_1^*,\omega_1^*)-M_i(V,\omega)M_j(V_1,\omega_1)\right]\right| =$$
$$=\frac{d^2}{2}\left|\int\limits_{\mathbb{R}^3}dV_1\int\limits_{\mathbb{R}^3}d\omega_1\int\limits_{\Sigma}d\alpha B(V-V_1,\alpha)\left[M_i(V^*,\omega^*)M_j(V_1^*,\omega_1^*)-M_i(V,\omega)M_j(V_1,\omega_1)\right]\varphi_i\varphi_j\right| \leqslant$$
$$\leqslant\frac{d^2}{2}\int\limits_{\mathbb{R}^3}dV_1\int\limits_{\mathbb{R}^3}d\omega_1\int\limits_{\Sigma}d\alpha B(V-V_1,\alpha)\left|\left[M_i(V^*,\omega^*)M_j(V_1^*,\omega_1^*)-M_i(V,\omega)M_j(V_1,\omega_1)\right]\varphi_i\varphi_j\right| \leqslant$$
$$\leqslant\frac{d^2}{2}\int\limits_{\mathbb{R}^3}dV_1\int\limits_{\mathbb{R}^3}d\omega_1 4\pi\left(|V|+|V_1|\right)\cdot 2M_i(V,\omega)M_j(V_1,\omega_1)\varphi_i\varphi_j =$$
$$=4\pi d^2\left(|V|\varphi_i M_i(V,\omega)\int\limits_{\mathbb{R}^3}dV_1\int\limits_{\mathbb{R}^3}d\omega_1 M_j(V_1,\omega_1)\varphi_j+\right.$$
$$\left.+\varphi_i M_i(V,\omega)\int\limits_{\mathbb{R}^3}dV_1\int\limits_{\mathbb{R}^3}d\omega_1 M_j(V_1,\omega_1)\varphi_j\right)$$

hold, which is the result of multiplying series of the form $\sum_{i=1}^{\infty}\varphi_i M_i$ and $\sum_{j=1}^{\infty}\varphi_j M_j|V|$ and integrating one of them, but only members are used with $i\neq j$. The convergence of the product is guaranteed by the condition (11), then it is obvious that the second series in (16) converges uniformly in the whole space $\mathbb{R}^7$.

We now integrate the inequality (16) by velocity spaces V and ω, which is possible due to the condition (11), which ensures the uniform convergence of the series on the right-hand side of the last inequality, as was verified above:

$$\int\limits_{\mathbb{R}^3}dV\int\limits_{\mathbb{R}^3}d\omega\Big|D(f)-Q(f,f)\Big| \leqslant$$
$$\leqslant\sum_{i=1}^{\infty}\int\limits_{\mathbb{R}^3}dV\int\limits_{\mathbb{R}^3}d\omega M_i\left|\frac{\partial\varphi_i}{\partial t}+\left(V,\frac{\partial\varphi_i}{\partial x}\right)\right|+\sum_{\substack{i,j=1\\ i\neq j}}^{\infty}\int\limits_{\mathbb{R}^3}dV\int\limits_{\mathbb{R}^3}d\omega\varphi_i\varphi_j|Q(M_i,M_j)|. \tag{17}$$

As was demonstrated in [14]

$$\int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega Q(M_i, M_j) = 0$$

and, taking into account the expansion $Q(M_i, M_j)$ to "gain"(G) and "loss"(L) members ([6]):

$$Q(f, g) = G(f, g) - fL(g),$$

where

$$G(f, g) = \frac{d^2}{2} \int\limits_{\mathbb{R}^3} dV_1 \int\limits_{\mathbb{R}^3} d\omega_1 \int\limits_{\Sigma} d\alpha B(V - V_1, \alpha) f(t, V_1^*, x, \omega_1^*) g(t, V^*, x, \omega^*),$$

$$L(g) = \frac{d^2}{2} \int\limits_{\mathbb{R}^3} dV_1 \int\limits_{\mathbb{R}^3} d\omega_1 \int\limits_{\Sigma} d\alpha B(V - V_1, \alpha) g(t, V_1, x, \omega_1),$$

we have

$$\int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega G(M_i, M_j) = \int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega M_i L(M_j).$$

