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## KLEIN VS MEHRTENS: RESTORING THE REPUTATION OF A GREAT MODERN

*This may be regarded as a continuation of the Klein Erlanger Programm, in the sense that a geometrical space with its group of transformations is generalized to a category with its algebra of mappings. Eilenberg–MacLane [16, p. 237] in 1945*

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Historian Herbert Mehrtens sought to portray the history of turn-of-the-century mathematics as a struggle of modern *vs* countermodern, led respectively by David Hilbert and Felix Klein. Some of Mehrtens' conclusions have been picked up by both historians (Jeremy Gray) and mathematicians (Frank Quinn).

We argue that Klein and Hilbert, both at Göttingen, were not adversaries but rather modernist allies in a bid to broaden the scope of mathematics beyond a narrow focus on arithmetized analysis as practiced by the Berlin school.

Klein's Göttingen lecture and other texts shed light on Klein's modernism. Hilbert's views on intuition are closer to Klein's views than Mehrtens is willing to allow. Klein and Hilbert were equally interested in the axiomatisation of physics. Among Klein's credits is helping launch the career of Abraham Fraenkel, and advancing the careers of Sophus Lie, Emmy Noether, and Ernst Zermelo, all four surely of impeccable modernist credentials.

Mehrtens' unsourced claim that Hilbert was interested in production rather than meaning appears to stem from Mehrtens' marxist leanings. Mehrtens' claim that [the future *SS-Brigadeführer*] "Theodor Vahlen . . . cited Klein's racist distinctions within mathematics, and sharpened them into open antisemitism" fabricates a spurious continuity between the two figures mentioned and is thus an odious misrepresentation of Klein's position.

**1.1. Felix Klein.** Historian Herbert Mehrtens sought to portray the history of turn-of-the-century mathematics as a struggle of modern *vs* countermodern, represented respectively by David Hilbert and Felix Klein; see e.g., Mehrtens [39]. Some of Mehrtens' conclusions have been picked up by both historians (e.g., Jeremy Gray [18]) and mathematicians (e.g., Frank Quinn [46]).

To be sure, notable differences in outlook existed between Hilbert and Klein. Thus, Hilbert did not share Klein's intense interest in the history of mathematics (see e.g., Rowe [53, p. 192]), and Klein was less interested in axiomatics than Hilbert (see e.g., Weyl [63, p. 16]). However, such differences though undeniable were extrapolated by scholars like Mehrtens to extravagant proportions.

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Against Mehrrens, we argue that Klein and Hilbert were not adversaries but rather modernist allies in a bid to broaden the scope of mathematics beyond a narrow focus on arithmetized analysis as practiced by the Berlin school, so as to include set theory, axiomatization of physics, and other innovative directions. To this end, we analyze Klein’s Göttingen lecture [30] and other texts.

Felix Klein’s 1895 Göttingen address was as influential as it is apparently controversial, as we will see in later sections.

## 1.2. The Göttingen address. Here Klein spoke of

an important mathematical tendency which has as its chief exponent Weierstrass. . .  
I refer to the *arithmetizing* of mathematics. [30, p. 241] (emphasis in the original)

The passage makes clear Klein’s appreciation of the significance of the framework developed by Weierstrass and others. According to Klein, the framework constituted an advance over earlier reliance on spatial intuition as a basis for proofs:

Gauss, taking for granted the continuity of space, unhesitatingly used space intuition as a basis for his proofs; but closer investigation showed not only that many special points still needed proof, but also that space intuition had led to the too hasty assumption of the generality of certain theorems which are by no means general. Hence arose the demand for exclusively arithmetical methods of proof. . . (ibid.)

The break with Gauss’ view is particularly significant and underscores the modernity of Klein’s. Such “arithmetical methods” meant that

nothing shall be accepted as a part of the science unless its rigorous truth<sup>1</sup> can be clearly demonstrated by the ordinary operations of analysis. (ibid.)

Although vague notions, like magnitude, continuous variable, etc., were still in use in these new developments, Klein believed that further refinements could be introduced through the limitations on the notion of quantity (as in Kronecker’s approach) or through the application of symbolic language (the approach of Peano and his school). Klein continued:

Thus, as you see, while voluntarily acknowledging the exceptional importance of the tendency, I do not grant that the arithmetized science is the essence of mathematics. . . I consider that the essential point is not the mere putting of the argument into the arithmetical form, but the mere rigid logic obtained by means of this form. (ibid., p. 242)

Klein felt that the pursuit of abstraction is an open-ended process that need not stop with Weierstrass. Klein’s enthusiasm for the arithmetization of analysis was evident. Also in evidence is his appreciation of new logical and (as we will show below) foundational studies,

<sup>1</sup>The phrase “rigorous truth” is meaningless according to contemporary usage in modern mathematics, and is a mistranslation found in Isabel Maddison’s (on the whole adequate) translation. Felix Klein wrote, in remarkably modern phrasing: “. . . die Forderung *ausschließlich arithmetischer Beweisführung [ist:] Als Besitzstand der Wissenschaft soll nur angesehen werden, was durch Anwendung der gewöhnlichen Rechnungsoperationen als identisch richtig klar erwiesen werden kann.*” [emphasis added] A correct translation of Klein’s words is ‘. . . the demand of *exclusively arithmetical proofs [is:] only those propositions are to be considered the secure possession of science which can clearly be demonstrated to be identically valid by applying the usual arithmetical operations.*’ In particular, Klein’s ‘identisch richtig’ (which Maddison rendered as ‘rigorous truth’) is the German equivalent of the English ‘identically valid’.

even though the term *logic* did not have the meaning we attach to it today.<sup>2</sup> It is in this context that we should view Klein's further claim to the effect that

it is not possible to treat mathematics exhaustively by the method of logical deduction alone, but that, even at the present time, *intuition* has its special province. (ibid., p. 242) (emphasis added)

We will deal with Klein's stance on intuition (mentioned in the comment just cited) in Section 1.3 immediately following.

**1.3. Spatial intuition.** In the remainder of his address, Klein seeks to place the role of spatial intuition in relation to logical and axiomatic developments. Klein is somewhat ambiguous as to the meaning he attaches to the term *intuition*. One can single out three possible meanings:

1. intuition as an indispensable tool in research;<sup>3</sup>
2. intuition as an indispensable tool in teaching;<sup>4</sup>
3. axiomatic accounts are insufficient and intuition must play its role in mathematical arguments.<sup>5</sup>

What we wish to emphasize is that even if meaning (3) occurs here at all, it occurs on a sophisticated level as indicated by Klein's endorsement of the arithmetic foundations for analysis (therefore no more intuitive talk of real numbers) and his comments on Green's theorem and electricity (emphasizing that physical intuition is insufficient to prove mathematical theorems); see Section 1.6. Klein's comment on Gauss quoted in Section 1.2 indicates that Klein clearly distanced himself from a reliance on spatial intuition as replacement for analysis. Even if at some sophisticated level Klein thought that axiomatic approach will be insufficient, no investigation of such a possible level of Klein's term *intuition* appears in Mehrtens's book, which contains little indication that he would actually have the mathematical wherewithal to carry out such an investigation. The level of mathematical competence possessed by Mehrtens is illustrated by his comment that "if it were possible to represent  $\pi$  by an algebraic equation, then the construction [i.e., squaring the circle] would be possible with compass and straightedge" [39, p. 111]. This amounts to an incorrect claim to the effect that every algebraic number is constructible ( $\sqrt[3]{2}$  is algebraic but non-constructible).

Klein felt that arithmetization of geometry meant that geometrical objects are given by formulas and are dealt with by means of analytic geometry and analytic methods:

<sup>2</sup>In this area Klein was more of an *enabler* of new mathematics than a direct *contributor*; he expressed his personal preference as follows: "symbolic methods... this subject does not appeal to me personally" (ibid., p. 243).

<sup>3</sup>As when Klein writes: "I might now introduce a historical excursus, showing that in the development of most of the branches of our science, intuition was the starting point, while logical treatment followed" [30, p. 246].

<sup>4</sup>As when Klein writes: "Among the teachers in our Gymnasia the need of mathematical instruction based on intuitive methods has now been so strongly and universally emphasized that one is compelled to enter a protest, and vigorously insist on the necessity for strict logical treatment. This is the central thought of a small pamphlet on elementary geometrical problems which I published last summer. Among the university professors of our subject exactly the reverse is the case; intuition is frequently not only undervalued, but as much as possible ignored" [30, p. 248].

<sup>5</sup>As when Klein writes: "On the other hand I have to point out most emphatically—and this is the negative part of my task—that it is not possible to treat mathematics exhaustively by the method of logical deduction alone, but that, even at the present time, intuition has its special province" [30, p. 242].