Taking this into account, we continue the estimation (17):

$$\int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega \Big| D(f) - Q(f, f) \Big|$$
$$\leqslant \sum_{i=1}^{\infty} \int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega M_i \left| \frac{\partial \varphi_i}{\partial t} + \left(V, \frac{\partial \varphi_i}{\partial x} \right) \right| + 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{\infty} \int\limits_{\mathbb{R}^3} dV \varphi_i \varphi_j G(M_i, M_j). \tag{18}$$

In the work [13] was shown that

$$\int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega G(M_i, M_j) = \frac{d^2 \rho_i \rho_j}{\pi^2} \int\limits_{\mathbb{R}^3} dq \int\limits_{\mathbb{R}^3} dq_1 e^{-q^2 - q_1^2} \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \overline{V}_i - \overline{V}_j \right|. \tag{19}$$

Using equality (19), the estimation (18) takes the following form:

$$\int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega \Big| D(f) - Q(f, f) \Big| \leqslant \sum_{i=1}^{\infty} \int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega M_i \left| \frac{\partial \varphi_i}{\partial t} + \left(V, \frac{\partial \varphi_i}{\partial x} \right) \right| +$$
$$+ 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{\infty} \frac{d^2 \rho_i \rho_j}{\pi^2} \varphi_i \varphi_j \int\limits_{\mathbb{R}^3} dq \int\limits_{\mathbb{R}^3} dq_1 e^{-q^2 - q_1^2} \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \overline{V}_i - \overline{V}_j \right|. \tag{20}$$

Further, we pass to the supremum in the last inequality (20) by the whole space $(t, x) \in \mathbb{R}^4$, in view of the condition (11) of theorem, and taking into account the type of error (10) we see that the inequality (12) is satisfied for the quantity Δ' of the following form:

$$\Delta' = \sum_{i=1}^{\infty} \rho_i \left(\frac{\beta_i}{\pi} \right)^{3/2} \int\limits_{\mathbb{R}^3} dV e^{-\beta_i (V - \overline{V}_i)^2} \sup_{(t,x) \in \mathbb{R}^4} \left| \frac{\partial \varphi_i}{\partial t} + \left(V, \frac{\partial \varphi_i}{\partial x} \right) \right| +$$

$$+2\sum_{\substack{i,j=1\\i\neq j}}^{\infty}\frac{d^2\rho_i\rho_j}{\pi^2}\int\limits_{\mathbb{R}^3}dq\int\limits_{\mathbb{R}^3}dq_1e^{-q^2-q_1^2}\left|\frac{q}{\sqrt{\beta_i}}-\frac{q_1}{\sqrt{\beta_j}}+\overline{V}_i-\overline{V}_j\right|\sup_{(t,x)\in\mathbb{R}^4}(\varphi_i\varphi_j),$$

after integration over the space of angular velocities $\omega\in\mathbb{R}^3$.

In the integral of the first functional series, we will perform a change of variables

$$p=\sqrt{\beta_i}(V-\overline{V}_i),$$

whose Jacobian is $J=\beta_i^{-3/2}$ and now $V=\frac{p}{\sqrt{\beta_i}}+\overline{V}_i$, then

$$\Delta'=\pi^{-3/2}\sum_{i=1}^{\infty}\rho_i\int\limits_{\mathbb{R}^3}dVe^{-p^2}\sup_{(t,x)\in\mathbb{R}^4}\left|\frac{\partial\varphi_i}{\partial t}+\left(\frac{p}{\sqrt{\beta_i}}+\overline{V}_i,\frac{\partial\varphi_i}{\partial x}\right)\right|+$$
$$+2\sum_{\substack{i,j=1\\i\neq j}}^{\infty}\frac{d^2\rho_i\rho_j}{\pi^2}\int\limits_{\mathbb{R}^3}dq\int\limits_{\mathbb{R}^3}dq_1e^{-q^2-q_1^2}\left|\frac{q}{\sqrt{\beta_i}}-\frac{q_1}{\sqrt{\beta_j}}+\overline{V}_i-\overline{V}_j\right|\sup_{(t,x)\in\mathbb{R}^4}(\varphi_i\varphi_j).\tag{21}$$

Carrying out the following re-designation

$$\gamma_i=\frac{1}{\sqrt{\beta_i}},\tag{22}$$

we get the following result for Δ'

$$\pi^{-3/2}\sum_{i=1}^{\infty}\rho_i\int\limits_{\mathbb{R}^3}dVe^{-p^2}\sup_{(t,x)\in\mathbb{R}^4}\left|\frac{\partial\varphi_i}{\partial t}+\left(p\gamma_i+\overline{V}_i,\frac{\partial\varphi_i}{\partial x}\right)\right|+$$
$$+2\sum_{\substack{i,j=1\\i\neq j}}^{\infty}\frac{d^2\rho_i\rho_j}{\pi^2}\int\limits_{\mathbb{R}^3}dq\int\limits_{\mathbb{R}^3}dq_1e^{-q^2-q_1^2}\left|q\gamma_i-q_1\gamma_j+\overline{V}_i-\overline{V}_j\right|\sup_{(t,x)\in\mathbb{R}^4}(\varphi_i\varphi_j).\tag{23}$$

The limiting transition in (21) $\beta_i\to+\infty$ is equivalent to $\gamma_i\to+0$ in (23) for which the continuity of expression is necessary (23) at zero, which is ensured by the condition of uniform convergence (11) and the obvious estimation $|\gamma_i|\leqslant\frac{1}{\sqrt{\theta_i+c}}$ (see the Remark 1). Then, using the lemma from [8] on the continuity of the supremum with respect to the parameter and on the theorems on the continuity of the integral and the functional series with respect to the parameter, we have

$$\lim_{\beta_i\to+\infty}\Delta'=\pi^{-3/2}\sum_{i=1}^{\infty}\rho_i\int\limits_{\mathbb{R}^3}dpe^{-p^2}\sup_{(t,x)\in\mathbb{R}^4}\left|\frac{\partial\varphi_i}{\partial t}+\left(\overline{V}_i,\frac{\partial\varphi_i}{\partial x}\right)\right|+$$
$$+2\sum_{\substack{i,j=1\\i\neq j}}^{\infty}\frac{d^2\rho_i\rho_j}{\pi^2}\int\limits_{\mathbb{R}^3}dq\int\limits_{\mathbb{R}^3}dq_1e^{-q^2-q_1^2}\left|\overline{V}_i-\overline{V}_j\right|\sup_{(t,x)\in\mathbb{R}^4}(\varphi_i\varphi_j)$$

and, calculating the integrals in the sums, we have (13), which proves the theorem. □

Corollary 1. *Let functions* φ_i *have the form:*

$$\varphi_i(t,x) = C_i(x - \overline{V}_i t) \tag{24}$$

or

$$\varphi_i(t,x) = E_i\left(x \times \overline{V}_i\right) \tag{25}$$

with functions C_i, E_i *such that they are satisfy the conditions (11) of the Theorem 1. Also, let at least one of the following requirements holds:*

$$\overline{V}_i = \overline{V}_j, \tag{26}$$

$$\operatorname{supp}\varphi_i \cap \operatorname{supp}\varphi_j = \varnothing \quad (i \neq j), \tag{27}$$

$$d \to 0, \tag{28}$$

then the error (10) for the distribution (7) can be made arbitrarily small.

Proof. If functions φ_i have the form (24), then:

$$\frac{\partial \varphi_i}{\partial t} = -(\overline{V}_i, C_i'), \quad \frac{\partial \varphi_i}{\partial x} = C_i',$$

therefore the first sum in (13) will be equal to zero, and the fulfillment of one of the conditions (26), (27) or (28) eliminates the second term too. Taking into account inequality (12), we have the infinitesimality of the uniform-integral error considered in (10) for the distribution (7) with coefficient functions φ_i of the form (24).

If the coefficient functions φ_i have the form (25), then the derivative with respect to time is immediately equal to zero, the gradient by x of the product $\left(\overline{V}_i, \frac{\partial \varphi_i}{\partial x}\right)$ vanishes, and again under one of the conditions (26), (27) and (28) we will find that the error (10) can be made arbitrarily small. □

Remark 2. Taking into account that the distribution (7) considered above contains the sum of an infinite number of Maxwellians, we see that for the infinitesimality of the uniform-integral error (10) one can choose not only one of the conditions (26), (27), but several simultaneously for different sets of Maxwellians.

Remark 3. From the physical point of view, the constructed approximate solution describes the interaction of an infinite set of flows in a gas from rough spheres, each of which corresponds to either a clot of a gas (24), or cylindrical distribution (25). Each two of these streams have either the same linear velocities, or do not intersect in the space, or all are correspond to the near-Knudsen gas.