The arithmetizing of mathematics began originally, as I pointed out, by ousting space intuition; the first problem that confronts us as we turn to geometry is therefore that of reconciling the results obtained by arithmetical methods with our conception of space. By this I mean that we accept the ordinary principles of analytical geometry, and try to find from these the geometrical interpretation of the more modern analytical developments. [30, p. 243]

The obvious advantage consists in the refinement of space intuition:

The net result is, on the one hand, a refinement of the process of space intuition; and on the other, an advantage due to the clearer view that is hereby obtained of the analytical results considered, with the consequent elimination of the paradoxical character that is otherwise apt to attach itself to them. (ibid., p. 243)

The next advantage is *idealisation*, namely a mathematical form of an imprecise intuition:

We ultimately perceive that space intuition is an inexact conception, and that in order that we may subject it to mathematical treatment, we *idealize* it by means of the so-called axioms, which actually serve as postulates. (ibid., p. 243–244; emphasis added)

**1.4. Idealisation in physics.** As for idealisation in mechanics and mathematical physics, Klein writes:

Throughout applied mathematics, as in the case of space intuition, we must idealize natural objects before we can use them for purposes of mathematical argument; but we find continually that in one and the same subject we may idealize objects in different ways, according to the purpose that we have in view. (ibid., p. 244)

In the realm of practical physics, idealisation provides precisely defined objects like Green's function (see Section 1.6), that enable new physical insights as well as abstract mathematical arguments. Yet Klein's intuition is twofold:

... [1] the cultivated intuition just discussed, which has been developed under the influence of logical deduction and might almost be called a form of memory; but rather of [2] the naïve intuition, largely a natural gift, which is unconsciously increased by minute study of one branch or other of the science. (ibid., p. 245–246)

Klein elaborates as follows:

The word intuition (*Anschauung*) is perhaps not well chosen; I mean it to include that instinctive feeling for the proportion of the moving parts with which the engineer criticises the distribution of power in any piece of mechanism he has constructed; and even that indefinite conviction the practiced calculator possesses as to the convergence of any infinite process that lies before him. I maintain that mathematical intuition - so understood - is always far in advance of logical reasoning and covers a wider field. (ibid.)

Thus Klein's intuition [1] is *cultivated* (under the influence of logical deduction), while intuition [2] is the most basic in his vision of mathematics, namely prelogical:

Logical investigation is not in place until intuition has completed the task of idealisation. (ibid., p. 247)

Klein finds confirmation of the idea that intuition was the starting point, whereas logical treatment followed, not only in the historical origins of infinitesimal calculus, but also in Minkowski's development of the theory of numbers.

As for the intuition [2] and pedagogy, Klein writes that

two classes at least of mathematical lectures must be based on intuition; the elementary lectures which actually introduce the beginner to higher mathematics - for the scholar must naturally follow the same course of development on a smaller scale, that the science itself has taken on a larger - and the lectures which are intended for those whose work is largely done by intuitive methods, namely, natural scientists and engineers. (ibid., p. 248)

To be sure, Klein's speculations on intuition are not profound; modern philosophers may find them problematic. Our goal here is not to argue that Klein was a great philosopher but rather to indicate that his preoccupations were those of modern mathematicians. If Mehrtens wishes to champion the cause of specifically *modern* mathematics, he cannot easily dismiss today's mathematicians as being just as countermodern with regard to intuition as he claims Klein is. Today mathematicians are still struggling with the role of intuition in the creative process.

**1.5. Attitude toward logic and foundations.** Klein recognized the significance of the contemporary developments in logic associated with the names of Peano and others, when he spoke of

efforts made to introduce symbols for the different logical processes,<sup>6</sup> in order to get rid of the association of ideas, and the lack of accuracy which creeps in unnoticed, and therefore not allowed for, when ordinary language is used. In this connection special mention must be made of an Italian mathematician, Peano, of Turin. . . [30, p. 242]

Klein's warm relationship with Pasch (see Schlimm [55]) further attests to Klein's visionary appreciation of contemporary developments in axiomatic foundations. At the same time, Klein voiced a cautionary note: "while voluntarily acknowledging the exceptional importance of the tendency, I do not grant that the arithmetized science is the essence of mathematics" (see Section 1.2 for a longer quotation). To Klein, the significance of the new methodology goes hand in hand with a rejection of 19th century methodology:

From this outline of the new geometrical programme you see that it differs greatly from any that was accepted during the first half of this [i.e., 19th] century. . . (ibid., p. 244)

Contrary to earlier ideas of mathematics as being a representation of reality, Klein is clearly aware of the idealizing nature of mathematical treatments, as shown by the passage quoted at the beginning of Section 1.4. As an example, Klein gives the possibility of idealizing matter by either continuous or discrete representations:

we treat matter either as continuous throughout space, or as made up of separate molecules, which we may consider to be either at rest or in rapid motion. (ibid., p. 245)

<sup>6</sup>In the original: "logischen Verknüpfung". A better translation would be "logical connectives". While *Verknüpfung* is often translated as *operation* in *algebraic* contexts, the most accurate translation in contemporary discussions of logic is *connective*.

If matter could admit *distinct* mathematical representations according to Klein, then clearly Klein did not share the naive earlier view of mathematics as a straightforward representation of reality.

**1.6. Klein on Green’s function.** Klein goes on to note the distinction between, on the one hand, a physical phenomenon and, on the other, a mathematical theorem attempting to capture the latter. He illustrates such a distinction by citing the example that “in electricity. . . a conductor under the influence of a charged point is in a state of electrical equilibrium” whose mathematical counterpart is the existence of Green’s function, and concludes:

You see here what is the precise object of these renewed investigations; not any new physical insight, but abstract *mathematical argument* in itself, on account of the clearness and precision which will thereby be added to our view of the experimental facts. (ibid.; emphasis added)

It emerges that according to Klein the ultimate criterion of the validity of a theorem is “mathematical argument” rather than physical insight.

**1.7. Pedagogy.** Going on to pedagogy, Klein notes: “I must add a few words on mathematics from the point of view of pedagogy” (ibid., p. 247). University professors bear the brunt of Klein’s critique:

Among the university professors of our subject exactly the reverse is the case; intuition is frequently not only undervalued, but as much as possible ignored. This is doubtless a consequence of the intrinsic importance of the arithmetizing tendency in modern mathematics. But the result reaches far beyond the mark. It is high time to assert openly once for all that this implies, not only a false pedagogy, but also a *distorted view of the science*. (ibid., p. 248) (emphasis added)

While cautious about possible deleterious effects on pedagogy, Klein welcomed the arithmetizing tendency where appropriate, and made common cause with Hilbert in most cases.

The visionary nature of Klein’s *Erlangen program* (EP) is widely known and acknowledged even by Gray (see Section 4.1) though not by Mehrtens (see Section 3). A major weakness of Mehrtens’ book is his failure to address the importance of category theory in modern mathematics. The EP focused on transformations of objects rather than objects themselves, a viewpoint recognizable as a foundation rock of the category-theoretic approach; see Marquis [37]. What is remarkable is that even Mehrtens’ nemesis Bieberbach acknowledged that the EP was a precursor of the axiomatic method (see Segal [57, p. 347, note 56]). The EP furnishes clear evidence in favor of Klein’s modernism and lack of recognition of this by Mehrtens constitutes massaging of evidence.

**1.8. Klein on infinitesimal analysis.** Klein’s foresight of the eventual success of infinitesimal analysis in modern mathematics is similarly remarkable. Thus, Klein formulated a criterion for what it would take for a continuum incorporating infinitesimals to furnish a successful framework, in terms of the availability of a mean value theorem in the framework; see Klein [31, p. 213]. Klein’s criterion was endorsed by Abraham Fraenkel [17, pp. 116–117]. Such a continuum was eventually developed by Fraenkel’s student Abraham Robinson [49]; see Kanovei et al. [27] for details.

The proposal of Vinsonhaler [61], Katz–Polev [28], and others for teaching calculus with infinitesimals relies on their intuitive appeal and would have likely met with Klein’s approval. Modern frameworks for infinitesimal analysis occasioned a re-evaluation of its history; see e.g., Bair et al. [1], Bascelli et al. [3], [4], Błaszczuk et al. [8].

### 1.9. Courant on the *Erlangen Program*. Courant wrote:

This so-called *Erlangen Program*, entitled ‘Comparative Considerations on recent geometrical research’ has become perhaps the most influential and most-read mathematical text of the last 60 years.

Since the end of the 18th century, geometry had extraordinarily thrived in France and Germany. Alongside old elementary geometry and analytical geometry, a large number of geometrical considerations had [been] developed, which stood side-by-side unmotivated and without mutual connections, and in this tangle [of ideas] even an expert could hardly orient themselves anymore. Klein felt the need to bring a uniform ordering principle into this chaos, and he has solved this task for the whole of geometry in way which one currently cannot imagine coming nearer to being complete. The magic wand, with which here Klein created order, was the group concept.

It [the group concept] permits to conceive every class of geometrical investigations (like Euclidean and Non-Euclidean geometry, projective geometry, line- and sphere-geometry, Riemannian geometry and topology) as invariant theory relative to a given group of geometrical transformations. The *Erlangen Program* constitutes for geometry a similarly forceful ordering-principle as the periodic system is for chemistry.

To this day, no geometrical theory can be considered finished if it cannot clearly assert its place within the framework of the *Erlangen Program*. Klein lived to have the satisfaction, fifty years later, to be able to very substantially contribute to the clarification of the mathematical foundations of relativity theory, simply by, essentially, applying his old thoughts from the *Erlangen program* to the new questions. (Courant [15, p. 200])

Courant’s comments clearly indicate the modern nature of Klein’s EP.

### 1.10. Hermann Weyl on Riemann surfaces. Hermann Weyl comments as follows on the significance of the work of Riemann on what are now called Riemann surfaces, and its clarification by Klein:

It has to be admitted, of course, that Riemann himself slightly disguised the true relationship between [complex] functions and Riemann surfaces, by the form of his presentation—perhaps for the one reason only that he did not want to visit overly alien ideas upon his contemporaries; he disguised this relationship in particular by only speaking of those multi-sheeted Riemann-surfaces covering the plane which have finitely many ramification points, which [by the way] even today are those which one primarily thinks of when the topic of Riemann surfaces is mentioned, and that *Riemann did not use the more general idea (which only later was developed to transparent clarity by K l e i n)*, an idea whose characteristic property can be described as follows: any connection with the complex plane, and generally any connection with three-dimensional point-spaces has been severed, and severed in principle. And yet, without any possible doubt, *only with Klein’s conception* does the basic thought of Riemann come to life and is the natural simplicity, vitality and effective force of these ideas made visible. The present work is based on these [i.e., Klein’s] thoughts. (Weyl [62, p. V]; emphasis added)

Klein's role in Weyl's project was instrumental:

The most important of the basic thoughts [in Weyl's book] are due to the man to whom I was permitted to dedicate this book in sincere and profound reference. Geheimrat<sup>7</sup> Klein has insisted, despite being overburdened by other tasks, and despite his failing health, to discuss the entire subject matter in frequent meetings with me; I owe a great debt of gratitude to him for his remarks which on several occasions have caused me to replace my initial presentation with a more correct and suitable one. (ibid., p. IX)

Weyl makes it clear that Klein made great *sacrifices of time and energy* to support the first *abstract* and *axiomatic* (and hence textbook-modern) presentation of the theory of Riemann surfaces. This is a factual argument against Mehrtens' counter-Klein thesis analyzed in Section 3.

**2. Karl Weierstrass (1815–1897).** The seminal contribution of Karl Weierstrass to the development of modern analysis is well known and requires no special comment. In this section we will focus on Weierstrass' comments on intuition and ethnicity, and compare them to Klein's.

**2.1. Letter to Kovalevskaya.** Sanford Segal notes that Weierstrass' 1883 letter to his student Kovalevskaya was cited by SA footsoldier Teichmüller in a 1938 lecture; see Segal [57, p. 405]. In his letter, Weierstrass expressed himself as follows:

Among the older mathematicians there are different sorts of human beings; this is a trivial proposition, which nevertheless explains much. My dear friend Kummer, for example, was not interested in what happened in mathematics as a whole, neither while he applied all his energy to find the proofs of the higher reciprocity laws, nor, and then less than ever, after he had expended his energies in these pursuits. His attitude [to new developments in mathematics] is, if not dismissive, indifferent. If you tell him that Euclidean Geometry is based on an unproved hypothesis, then he [readily] admits this; [but] to proceed from this insight to asking the question 'How does geometry develop without this hypothesis?', now that is contrary to his [mental] nature, and the exertions aimed at this question, and the general investigations which follow from this question and which liberate themselves from what is empirically given or assumed, are moot speculations to him, or even an abomination. (from an 1883 letter from Weierstrass to Kovalevskaya reproduced in Mittag-Leffler [43, p. 148–150]; translation ours)

Having thus analyzed Kummer, Weierstrass goes on to provide his insights into Kronecker's temperament:

Kronecker is different, he familiarizes himself quickly with everything that is new, his facility of perception enables him to do so, but it does not happen deeply; he does not have the talent to apply the same interest to good work of others as to his own work. (ibid.)

Weierstrass then proposes the following analysis of such differences in mathematical temperaments:

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<sup>7</sup>Often translated as *privy councillor*.

This is compounded by a defect which can be found in many very intelligent people, especially those from the *semitic tribe*[:]<sup>8</sup> he [i.e., Kronecker] does not have sufficient imagination (I should rather say: *intuition*) and it is true to say that a mathematician who is not a little bit of a poet, will never be a consummate mathematician. Comparisons are instructive: the all-encompassing view which is directed towards the Highest, the Ideal,<sup>9</sup> marks out, in a very striking manner, Abel as better than Jacobi, marks out Riemann as better than all his contemporaries (Eisenstein, Rosenhain), and marks out Helmholtz as better than Kirchhoff (even though the latter did not have a droplet of *semitic blood*).<sup>10</sup> (ibid.; emphasis added)

Notes Rowe: “These remarks, echoed in the very same stereotypes set forth by anti-Semites like Dingler and Bieberbach fifty years later, are a good illustration of how deeply rooted such thinking was in German culture” (Rowe [52, p. 443]).

Weierstrass’ speculations as to the insufficient endowment in *intuition* on the part of the *semitic tribe* marks out Weierstrass as a better candidate than Klein (who never proffered such speculations) for countermodern leadership, by Mehrtens’ own standards. We would like to suggest that Mehrtens’ book is critically flawed in having misidentified its protagonists: while Hilbert can serviceably provide modern leadership, his opposite numbers at the helm of the *intuitive countermoderns* should have been Weierstrass and his students (see Bair et al. [2]) rather than Klein.

We would like to clarify that Weierstrass made these comments in a private letter, and did not anticipate that his classification will be amplified through publication by Mittag-Leffler. Klein, on the other hand, made his comments on Teutonic, Latin and Hebrew characteristics (see Section 4.9) in a public address (Klein [29]). It is unclear whether Mehrtens would characterize Weierstrass’ comments as *biologistisch-rassistische*.<sup>11</sup>

**2.2. Weierstrass and Klein on Kronecker.** In Section 2.1 we observed that Karl Weierstrass attributed perceived shortcomings of Kronecker’s mathematical outlook to the latter’s membership in the *semitic tribe* with an attendant deficiency in *intuition*. Weierstrass’ remarks on Kronecker can be profitably compared with those made by Klein, in a letter to Friedrich Althoff, “the kingpin of the Prussian university system” (see Rowe [52, p. 424]), concerning mathematics at Berlin:

Without question the positive aspects have been borne primarily by Kronecker. In this respect I must not withhold my praise. That Kronecker, even his last years of life, was able to bring new ideas to our science with such youthful ambition, and thereby to uphold Berlin’s old fame as a center for mathematical research in

<sup>8</sup>Rowe translates *Stamm* as “stock” but “tribe” seems both more accurate and more consistent with Weierstrass’ tone in the letter.

<sup>9</sup>Rowe translates Weierstrass’ phrase “allumfassende auf das höchste, das Ideale gerichteten Blick” as “all-embracing vision focused on the loftiest of ideals” but the translation is incorrect. Weierstrass is *not* using ‘höchste’ as an *adjective* modifying ‘Ideale’; rather, Weierstrass uses a juxtaposition of a *neuter nominalized superlative* (‘das höchste’) with a *neuter nominalized adjective* (‘das Ideale’), separating the two by a comma instead of the conjunction ‘and’, to create a rhetorical effect in his letter.

<sup>10</sup>In the original: “Dazu kommt ein Mangel, der sich bei vielen höchst verständigen Menschen, namentlich bei denen semitischen Stammes findet[:] er besitzt nicht ausreichend Phantasie (Intuition möchte ich lieber sagen) und es ist wahr, ein Mathematiker, der nicht etwas Poet ist, wird nimmer ein vollkommener Mathematiker sein. Vergleiche sind lehrreich: Der allumfassende auf das höchste, das Ideale gerichtete Blick zeichnet Abel vor Jacobi, Riemann vor allen seinen Zeitgenossen (Eisenstein, Rosenhain), Helmholtz vor Kirchhoff (obwohl bei dem letztern kein Tröpfchen semitischen Blutes vorhanden) in ganz eclatanter Weise aus.”

<sup>11</sup>See note 25 in Section 4.3.

a new form, that is an accomplishment one can only admire without reservation. My critique merely concerns the one-sidedness with which Kronecker, from a philosophical standpoint, fought against various scientific directions that were remote from his own. . . This one-sidedness was probably less grounded in Kronecker's original talents than it was in the disposition of his character. Unconditional mastery, if possible over all of German mathematics, became more and more the goal which he pursued with all the cleverness and tenacity he could muster. Little wonder that there is no one to take his place now that he has left the arena. (Klein to Althoff, as quoted in Rowe [52, p. 442])

Regardless of whether one agrees with Klein's assessment of Kronecker's mathematics and/or politics, one has to acknowledge that from the viewpoint of racial harmony, Klein's remarks are unobjectionable; Kronecker's relation to what Weierstrass referred to as the *semitic tribe* is not mentioned. Whenever Klein did mention a mathematician's Jewishness, it was in laudatory terms like the following:

Personally, [James Joseph] Sylvester was extremely engaging, witty and effervescent. He was a brilliant orator and often distinguished himself by his pithy, agile poetic skill, to the mirth of everyone. By his brilliance and agility of mind he was a genuine representative of his race; he hailed from a purely Jewish family, which, having been nameless before, had adopted the [sur]name Sylvester only in his generation. (Klein [32, p. 163])

Klein's philosemitic comments in this area stand in contrast with those penned by Weierstrass and cited in Section 2.1, as well as those penned by Vahlen and cited in Section 4.6.