Next, consider a stationary, inhomogeneous Maxwellians [1,5,6], those in contrast to the Maxwellians, which depend only on the linear V and angular ω velocities, have the dependence on the spatial coordinate x too. Structure of the Maxwellian M_i (8) remains unchanged, but in contrast to global case, the density ρ_i now has the form:

$$\rho_i = \rho_{0i} e^{\beta_i \overline{\omega}_i^2 r_i^2}, \tag{29}$$

where ρ_{0i} is a nonnegative scalar constant, and the square of the distance r_i^2 is given by:

$$r_i^2 = \frac{1}{\overline{\omega}_i^2} \left(\overline{\omega}_i \times (x - x_{0i})\right)^2, \tag{30}$$

which specifies the distance from the molecule to the axis of rotation x_{0i} at the time $t = 0$, for which the following representation is true:

$$x_{0i} = \frac{1}{\overline{\omega}_i^2}\left(\overline{\omega}_i \times \widehat{V}_i\right) \tag{31}$$

where $\overline{\omega}_i$ is the angular velocity of the gas flow as a whole, and $\widehat{V}_i$ is an arbitrary vector constant. Also, unlike (8) the mass velocity $\overline{V}_i$ depends on the position of the molecule as follows:

$$\overline{V}_i = \widehat{V}_i + (\overline{\omega}_i \times x). \tag{32}$$

Stationary, inhomogeneous Maxwellian $M_i(V, \omega, x)$ describes the helical motion of a gas stream that rotates about an axis x_{0i}.

As before, the solution of equations (1)–(3) we will seek in the form (7) except that the Maxwellians M_i depend now on x, and with the new density representation (29) and the mass velocity (32).

Theorem 2. *Let the coefficient functions $\varphi_i(t,x)$ have the following form:*

$$\varphi_i(t,x) = e^{-\beta_i \overline{\omega}_i^2 r_i^2}\psi_i(t,x), \tag{33}$$

where $\psi_i(t,x)$ are non-negative functions, bounded, smooth in the whole space $\mathbb{R}^4$. Suppose that the conditions (11) are satisfied with substitution the function ψ_i instead of φ_i, and besides the values

$$\psi_i|x|, \qquad \left((\overline{\omega}_i \times x), \frac{\partial \psi_i}{\partial x}\right) \tag{34}$$

are bounded and functional series with such a common term converge uniformly on $\mathbb{R}^4$.

Also, let the following condition be fulfilled:

$$\overline{\omega}_i = \overline{\omega}_{0i}\beta_i^{-k_i}, \tag{35}$$

where $k_i \geqslant \frac{1}{2}$. Then there exists a quantity Δ', such that the estimation (12) holds, but instead of (13) low-temperature limit Δ' we have the form:

a) if $k_i > \frac{1}{2}$, then

$$\lim_{\beta_i \to +\infty} \Delta' =$$

$$= \sum_{i=1}^{\infty} \rho_{0i} \sup_{(t,x)\in\mathbb{R}^4} \left| \frac{\partial \psi_i}{\partial t} + \left(\widehat{V}_i, \frac{\partial \psi_i}{\partial x}\right)\right| + 2\pi d^2 \sum_{\substack{i,j=1 \\ i\neq j}}^{\infty} \rho_{0i}\rho_{0j}\left|\widehat{V}_i - \widehat{V}_j\right| \sup_{(t,x)\in\mathbb{R}^4} (\psi_i\psi_j); \tag{36}$$

b) if $k_i = \frac{1}{2}$, then on the right-hand side of (36) an additional summand appears:

$$4\pi \sum_{i=1}^{\infty} \rho_{0i}\left|(\overline{\omega}_{0i} \times \widehat{V}_i)\right| \sup_{(t,x)\in\mathbb{R}^4} \psi_i. \tag{37}$$

Proof. The inequality (20), which was obtained in the proof of the previous theorem remains true, therefore, using a new form of density ρ_i and mass velocity $\overline{V}_i$, taking into account the integration over the angular velocity space ω, we obtain the following estimation

$$\int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega \left| D(f) - Q(f,f)\right| \leqslant$$

$$\leqslant \sum_{i=1}^{\infty} \rho_{0i} \left(\frac{\beta_i}{\pi}\right)^{3/2} e^{\beta_i \overline{\omega}_i^2 r_i^2} \int\limits_{\mathbb{R}^3} dV e^{-\beta_i \left(V-\widehat{V}_i-[\overline{\omega}_i,x]\right)^2} \left|\frac{\partial \varphi_i}{\partial t} + \left(V, \frac{\partial \varphi_i}{\partial x}\right)\right| +$$