**2.3. Letter to Schwarz.** Weierstrass opposed the looming appointment of Sophus Lie at Leipzig on the grounds that the foreigner Lie was not of such consequence as to warrant a passing over of actual *countrymen* (i.e., compatriots); see Stubhaug [60, p. 317]. Weierstrass went on to express his scorn in a 1885 letter written from Lake Geneva to his student Schwarz at Göttingen, in the following terms:

Du Bois-Reymond really sometimes nails it; years ago already he called the *Trifolium* of Klein–Lie–Mayer an ‘acolyte society.’<sup>12</sup> (Weierstrass as cited in Confalonieri [10, p. 288])

The *nailing* comment is followed by a piece of advice for Schwarz:

But, dear friend, don't let yourself be led astray, go on to correctly walk your walk, teach and work in your thorough style—thus you will best counteract the swindle,<sup>13</sup> even though *initially*, you will have to brace yourself for this, the big crowd<sup>14</sup> will turn to the newly rising sun.<sup>15</sup> (Weierstrass as cited in Confalonieri

<sup>12</sup>In the original: “Du Bois-Reymond trifft doch zuweilen den Nagel auf den Kopf, er nannte vor Jahren schon das Trifolium Klein–Lie–Mayer ‘société thuriféraire’.”

<sup>13</sup>In the original: “Schwindel”. Weierstrass' term apparently refers to the activity of the Klein–Lie–Mayer society and connects well with Weierstrass' epithet *Blender* used by Weierstrass in 1892 in reference to Klein.

<sup>14</sup>The words “der große Heifer” in Confalonieri's transcription are possibly a corruption of “der große Haufen”. Weierstrass' manuscript shows a black speck above the squiggly line representing this word. The speck is most likely an overline that was once used in German to distinguish a handwritten ‘u’ from a handwritten ‘n’. Confalonieri seems to have mistaken the speck for a dotted ‘i’, leading him to guess ‘Heifer’ (not a word in German).

<sup>15</sup>The words “der neu mitgehenden Sonne” in Confalonieri's transcription are possibly a corruption of “der

[10, p. 288])

Weierstrass appears to be mixing metaphors here. The sunflower follows the sun, just as a crowd blindly follows a new fad. Weierstrass' *newly rising sun* appears to refer to Klein.

These comments by Weierstrass are significant because they indicate that he had a fundamental disagreement with Klein over the nature and future of mathematics. Weierstrass seems to have viewed Klein's interests such as the *Erlangen program* as a passing fad (that goes away as quickly as the sun sets). Huygens had similar feelings about Leibniz' calculus, in that he thought it was merely a repackaging of existing techniques.

**3. Mehrtens on Klein.** In November 1933 philosopher Hugo Dingler filed a pamphlet against Klein (1849–1925).

**3.1. Dingler's pamphlet; J-type and S-type.** In his pamphlet, Dingler claims that Klein was half-Jewish, that Klein filled Göttingen with Jews and foreigners, and that Klein was hungry for power to control German mathematics and re-make it along Jewish lines and in sum was *un-German*; see Rowe [52].

**3.2. Bieberbach versus Dingler.** A quarter century after defending his thesis under Klein, Ludwig Bieberbach made a claim *contrary* to Dingler's regarding Klein, with the latter now becoming emphatically *German*; see Bieberbach [6]. Thus Bieberbach sought to attribute his own (Bieberbach's) views concerning *German mathematics* to Klein himself, and to co-opt the latter in the service of an unsavory ideology.

Bieberbach relied on a dichotomy of *S-type* versus *J-type* borrowed from Jaensch [26]. Here *S-type* (for *Strahltypus*) refers to a "radiating" type that "only values those things in Reality which his intellect infers in it" and moreover "denies the connection to an outer reality that is not mentally constructed" (see Segal [57, p. 365]).

In contrast, the *J-type* (or *I-type*, for *Integrationstypus*) refers to one who "is wide-open to Reality" and "lets the influence of experience stream into him" (see Segal [57, 362–363] for details).

The Jews Jacobi and Landau among others was construed as *S-types*, whereas Klein as a *J-type*. On occasion, Bieberbach used the terms *Aryan* and *non-Aryan* [7, p. 177]; for further discussion see Segal [57, p. 380].

**3.3. Mehrtens' choice; C-type and M-type.** Over half a century later, historian Herbert Mehrtens chose to back Bieberbach in the Dingler–Bieberbach disagreement over Klein.<sup>16</sup> The historian confidently announced that "Klein is indeed a representative of counter-modernism" (Mehrtens [41, p. 520]). What Mehrtens found countermodern in Klein is Klein's mathematical *outlook* rather than his mathematical *output*.

To contrast Klein with Hilbert, Mehrtens introduced a dichotomy of *countermodern* (*C-type* for short) and *modern* (*M-type* for short), with Klein construed as *C-type* and Hilbert as *M-type* (*C-* and *M-* notation ours).

Mehrtens not only claimed that Klein was *C-type* but also dropped both damaging innuendo that Klein may have ultimately been an enabler of the national-socialist ideology and odious allegations of racism, as analyzed in Sections 3.4 and 4.3.

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neu aufgehenden Sonne". Note that the word in question starts with the glyph that Weierstrass uses for 'a'.  
<sup>16</sup>This is not to imply that Mehrtens endorsed either Dingler's or Bieberbach's views in the sense of their unsavory political or philosophical orientations. Nor is it an assertion that Mehrtens' own political or philosophical orientations are *not* unsavory.

Note that the onus is on Mehrtens to convince the reader that he is pursuing a meaningful dichotomy of *C-type* versus *M-type*. We argue that by pursuing extraneous issues, Mehrtens obscures the true vital issues of the period, and moreover reveals his ideological *parti pris* in the process.

**3.4. Racism.** Mehrtens' page 520 contains no fewer than six occurrences of the term *racism*; thus, we find:

The case of Bieberbach as well as the behaviour of some other mathematicians suggest the thesis that mathematical counter-modernism is correlated to nationalism and eventually also to *racism*. (ibid.; emphasis added)

Furthermore,

Klein is indeed a representative of counter-modernism, but his *racism* is more of the theoretical type. (ibid.; emphasis added)

Mehrtens' procedure here is objectionable on several counts:

- The juxtaposition of the two passages (one on Bieberbach and one on Klein) in close proximity suggests a spurious affinity between them;
- the claim of being a “racist of a theoretical type” presupposes being a racist in the first place, an allegation against Klein that Mehrtens has not yet established.

Now the term *racist* can be given (at least) two distinct meanings:

1. someone interested in analyzing differences in intellectual outlook among distinct ethnicities (*racist1*);
2. someone who believes in the inferiority of one ethnicity to another based on such differences, and advocates corrective action (*racist2*).

Klein's discussions of ethnic differences possibly make him *racist1*. Yet Mehrtens' comments on Bieberbach clearly indicate that Mehrtens has *racist2* in mind when he uses the term:

When the National Socialists came to power in 1933, [Bieberbach] attempted to find political backing for his counter-modernist perspective on mathematics, and declared both, intuition and concreteness, to be the inborn characteristic of the mathematician of the German race, while the tendency towards abstractness and unconcrete logical subtleties would be the style of Jews and of the French (Mehrtens 1987). He thus turned counter-modernism into outright *racism* and anti-modernism. (Mehrtens [41, p. 519]; emphasis added)

Thus the *racism* case against Klein as found in Mehrtens [41] is based on equivocation on the meaning of the term *racist*. The remaining four occurrences of the term *racism* on page 520 in Mehrtens are less tendentious; yet Mehrtens' reader can well wonder why the issue is being discussed in such detail at all, if pinning a *latent racism* slur on an allegedly *C-type* Klein were not one of Mehrtens' intentions. Mehrtens goes so far as to imply a connection between Klein's comments and the *Nationalsozialismus*; see Section 4.3.

The historical record indicates that Klein struggled valiantly to hire Jewish mathematicians like Hurwitz and Schoenflies, and conducted a warm correspondence with Pasch, Gordan,

and others. Klein at times put his own reputation on the line to do so, as in his correspondence with Althoff concerning the promotion of Schoenflies.<sup>17</sup>

Mehrtens' strategy in the face of such facts is

1. to acknowledge that Klein appreciated some Jewish mathematicians, but
2. to claim that this does not contradict Mehrtens' thesis that Klein, perhaps unwittingly and without foreseeing the criminal abuses that were to come, produced texts and personal statements which were grist to the mill of antisemitic tendencies in Germany at the time [39, p. 217].