$$+2 \sum_{\substack{i,j=1\\ i\neq j}}^{\infty} \varphi_i \varphi_j \frac{d^2 \rho_{0i}\rho_{0j}}{\pi^2} e^{\beta_i \overline{\omega}_i^2 r_i^2 + \beta_j \overline{\omega}_j^2 r_j^2} \int\limits_{\mathbb{R}^3} dq \int\limits_{\mathbb{R}^3} dq_1 e^{-q^2-q_1^2} \times$$

$$\times \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \widehat{V}_i - \widehat{V}_j + ((\overline{\omega}_i - \overline{\omega}_j) \times x) \right|. \tag{38}$$

In the first series (38) under the integral sign we will make the change of variables

$$p = \sqrt{\beta_i} \left(V - \widehat{V}_i - (\overline{\omega}_i \times x)\right), \tag{39}$$

from which it follows that

$$V = \frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + (\overline{\omega}_i \times x),$$

and the Jacobian is $J = \beta_i^{-3/2}$. Then the inequality (38) with taking into account (39) becomes the form

$$\int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega \left| D(f) - Q(f,f) \right| \leqslant$$

$$\leqslant \pi^{-3/2} \sum_{i=1}^{\infty} \rho_{0i} e^{\beta_i \overline{\omega}_i^2 r_i^2} \int\limits_{\mathbb{R}^3} dp e^{-p^2} \left| \frac{\partial \varphi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + (\overline{\omega}_i \times x), \frac{\partial \varphi_i}{\partial x}\right)\right| +$$

$$+2 \sum_{\substack{i,j=1\\ i\neq j}}^{\infty} \varphi_i \varphi_j \frac{d^2 \rho_{0i}\rho_{0j}}{\pi^2} e^{\beta_i \overline{\omega}_i^2 r_i^2 + \beta_j \overline{\omega}_j^2 r_j^2} \int\limits_{\mathbb{R}^3} dq \int\limits_{\mathbb{R}^3} dq_1 e^{-q^2-q_1^2} \times$$

$$\times \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \widehat{V}_i - \widehat{V}_j + ((\overline{\omega}_i - \overline{\omega}_j) \times x) \right|. \tag{40}$$

Using the form of the coefficient functions $\varphi_i(t,x)$, represented by the expression (33) we find the partial derivative $\varphi_i(t,x)$ by time t:

$$\frac{\partial \varphi_i}{\partial t} = e^{\beta_i \overline{\omega}_i^2 r_i^2} \frac{\partial \psi_i}{\partial t} \tag{41}$$

and the gradient of φ_i with respect to the spatial coordinate x:

$$\frac{\partial \varphi_i}{\partial x} = e^{\beta_i \overline{\omega}_i^2 r_i^2} \left(\frac{\partial \psi_i}{\partial x} + 2\beta_i \psi_i \left\{\overline{\omega}_i \times (\overline{\omega}_i \times (x - x_{0i}))\right\}\right).$$

As known, for arbitrary three-dimensional vectors a, b, c the following equality is true:

$$a \times (b \times c) = b(a,c) - c(a,b) \tag{42}$$

So taking into account the form of the axis x_{0i} (31):

$$\overline{\omega}_i \times (\overline{\omega}_i \times (x - x_{0i})) = \overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 x + \overline{\omega}_i \times \widehat{V}_i,$$

then

$$\frac{\partial \varphi_i}{\partial x} = e^{\beta_i \overline{\omega}_i^2 r_i^2} \left(\frac{\partial \psi_i}{\partial x} + 2\beta_i \psi_i \left(\overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 x + \overline{\omega}_i \times \widehat{V}_i \right) \right). \tag{43}$$

The obtained derivatives of the functions φ_i (41), (43) are substituting in the inequality (40)

$$\int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega \Big| D(f) - Q(f,f) \Big| \leqslant \pi^{-3/2} \sum_{i=1}^{\infty} \rho_{0i} \int\limits_{\mathbb{R}^3} dp e^{-p^2} \times$$
$$\times \left| \frac{\partial \psi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + \overline{\omega}_i \times x, \frac{\partial \psi_i}{\partial x} + 2\beta_i \psi_i \left(\overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 x + \overline{\omega}_i \times \widehat{V}_i \right) \right) \right| +$$
$$+ 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{\infty} \frac{d^2 \rho_{0i} \rho_{0j}}{\pi^2} \psi_i \psi_j \int\limits_{\mathbb{R}^3} dq \int\limits_{\mathbb{R}^3} dq_1 e^{-q^2 - q_1^2} \cdot \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \widehat{V}_i - \widehat{V}_j + (\overline{\omega}_i - \overline{\omega}_j) \times x \right|,$$

and after elementary transformations we finally have

$$\int\limits_{\mathbb{R}^3} dV \int\limits_{\mathbb{R}^3} d\omega \Big| D(f) - Q(f,f) \Big| \leqslant$$
$$\leqslant \pi^{-3/2} \sum_{i=1}^{\infty} \rho_{0i} \int\limits_{\mathbb{R}^3} dp e^{-p^2} \left| \frac{\partial \psi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + \overline{\omega}_i \times x, \frac{\partial \psi_i}{\partial x} \right) + \right.$$
$$\left. + 2p\sqrt{\beta_i} \psi_i \left(\overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 x + \overline{\omega}_i \times \widehat{V}_i \right) \right| +$$
$$+ 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{\infty} \frac{d^2 \rho_{0i} \rho_{0j}}{\pi^2} \psi_i \psi_j \int\limits_{\mathbb{R}^3} dq \int\limits_{\mathbb{R}^3} dq_1 e^{-q^2 - q_1^2} \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \widehat{V}_i - \widehat{V}_j + (\overline{\omega}_i - \overline{\omega}_j) \times x \right|. \tag{44}$$

Further in the previous inequality we make the transition to the supremum over the whole space $\mathbb{R}^4$, Whose existence is guaranteed by the conditions of the theorem and the lemma proved in [8]. We obtain the inequality (12) with the value Δ', which is defined as follows

$$\Delta' = \pi^{-3/2} \sum_{i=1}^{\infty} \rho_{0i} \int\limits_{\mathbb{R}^3} dp e^{-p^2} \sup_{(t,x) \in \mathbb{R}^4} \left| \frac{\partial \psi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + \overline{\omega}_i \times (x - x_{0i}), \frac{\partial \psi_i}{\partial x} \right) + \right.$$
$$\left. + 2p\sqrt{\beta_i} \psi_i \left(\overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 x + \left(\overline{\omega}_i \times \widehat{V}_i \right) \right) + 2\beta_i \psi_i \left(\left(\widehat{V}_i, \overline{\omega}_i \right) (\overline{\omega}_i, x) - \overline{\omega}_i^2 \left(\widehat{V}_i, x \right) \right) \right| +$$
$$+ 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{\infty} \frac{d^2 \rho_{0i} \rho_{0j}}{\pi^2} \sup_{(t,x) \in \mathbb{R}^4} \psi_i \psi_j \int\limits_{\mathbb{R}^3} dq \int\limits_{\mathbb{R}^3} dq_1 e^{-q^2 - q_1^2} \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \widehat{V}_i - \widehat{V}_j + (\overline{\omega}_i - \overline{\omega}_j) \times x \right|. \tag{45}$$

We will use the slowing down of the angular velocity of rotation $\overline{\omega}_i$, which ensures the condition (35) of the theorem and, performing the low-temperature limit transition, arguing its possibility similarly to what was done in the previous theorem, in the case $k_i > \frac{1}{2}$ we have the statement (36). The additional term (37) arises as a consequence of using of the triangle inequality, i.e.

$$\Delta' \leqslant \pi^{-3/2} \sum_{i=1}^{\infty} \rho_{0i} \int_{\mathbb{R}^3} dp e^{-p^2} \sup_{(t,x)\in\mathbb{R}^4} \left| \frac{\partial \psi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + \overline{\omega}_i \times x, \frac{\partial \psi_i}{\partial x} \right) + \right.$$

$$\left. + 2p\sqrt{\beta_i}\psi_i \left(\overline{\omega}_i(\overline{\omega}_i, x) - \overline{\omega}_i^2 x \right) \right| + 2\pi^{-3/2} \sum_{i=1}^{\infty} \rho_{0i} \int_{\mathbb{R}^3} dp e^{-p^2} \sup_{(t,x)\in\mathbb{R}^4} |p| \sqrt{\beta_i} \psi_i \left| \overline{\omega}_i \times \widehat{V}_i \right| +$$

$$+2 \sum_{\substack{i,j=1 \\ i\neq j}}^{\infty} \frac{d^2 \rho_{0i}\rho_{0j}}{\pi^2} 3\psi_i\psi_j \int_{\mathbb{R}^3} dq \int_{\mathbb{R}^3} dq_1 e^{-q^2-q_1^2} \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \widehat{V}_i - \widehat{V}_j + (\overline{\omega}_i - \overline{\omega}_j) \times x \right|, \qquad (46)$$

and the following condition

$$\overline{\omega}_i = \frac{\overline{\omega}_{0i}}{\sqrt{\beta_i}}. \qquad (47)$$