This stance of Mehrtens' amounts to assignment of guilt by association and therefore represents a character smear against Klein.

**3.5. Internationalism.** Mehrtens goes on to raise the issue of internationalism:

The converse proposition would be that modernism is related to liberalism and internationalism. Indeed, the leading representative of the modern style, David Hilbert, can be rated as a liberal and an internationalist. (Mehrtens [41, p. 520])

A reader would gather from this comment that Klein may perhaps *not* be rated as a liberal, being a *C-type* contrary to Hilbert's *M-type* (see Section 3.3). In his review of Mehrtens' approach, Albert Lewis aptly remarked:

The two world wars and their aftermaths brought out the stark contrast between *international* and *national* mathematical biases while the center of gravity of mathematical activity moved towards the United States. (Lewis [36]; emphasis added)

Indeed, relative to the *national* vs *international* dichotomy mentioned by Lewis, Felix Klein's outlook clearly belongs to the latter (as correctly pointed out by Dingler; see Section 3.1), which is the opposite of the countermodern role Mehrtens seeks to pin on Klein.

**3.6. What is mathematical modernism? Mehrtens '93.** Herbert Mehrtens and other historians "have attempted to give a Marxist analysis of the connection between mathematics and productive forces, and there have been philosophical studies about the communication processes involved in the production of mathematical knowledge" (Mehrtens et al. [42, pp. ix–x]). In a 1993 text, Mehrtens posits that

a scientific discipline exchanges its knowledge products plus political loyalty in return for material resources plus social legitimacy. (Mehrtens [40, p. 220])

Readers concerned that that the tools of a crude marxism may not provide sufficient discrimination to deal with delicate issues of the transformation that took place in mathematics at the beginning of the 20th century, will have their apprehensions confirmed when they reach Mehrtens' definition of modernism:

By 1930 it was quite clear what the term *modern* meant when applied to mathematics: the conceptual study of abstract mathematical concepts characterized by axioms valid for sets or other undefined elements and presented by proceeding from

<sup>17</sup>Schoenflies went on to do work fundamental for both group theory and modern spectroscopy. This work is widely used to this day. His correspondence with Fedorov aimed at correcting errors in earlier versions of their classification of crystallographic groups is one famous early example of the modern phenomenon of mathematicians from different linguistic communities collaborating incrementally to arrive at a valid result; see Schwarzenberger [56, pp. 162–163].

the elementary concepts to the more complicated structures; a hierarchical system of mathematical truths rigorously proved, the language applied having hardly any other function than to label the objects and to ensure the validity of statements. “Modern” mathematics in this sense had no extramathematical *meaning*, did not indicate possible fields or objects of application, was devoid of hints to the historical or heuristical background of the theory, and at most was in a highly implicit manner structured along didactical guidelines. (Mehrtens [40, p. 224]; emphasis added)

Among the problematic aspects of Mehrtens’ definition of modernism are the following:

1. Mehrtens makes no distinction between the levels of language and metalanguage, or mathematics and metamathematics, which are essential to understanding Hilbert’s finitist program;
2. Mehrtens rules out applications by definition (in his second sentence), as well as didactic concerns (little wonder Klein didn’t fare well in Mehrtens’ book);
3. *meaning* is ruled out by definition, which would exclude Hilbert from Mehrtens’ modernism;
4. Mehrtens’ dismissive comment on history (“devoid of hints to the historical . . . background of the theory”) flies in the face of the fact that Hilbert’s *Grundlagen der Geometrie* explicitly refers to Euclid’s *Elements*.

To elaborate on the last point, note that Hilbert deals not merely with the axioms but also with Euclid’s theory of area and theory of proportion. Hilbert incorporates the perspective of Euclid as an organic part of his presentation in the *Grundlagen* (see Hilbert [24]). Mehrtens goes on to claim that

The central cognitive definition of mathematics by its *pure, modern core* was related to those task-oriented fields [of applied mathematics and mathematical pedagogy]. (Mehrtens [40, p. 226]; emphasis added)

However, the positing of a “pure, modern core” by Mehrtens remains a mere assumption echoing those made two pages earlier. Mehrtens continues with a thinly veiled dig against Klein’s book [31]:

Lectures on “elementary mathematics from higher standpoint” served . . . to relate the pedagogical branch to the development of scientific mathematics, and to attempt to axiomatize mathematical theories for physics aimed at an immediate relation between this field of application and pure mathematics. (Mehrtens [40, p. 226])

Again, the exclusion of applications from the “pure, modern” realm is *assumed* rather than *argued* by Mehrtens. His reader may be justifiably shocked to learn that

In a lecture in 1926 [Bieberbach] sharply attacked [David] Hilbert, the dean of mathematical modernity, and depicted modern theories as ‘skeletons in the sand of the desert of which nobody knows whence they come and what they have served for.’ (Mehrtens [40, p. 227])

However, the suitably shocked reader may well feel it to be a weakness of Mehrtens’ depiction of Klein as a *C-type*, since obviously no similar anti-Hilbert lecture was ever given or imagined by Klein. Mehrtens wants us to believe that

the social system of mathematics was (and is) *interested not in meaning but rather in production*. The competing formalist style was obviously productive, and the corresponding foundational research program, Hilbert's 'metamathematics,' imposed no restrictions on existing mathematical theories except the demand for logical coherence. (Mehrtens [40, p. 230]; emphasis added)

Was Hilbert in fact interested in *production* rather than *meaning* as Mehrtens claims here? Mehrtens probably believes that, too. However, he presents no evidence to support such a sweeping claim that is more revealing of his (Mehrtens') ideological commitments than of Hilbert's views, as is Mehrtens' comment to the effect that

In [Hilbert's] program, mathematics is of the highest generality, and *meaning* and utility of new knowledge are hardly visible anymore.<sup>18</sup> (Mehrtens [39, p. 380]; emphasis added)

On Hilbert's commitment to meaning and utility, *pace* Mehrtens, see further in Section 4.11.

**3.7. What is mathematical modernism? Mehrtens '96.** In 1996 Mehrtens provides an additional definition of mathematical modernism: "The counter-modernist attitude arises with modernism" [41, p. 522] and

It is part of modernity of the modern world. Turning to the political side of my topic, I want to state very briefly the sociological argument. I take mathematical modernism to be the defining center of a socially modernized professional and autonomous mathematics business, concentrating and symbolizing itself as a research discipline. Modernism is, so to speak, the avant-garde of the profession, defining the real, the pure, and the most progressive type of mathematics. The business, however, includes much more than avant-garde research. Mathematicians of a more conservative type or *mathematicians in applications*, hybrid fields or in teaching cannot fully identify with the modernist attitude. (Mehrtens [41, p. 523]; emphasis added)

The problem with such a definition is that Klein becomes *tautologously* a *C-type* here, because "mathematicians in applications" are *defined* as incapable of fully identifying with the modernist attitude. The talk about "socially modernized professional and autonomous mathematics business" would indicate on the contrary that Klein is a modernist, since nobody contributed to the flourishing of professional mathematics institutions as he did.

Readers can wonder what "the real, the pure, and the most progressive type of mathematics" is exactly. Mehrtens defines modernist mathematics as *the real, the pure, and the most progressive type* but the definition seems circular. What is *progressive* mathematics if not *modern*?

Readers can have similar doubts about the *a priori* exclusion of applied mathematicians from the rarefied realm of the modernist species. Thus, Abraham Robinson published numerous articles in applied mathematics as well as a joint book called *Wing theory* [48] (yes, these are *airplane* wings, not a branch of *ring theory*). Yet his philosophical stance is close to Hilbert's formalism; see Robinson's essay "Formalism 64" [50] which would normally gain him admittance to the select modernist club if Mehrtens' enthusiasm for David Hilbert's formalism is any guide.

<sup>18</sup>In the original: "In seinem Programm ist die Mathematik von höchster Allgemeinheit, und Sinn und Nutzen neuen Wissens sind kaum mehr zu erkennen."

Even more problematic for Mehrtens is the fact that Hilbert, like Klein, was specifically interested in axiomatizing physics:

Another author who influenced Hilbert deeply during this Königsberg period was Heinrich Hertz, as reflected in [Hilbert's] sixth Paris problem, which pointed toward the axiomatization of mathematical physics. (Rowe [54, p. 180])

(See also Corry [11].) If applications are specifically excluded from the realm of mathematical modernism, what is one to make of this application to physics pursued by Mehrtens' modernist hero Hilbert?

When Mehrtens proclaims: "I take mathematical modernism to be the defining center of a socially modernized... mathematics business," anyone with a background in (mathematical) logic might wonder how one can define mathematical *modernism* in terms of *modernized* business without committing an elementary logical error of circularity. In short, what Mehrtens provides here is not a definition of modernism but rather an assortment of cherry-picked conditions designed specifically to exclude... Felix Klein. Such a technique could be described in popular parlance as moving the goalposts to score a point.

**3.8. Stereotyping.** Does it ever occur to Mehrtens that significant *differences* may exist among the alleged *C-types*? The answer is that it does, as when he speaks of

the individual who constructs himself as part of a higher order, and who, from that higher order, ... receives the gift of a mathematical talent and thus the gift of insight into that higher order. This applies to Poincaré, Klein, Bieberbach and others, but not so much to the more radical and pessimistic Brouwer. *I shall not discuss the individual differences here.*" (Mehrtens [41, pp. 525–526]; emphasis added).

Mehrtens' passage here is assorted with a footnote 6 offering tantalizing glimpses into what such undiscussed differences might be:

6. But I would like to recall the 'polythetic' character of concepts like 'fundamentalism'. One could, tentatively, divide the family resemblances into a 'technocratic', progress-oriented group (Klein, Poincaré), a traditionalist or 'mandarin' group (Kronecker), and a romantic, mystical variant, connecting mathematical fundamentalism with a critique of progress and civilization as Brouwer did. (Mehrtens [41, p. 526, note 6])

Alas, the reader never learns whether Bieberbach was fundamentalist, mandarin, mystical, or technocratic; perhaps all of the above. We therefore have the following classification into *C*-subtypes and their leading representatives following Mehrtens:

- $C_1$  Bieberbach (fundamentalist?);
- $C_2$  Brouwer (mystical);
- $C_3$  Kronecker (mandarin);
- $C_4$  Klein, Poincaré (technocrat).

We note that, *pace* Mehrtens, stereotyping mathematicians into such categories serves no useful purpose if the historian's goal is *meaningful* history, though it may well serve a goal of the *production* of marxist historiography. In the end Mehrtens' claim concerning Hilbert that he was interested in production rather than meaning applies only to the author of the claim.

Mehrtens' systematic stereotyping reminds one of nothing more than similar procedures adopted by his nemesis Bieberbach. Thus, Segal summarizes Bieberbach's racial theorizing as follows:

Gauss was also contrasted with Carl Gustav Jacobi (a Jew). Jacobi was "oriental" and had a "heedless will to push through his own personality." Gauss was characterized as "nordisch-falisch," a term borrowed from H.F.K. Gunther's racial theories; similarly, Euler was "ostisch-dinarisch," another similar term. (Segal [57, p. 363])

See there for a discussion of Bieberbach's classification of *J-types* into subtypes  $J_1, J_2, J_3$ . Mehrtens' final paragraph contains the following gem:

When the ideology of sound, realist common sense had crossed the border to Germany in the early thirties, mathematical modernism took the opposite route with a group of scholars that were to name themselves "Bourbaki" and to become the last *high priests* of mathematical modernism before the *post-modern* era. (Mehrtens [41, p. 527]; emphasis added)

It is unclear what meaningful historiographic purpose is served by stereotyping the Bourbaki as "high priests."

Mehrtens' claim that in mathematics, *modernism* was followed by something called *post-modernism* may strike many a reader as novel. Then again such a reader may be unfamiliar with the intricacies of the received academic lingo. What objective does it serve to postulate a specifically *post-modern* phase in the development of mathematics? We will venture an explanation in Section 3.9.

### 3.9. *Late capitalism* and Kramer's diagnosis. Writes Gray:

Mehrtens's critique was written in a post-Marxist spirit, influenced by such writers as Foucault. Modernization, for him, is not progress, it is also part of the catastrophe of Nazism. If he is less clear that the search for meaning and a place in the world has its good side, it is only because he sees more clearly the ways in which *late capitalism* is antithetical to all of that. (Gray [18, p. 10]; emphasis added)

Here Gray clearly acknowledges the marxist source of Mehrtens' inspiration. Gray goes on to state:

Two further avowedly speculative chapters close [Mehrtens'] book, which go further into *cultural criticism* than I need to follow here." (ibid., footnote 13; emphasis added)

The root of the dilemma, as far as *cultural criticism* is concerned, is that Mehrtens and similar-minded marxist academics have a problem with the symbiotic relationship between capitalism on the one hand, and modernism and high culture on the other. After its revolutionary beginnings as a radical movement,<sup>19</sup> modernism went on to enjoy a symbiotic relationship with what Gray refers to as *late capitalism*, providing the source of the enmity toward modernism on the part of marxist academics disenchanted with bourgeois society. The said academics felt betrayed by modernism, and can therefore speak approvingly only of

<sup>19</sup>In this connection, Mehrtens writes that *modernity* can be a *radical* stance: "die mögliche Radikalität der Moderne" [39, p. 182].

*postmodernism* (or pop art), never of modernism itself, as poignantly summarized by Hilton Kramer in his essay ‘Modernism and its enemies’:

... This view may be summed up as follows: Modernism claimed to be revolutionary, it claimed to be anti-bourgeois, it promised us a new world, but it turned out to be a coefficient of bourgeois capitalist culture, after all, and we therefore reject the claim of high culture and must work to destroy the privileged status it enjoys in the cultural life of the bourgeois democracies. (Kramer [34, p. 13])

To Mehrtens, *applied* mathematics is similarly a “coefficient of bourgeois capitalist culture” aiding and abetting not merely the bourgeois democracies but the NS regime as well:

The plan for an international congress for mechanics in Germany could be sold to the aircraft ministry as a necessity for productive aircraft research. (Mehrtens [40, p. 237])

Thus any advocacy of mathematical *modernism* by a marxist academic must start with defining away *applications* from the outset as *countermodern*—which is precisely what Mehrtens did in [41, p. 523] as discussed in Section 3.7. For further details on bellied capitalists see Section 4.7 (especially note 41).

**4. Mehrtens’ book.** In his 1990 book, Mehrtens describes the modernist transformation of mathematics in the early 20th century in Germany. According to Mehrtens, such a transformation is embodied in Hilbert’s formalism and Cantorian set theory.

**4.1. Selective modernist transformation.** Mehrtens frequently cites Cantor’s famous dictum on freedom being the essence of mathematics, but finesses the issues concerning the reality of mathematical objects where Cantor held decidedly unmodern views laced with both theology and metaphysics.

Oddly, Mehrtens does not mention Emmy Noether’s school of abstract algebra, and only briefly notes the impression that van der Waerden’s book *Moderne Algebra* had made on the young Dieudonné.

Whereas Gray treats Klein’s *Erlangen Program* (EP) as the first item of his chapter “Mathematical Modernism Arrives” in [18], Mehrtens sees Riemann’s *Habilitation* talk of 1854 as the beginning of modernism (but see Section 4.2), and Klein’s EP as the first move of the *Gegenmoderne*. Having described it in five pages, he devotes another 19 pages to the interpretation of footnote III of the EP, titled “Über den Wert räumlicher Anschauung,” i.e., the value of spatial intuition.

Here Mehrtens elaborates the key thesis of his book, namely the divide between mathematicians of respectively *C-type* and *M-type* (in our terminology; see Section 3.3). Mehrtens quotes, among others, Heidegger, Foucault, Marx, Kant and Einstein. This list of authors indicates that the divide has little to do with mathematics proper, but is concerned rather with the relation between mathematics and reality. While the *M-types* (according to Mehrtens) saw mathematics as a formal system detached from reality (Riemann had first developed an abstract theory, and then applied it to space), the *C-types* (Klein among them) insisted that “there is a true geometry which is not . . . intended to be merely an illustrative form of more abstract investigations.”

The struggle between *C-types* and *M-types* became more heated in the decades following, although Mehrtens fails to give a single example of Klein having actively opposed modernism. Accordingly, Mehrtens calls Klein an exponent of the *kooperative Gegenmoderne* [39, p. 207].

For Mehrtens, the *Gegenmoderne* culminated in the 1920s and 1930s with Brouwer's attack on Hilbert and with Bieberbach's *Deutsche Mathematik*, and petered out afterwards. As is well known, Bieberbach was hardly mainstream even among the German mathematicians of his time, as evidenced by the landslide victory of the counter-proposal to Bieberbach's proposal to adopt a *Führerprinzip* within the *Deutsche Mathematiker-Vereinigung* in the election held on 13 September 1934. According to Mehrtens [38], Bieberbach's proposal got 40 no-votes, 11 yes-votes, and 3 empty ballots, whereas the more moderate counterproposal got 38 yes-votes, 8 no-votes, with 4 abstentions.<sup>20</sup>

**4.2. Riemann, Klein, Heidegger, Klein, Riemann.** Riemann's viewpoint creates a problem for Mehrtens' simplistic dichotomy of *C-type* versus *M-type*. As Rowe points out,

the contrast [of Hilbert's viewpoint on Euclidean geometry] with Riemann's viewpoint could not be sharper. For the latter insisted that a refined understanding of Euclidean geometry was a dead end. In fact, at the very outset of his *Habilitationsvortrag*, Riemann proclaimed that the study of the foundations of geometry from Euclid to Legendre—seen as an empirical science—had remained in the dark, owing to a failure to explore crucial issues or hypotheses concerning physical measurements. Natural philosophers thus lacked a general theory of extended magnitudes or, to use modern language, an understanding of differential geometry in arbitrary dimensions. Axiomatics, on the other hand, plays virtually no role in Riemann's text, least of all speculations about a theory of parallels. (Rowe [54, p. 179])

Riemann's pervasive influence on modern mathematics ranges from the theory of Riemann surfaces and Riemannian geometry to the Riemann hypothesis and the deepest problems in number theory, and reaches as far as category theory (see Marquis [37]).