Proceeding to the low-temperature limit in the inequality (46) in the case (47) we obtain the statement (36) with the term (37) after calculating the integral:

$$\int_{\mathbb{R}^3} |p| e^{-p^2} dp = 2\pi,$$

which proves Theorem 2. □

Corollary 2. *Let the functions ψ_i are the functions C_i, E_i from Corollary 1(see (24), (25)). In addition, suppose that the conditions (26), (27) or (28) with a symbolic substitution of φ_i instead of ψ_i are valid. Then for the case (36) the error (10) can be made arbitrarily small.*

For the case $k_i = \frac{1}{2}$, it is necessary to additionally require the following condition

$$\overline{\omega}_{0i} || \widehat{V}_i \qquad (48)$$

and then the error (10) can be made arbitrarily small again.

The proof of this corollary is completely analogous to the verification of the corollary 1.

Theorem 3. *We renounce the condition (33), but let us require that the conditions imposed on the series (11) remains true after multiplication of each term by multiplier $e^{\beta_i \overline{\omega}_i^2 r_i^2}$. In addition, let*

$$\overline{\omega}_i = \overline{\omega}_{0i} \beta_i^{-k_i}, \quad k_i > \frac{1}{2} \qquad (49)$$

and the parallelism condition for the vectors $\overline{\omega}_{0i}$ and $\widehat{V}_i$ (48).

Then, as before, there is a quantity Δ' such that the inequality (12) holds true, and its low-temperature limit coincides with (36), where instead of ψ_i it is need to read φ_i.

Proof. The estimation (40) from the proof of Theorem 2 remains true. We pass to the supremum in the inequality (40), which allow the imposed conditions in this theorem, we obtain the inequality (12) and the form for the quantity Δ' :

$$\Delta' = \pi^{-3/2} \sum_{i=1}^{\infty} \rho_{0i} \sup_{(t,x)\in\mathbb{R}^4} e^{\beta_i \overline{\omega}_i^2 r_i^2} \int_{\mathbb{R}^3} dp e^{-p^2} \left| \frac{\partial \varphi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + \overline{\omega}_i \times x, \frac{\partial \varphi_i}{\partial x} \right) \right| +$$

$$+2\sum_{\substack{i,j=1\\ i\neq j}}^{\infty}\frac{d^2\rho_{0i}\rho_{0j}}{\pi^2}\cdot\sup_{(t,x)\in\mathbb{R}^4}\varphi_i\varphi_j e^{\beta_i\overline{\omega}_i^2r_i^2+\beta_j\overline{\omega}_j^2r_j^2}\times$$

$$\times\int\limits_{\mathbb{R}^3}dq\int\limits_{\mathbb{R}^3}dq_1e^{-q^2-q_1^2}\left|\frac{q}{\sqrt{\beta_i}}-\frac{q_1}{\sqrt{\beta_j}}+\widehat{V}_i-\widehat{V}_j+(\overline{\omega}_i-\overline{\omega}_j)\times x\right|. \tag{50}$$

We transform the exponent $\beta_i\overline{\omega}_i^2r_i^2$, using an expression for the distance (30)

$$\beta_i\overline{\omega}_i^2r_i^2=\beta_i\Big(\overline{\omega}_i\times(x-x_{0i})\Big)^2,$$

and some another formula from the vector algebra $(a\times b,c\times d)=(a,c)(b,d)-(a,d)(b,c)$, from which it follows:

$$\beta_i\Big(\overline{\omega}_i\times(x-x_{0i})\Big)^2=\beta_i\overline{\omega}_i^2(x-x_{0i})^2-\beta_i(\overline{\omega}_i,x)^2,$$

taking into account that from (31) should be $\overline{\omega}_i\perp x_{0i}$. Then, remembering, again (31) and the condition (48), finally we have:

$$\beta_i\overline{\omega}_i^2r_i^2=\beta_i\overline{\omega}_i^2x^2-\beta_i(\overline{\omega}_i,x)^2. \tag{51}$$