Determined to frame Klein as a *C-type*, Mehrtens faces the formidable challenge of Klein's solid reputation as member of the Riemannian tradition at Göttingen. Mehrtens' strategy to circumvent the challenge is a *tour de force* of obfuscation.

Unable to deny Riemann's sterling reputation as a modern, Mehrtens concedes in [39, p. 67] that Riemann made 'the first move of modernity'. However, he then proceeds to paint a different picture, seeking to portray Riemann as a romantic with pantheistic views. Mehrtens attempts to back up his picture by collating and misrepresenting some passages from fragmentary personal notes of Riemann's (published posthumously by Dedekind and Weber as [47]) and goes on to claim:

In Riemann's philosophical fragments there is talk of an *earth-soul*, which is a moving, multifaceted *thought-process*.<sup>21</sup> (Mehrtens [39, p. 69])

Unable to detect an occurrence of either of the term *Anschauung* or *Intuition* in these fragments that would provide a figleaf of respectability for Mehrtens' (partial) C-typing of Riemann, Mehrtens seeks to connect Riemann to Heidegger, whose name evokes the familiar sinister associations.

Heidegger's abrupt appearance on page 70 of Mehrtens' book is followed on page 71 by a quotation of the following difficult passage:

<sup>20</sup>The numbers reported by Mehrtens do not quite add up but the pattern is clear.

<sup>21</sup>In the original: "In Riemanns philosophischen Fragmenten ist die Rede von einer "Erdseele", die ein bewegter, vielfältiger "Denkprozess" ist."

Cognition is a kind of representational thinking [*Vorstellen*]. In this *presentation* [*Stellen*] something we encounter comes to stand [*Stehen*], to a standstill [*Stand*]. What is encountered and brought to a standstill in representational thinking is the *object* [*Gegenstand*].<sup>22</sup> (Heidegger [21, p. 23])

In his introduction, Mehrtens describes his procedure (including *Discourse Analysis*) as follows:

The sixth chapter, which sums up the interpretatory framework, and—using *Discourse Analysis*, Semiotics and Semiology, and Poetology—develops the leitmotifs, systematically and *within* the three images Eulenspiegel, the Golem, and Münchhausen, was the last chapter that I revised. (Mehrtens [39, p. 9]; emphasis added)

Examining pages 70–71 in [39] and applying such a ‘Discourse Analysis’ reveals that the names of Riemann (R), Klein (K), and Heidegger (H) occur here in close succession in the following order:

R, R, K, K, H, K, R.

Mehrtens juxtaposes the names and accompanies them by obscure passages from fragments of Riemann’s personal notes and cryptic phrases from Heidegger taken out of context, skillfully creating a *floeu artistique* that suggests an affinity among the three protagonists, with an undercurrent of damaging innuendo implicating both Riemann and Klein in the swamp of Heidegger’s well-known forays into politics.<sup>23</sup>

**4.3. Theodor Lessing and race.** Mehrtens inserts some quotes from an idiosyncratic article by philosopher Theodor Lessing in the midst of a discussion of Felix Klein’s views in (Mehrtens [39, pp. 216–218]), including Lessing’s unsourced speculations in [35, p. 235] regarding possible Jewish roots of Riemann, Weierstrass, and Klein himself (Lessing’s article, highly critical of Klein’s educational reform, was naturally discussed in Klein’s seminar).<sup>24</sup> Lessing’s speculations on race in this passage are rather wild-eyed by modern standards. Without providing an evaluation of Lessing’s idiosyncratic speculations, Mehrtens abruptly returns to analyzing Klein’s views, thereby implying guilt by association on Klein’s part. A further example of assigning guilt by association is the following passage:

The biologicistic-racist discourse, which was an everyday phenomenon in Wilhelmine Germany, and not only there, and which Klein served and amplified, was continued in the *Nationalsozialismus*.<sup>25</sup> [39, p. 217]

The issue of Klein’s alleged racism was dealt with in Section 3.4. For a comparison of Klein’s position with that of Weierstrass see Section 2; for Vahlen see Section 4.6. We will deal with the *biologicistic* issue separately in Section 4.4 immediately following.

<sup>22</sup>In the original: “Das Erkennen gilt als eine Art des Vorstellens. In *diesem* Stellen kommt etwas, was uns begegnet, zum Stehen, zum Stand. Das im Vorstellen zum Stand gebrachte Begegnende ist der *Gegenstand*” (Heidegger [20, p. 46]). The first sentence does not appear in Mehrtens’ quotation of Heidegger, obscuring the meaning of the already difficult passage.

<sup>23</sup>In a similar vein, Mehrtens goes as far as to link both Poincaré and Klein with the tainted term *Führer* in (Mehrtens [39, p. 577]). Related innuendo exploiting the tainted term occurs also on pages 161, 252, 253, 254, 428, 576 in [39].

<sup>24</sup>Did Dingler’s claim that Klein was half-Jewish (see Section 3.1) have Lessing’s speculation at its origin? This requires further research.

<sup>25</sup>In the original: “Der im wilhelminischen Deutschland und nicht nur dort alltägliche biologicistisch-rassistische Diskurs, den Klein bediente und verstärkte, wurde im Nationalsozialismus fortgeführt.”

**4.4. A nonconventional arsenal: the *biologistischer* stockpile.** In his book, Mehrrens repeatedly exploits the sinister adjective *biologistisch*:

1. “Time and again mathematicians have observed that there is a generalizable difference between geometric-*anschaulich* and logical-algebraic thought. . . . Klein conceived of this difference as a difference of psychic dispositions, and he moreover takes the step to biologisation when he writes, etc.”<sup>26</sup> [39, p. 215];
2. ‘biologistic-racist discourse’<sup>27</sup> [39, p. 217];
3. “The ‘pure gift of inspiration’ links the transcendental giver with the biologistic conception of ‘gift’ and ‘selection’.”<sup>28</sup> [39, p. 219];
4. “The nationalistic or biologistic discourses/narratives of *Volk* and *Rasse*, as well as the aggressively-heroic pathos of soldierly masculinity were suitable to be connected with the self-image of scientists, and also with the [prevailing] idea of science, this idea having been used by/in countermodernity to defend meaning and to defend relevance-for-reality.”<sup>29</sup> [39, p. 313];
5. “With hindsight it is all too evident that this<sup>30</sup> had the potential to biologistically articulate the possible conflict among these civilized peoples.”<sup>31</sup> [39, p. 345];
6. “However, the reform-movement with [its newly coined terms of] the ‘modern culture’ and the ‘functional thinking’, and with its biologistic philosophical outlook, does not have anything to do with the self-definition of science and its fields of application, but is aimed at gaining legitimacy in the eyes of the general public and [is also aimed at] the cultural representation of the new realities.”<sup>32</sup> [39, p. 376]

Mehrens’ *biologistisches* leitmotif creates a (clearly intended) impression of continuity between, on the one hand, Mehrrens’ case against Klein and, on the other, Mehrrens’ case against the abuses of the NS era. Such alleged continuity is spurious (see further in Section 4.5). The kind of role Mehrrens sees for biology as far as NS ideology is concerned is made crystal clear by the following passage:

<sup>26</sup>In the original: “Daß es jedoch einen verallgemeinerbaren Unterschied zwischen geometrisch-anschaulich und logisch-algebraischem Denken gebe, ist von Mathematikern immer wieder beobachtet worden. . . . Klein faßt sie naturalistisch als eine Verschiedenheit psychischer Dispositionen auf und geht noch den Schritt zur Biologisierung weiter, wenn er schreibt, etc.”

<sup>27</sup>In the original: “biologistisch-rassistische[r] Diskurs.” This item already appeared in Section 4.3.

<sup>28</sup>In the original: “Die ‘rein geschenkte Eingebung’ verbindet den transzendenten Schenker mit der biologistischen Konzeption von Begabung und Auslese.” Note that the German word *Auslese* (‘selection’) was routinely used in NS-ideology.

<sup>29</sup>In the original: “Die naturalistischen oder biologistischen Reden von Volk und Rasse, ebenso wie das aggressiv-heroische Pathos soldatischer Männlichkeit ließen sich mit dem Selbstbild der Wissenschaftler und dem Verständnis von Wissenschaft verknüpfen, mit dem in der Gegegenmoderne Sinn und Wirklichkeitsbezug der theoretischen Wissenschaft verteidigt worden waren.”

<sup>30</sup>Here Mehrrens refers to Klein’s comments at Evanston in 1893 (see Section 4.9); only a single sentence earlier, Klein’s name is mentioned, and associates Klein with “the imperialist racism of the dominating ‘civilized peoples’” (the single quotation marks around “civilized peoples’ are in the original).