Using the condition (49) of the theorem, we have that

$$\lim_{\beta_i\to+\infty}\beta_i\overline{\omega}_i^2r_i^2=0. \tag{52}$$

Then, by performing the low-temperature limit in inequality (50), arguing in a completely analogous manner to the way it was done in the first theorem with allowance for the condition (49) and limit (51), we obtain the statement of this theorem. □

Remark 4. Since the assertion of Theorem 3, with the imposed conditions, preserves the assertion (36), where instead of functions ψ_i substitute functions φ_i (Here it means that $\psi_i\equiv\varphi_i$), then we actually obtain the assertion of Theorem 1, namely the limit (13), where $\overline{V}_i\equiv\widehat{V}_i$, which allows us to use Corollaries 1 and 2, if we take into account the conditions of Theorem 3.

Remark 5. Theorems 2, 3 and Corollary 2 have a physical meaning similar to that described in Remark 3, but flows that are considered to have also a rotational velocity that is either parallel to linear (see (48)), or with one degree or another, slow down their rotation at low temperatures (see (47),(49)).

3. Conclusions. Some approximate solutions of the Boltzmann equation for a model of rough spheres have been constructed in a form of infinite sum of Maxwellians with coefficient functions of time t and spatial coordinates x. Two cases of Maxwellian modes are considered: the global Maxwellian and one of the local ones, namely, the screw, the analytic expression for which, unlike the global one, depends not only on the linear and angular velocities of the molecule, but also on the spatial coordinate. Some sufficient conditions are obtained for the hydrodynamic parameters of the distribution, which makes it possible to make arbitrarily small the considered uniformly-integral error (10) between the parts of the Bryan-Pidduck equation (1)–(3).

REFERENCES

1. S. Chapman, T.G.Cowling, The mathematical theory of non-uniform gases, Cambridge Univ. Press, Cambridge, 1952.
2. C. Cercignani, M. Lampis, *On the kinetic theory of a dense gas of rough spheres,* J. Statist. Phys., **53** (1988), 655–672.
3. V.D. Gordevskyy, *Explicit approximate solutions of the Boltzmann equation for the model of rough spheres,* Reports of the National Academy of Sciences of Ukraine, **4** (2000), 10–13. (in Ukrainian)
4. V.D. Gordevskyy, *Approximate billow solutions of the kinetic Bryan-Pidduck equation,* Math. Meth. Appl. Sci., **23** (2000), 1121–1137.
5. V.D. Gordevskyy, A.A. Gukalov, *Maxwell distributions in a model of rough spheres,* Ukr. Mat. Zh., **63** (2011), №5, 629–639. (in Russian)
6. C. Cercignani, The Boltzman Equation and its Applications, Springer, New York, 1988.
7. T. Carleman, Problems Mathematiques dans la Theorie Cinetique des Gas, Almqvist & Wiksells,Uppsala, 1957.
8. H. Grad, *On the kinetic theory of rarefield gases,* Comm. Pure and Appl. Math., **2** (1949), №4, 331–407.
9. O.G. Fridlender, *Local Maxwellian Solutions of the Boltzmann Equation,* J. Appl. Math. Mech., **29** (1965), №5, 973–977. (in Russian)
10. V.D. Gordevskyy, *On the non-stationary Maxwellians,* Math. Meth. Appl. Sci., **27** (2004), №2, 231–247.
11. V.D. Gordevskyy, *Biflow distributions with screw modes,* Theor. Math. Phys., **126** (2001), №2, 234–249.
12. A.A. Gukalov, *Interaction between "Accelarating-Packing" Flows for the Bryan-Pidduck Model,* J. of Math. Phys., Anal., Geometry, **9** (2013), №3, 316–331.
13. V.D. Gordevskyy, *Approximate Biflow Solutions of the Kinetic Bryan-Pidduck Equation,* Math. Meth. Appl. Sci., **23** (2003), 1121–1137.
14. V.D. Gordevskyy, A.A. Gukalov, *Interaction of the Eddy Flows in the Bryan–Pidduck Model,* Vysnik Kharkiv Univ., Ser. Mat. Prykl. Mat. Mech., **64** (2011), №2, 27–41. (in Russian)

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