<sup>31</sup>In the original: “Daß darin das Potential lag, den möglichen Konflikt unter jenen ‘Kulturnationen’ biologistisch zu artikulieren, ist im nachhinein allzu deutlich.”

<sup>32</sup>In the original: “Die Reformbewegung mit der ‘modernen Kultur’ und dem ‘funktionalen Denken’, auch mit dem biologistischen Weltbild, aber hat gerade nicht mit der Selbstdefinition von Wissenschaft und deren Praxisfeldern zu tun, sondern zielt auf die öffentliche Legitimität und kulturelle Repräsentation der neuen Wirklichkeiten.”

A more obvious example from biology is the concept ‘instinct,’ which was productive in Nazi Germany, both politically and scientifically, and served to mutually reinforce parts of Nazi ideology and *biological* ethology. . . (Mehrtens [40, p. 229]; emphasis added)

Mehrtens’ indiscriminate use of the *biologistischer* stockpile against Klein is not consistent with standards of meaningful historical scholarship.

**4.5. Kleinian continuities according to Mehrstens.** Klein’s rather commonplace description of mathematical platonism appears in his 1926 text:

One faction among the mathematicians thinks themselves unrestricted autocrats in their respective realm, this realm being created of their own accord, according to their own whim, by logical deductions; the other faction proceeds from the belief that science pre-exists [the scientist], being in a state of ideal perfection, and that all that falls to our lot is to discover, in lucky instants, a limited new territory, as a piece [of that pre-existing science].<sup>33</sup> (Klein [32, p. 72])

Mehrtens reacts to Klein’s comments on mathematical Platonism as follows:

It transpires from the evaluative adjectives [employed by Klein] how Klein places himself; this also agrees with the ‘*Kleinian continuities*’, and with the counter-modern discourse: the “logic” and the “whim” of modernity are the *abhorrent* antithesis to the *rechter* German mathematician, who in pious modesty is receiving donations of ‘eternal truth’, piecemeal, by way of “inspiration” (read: ‘Anschauung’, ‘Intuition’).<sup>34</sup> [39, p. 219] (emphasis added)

Mehrtens’ conclusion introduces a theological element into the discussion:

The tone [of Klein’s voice] is religious. The devil is [incarnated in] other people; the ‘fall of man’ consists of the [choice to use one’s] whim, and this whim does not bow to the facts of truth and power.<sup>35</sup> (ibid.)

The discrepancy between Klein’s comment and Mehrstens’ intemperate reaction to it requires no amplification. The Talmudic dictum “kol haposel, bemumo posel” (whoever disqualifies others, [it is] in his own blemish [that he] disqualifies [them]) provides an insight into this passage, where *Mehrtens* himself coins the expression *wertende Beiworte*: Klein never used the adjective *abscheulich* [‘abhorrent’] with regard to any of the ideas mentioned: ‘logic’, ‘whim/arbitrariness’, or ‘modernity’.<sup>36</sup>

<sup>33</sup>In the original: “Die eine Gruppe von Mathematikern hält sich für unbeschränkte Selbstherrscher in ihrem Gebiet, das sie nach eigener Willkür logisch deduzierend aus sich heraus schaffen; die andere geht von der Auffassung aus, daß die Wissenschaft in ideeller Vollendung vorexistiere, und daß es uns nur gegeben ist, in glücklichen Augenblicken ein begrenztes Neuland als Stück davon zu entdecken. Nicht Erfinden nach Gutdünken, sondern Auffinden des ewig Vorhandenen, nicht die selbstbewußte Tat, sondern die vom Bewußtsein und Willen unabhängige, rein geschenkte Eingebung erscheint ihnen als das Wesen des Schaffens.”

<sup>34</sup>In the original: “Wo er sich selbst einordnet, ist in den wertenden Beiworten deutlich und fügt sich in *Kleins Kontinuitäten* und in den *gegenmodernen Diskurs*: Die “Logik” und die “Willkür” der Moderne sind der abscheuliche Gegensatz zum *rechten deutschen Mathematiker*, der in frommer Bescheidenheit die ‘ewige Wahrheit’ stückchenweise aber “rein” durch “Eingebung” (sprich: Anschauung, Intuition) geschenkt bekommt” (emphasis added).

<sup>35</sup>In the original: “Der Tonfall ist religiös. Der Teufel, das sind die anderen; der Sündenfall ist die Willkür, die vor den Gegebenheiten der Wahrheit und der Macht den Nacken nicht beugt.”

<sup>36</sup>Nor did Klein refer to non-formalist mathematicians as the ‘right’ mathematicians. Mehrstens wishes to see a political statement in places where there is none. In keeping with an apparently irrepressible urge to paint

The *continuity* alleged in Mehrtens' passage is all Mehrtens', not Klein's. Mehrtens' procedure here is another example of *massaging the evidence*.

#### 4.6. The butterfly model from Klein to Vahlen. Writes Mehrtens:

Theodor Vahlen, who after 1933 was an executive official in the ministry, and a professor in Berlin, gave, on [15 may] 1923, an address on assuming his office as rector [of University of Greifswald], in the usual tradition, speaking on 'Value and Essence' of mathematics, and cited *Klein's racist distinctions* within mathematics, and sharpened them into open antisemitism.<sup>37</sup> [39, p. 310–311] (emphasis added)

What we object to most is Mehrtens' judgmental phrase 'Klein's racist distinctions.' Here Mehrtens again equivocates on the meaning of a loaded term (see Section 3.4), and furthermore implies a continuity (and perhaps even organic necessity) between Klein's remarks on ethnic differences on the one hand, and Vahlen's "sharpened" open antisemitism, on the other. In the background of Mehrtens' remarks is an unspoken endorsement of the *butterfly model* of the evolution of ideas, contrasted with the *Latin model* (see Ian Hacking [19, p. 119]).

Mehrtens' allegation of continuity between the views of Klein and those of Vahlen (who eventually reached the rank of *SS-Brigadeführer*)<sup>38</sup> is an additional instance of assigning guilt by association, as well as a character smear against Klein.

In line with the Talmudic dictum cited in Section 4.5, Mehrtens seeks to argue a case of *biological* inevitability between the cocoon of Klein's remarks on intuition and race, on the one hand, and the dragon-butterfly (see e.g., Benisch [5]) of the *Nationalsozialismus*, on the other.

#### 4.7. Imperialist fight and a 1908 cartoon. Mehrtens implicates Klein in no less than an "imperialist competitive fight."

In the same year [1901, in a speech] before the 'support club'<sup>39</sup> Klein emphasizes "the importance of [higher] education in mathematics and the sciences in the competitive fight of the nations, and of the general cultural significance of high school education, and of a capable class of high school teachers supported by public trust." If the higher totality is culture, then the concrete totality is the nation in the *imperialist competitive fight*. . . . The goal was to define a *modern* culture in such a way, and to make it a state affair in such a way, that the professionals in natural- and social sciences, economics and bureaucracy rise to the rank of a state

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Klein as a right-wing precursor of worse things to come, Mehrtens plays on an ambiguity of the German adjective *recht* (it could mean either right-minded, righteous, or right-wing). Furthermore, Klein did not use the religious term *fromm* anywhere, nor *ewig*, nor *Teufel*, nor *Sündenfall*, nor *Macht*. Mehrtens' claim to the contrary is sheer fabrication. It is not Klein but Mehrtens who struck a religious tone, contrasting with the merely old-fashioned tone of Klein's comment.

<sup>37</sup>In the original: "Theodor Vahlen, nach 1933 leitender Ministerialbeamter und Professor in Berlin, hielt 1923 eine Rektoratsrede in einschlägiger Tradition 'Wert und Wesen' der Mathematik und zitierte *Kleins rassistische Unterscheidungen* in der Mathematik und verschärfte sie zu offenem Antisemitismus" (emphasis added). Mehrtens' reference to "Professor in Berlin, hielt 1923 eine Rektoratsrede, etc." may give the impression that Vahlen gave the antisemitic address at Berlin, since the word 'Greifswald' is relegated to a footnote, and 'Universitätsreden' is omitted, making it appear more significant than it actually was (Greifswald is a small city).

<sup>38</sup>See Stöwer [59, p. 145, note 349].

<sup>39</sup>This is a reference to an association charged with carrying out the reforms in the teaching of mathematics and science in German-speaking high school teaching.

elite.<sup>40</sup> [39, p. 361] (emphasis on ‘modern’ in the original; emphasis on ‘imperialist competitive fight’ added)

The tone of the above passage is consistent with the identification of the source of the enmity toward modernism on the part of marxist academics and Hilton Kramer’s diagnosis thereof (see Section 3.9).

That the driving force behind Mehrtens’ animus toward Klein is marxist detestation of capitalism is confirmed by Mehrtens’ analysis of a cartoon reproduced at [39, p. 381]. The cartoon represents the activities of the *Göttinger Vereinigung* founded by Klein, formed to encourage interactions between the Academy and Industry. The cartoon depicts recognizably professorial types exchanging books for proverbial moneybags brought by recognizably capitalist types.<sup>41</sup> Hovering over the scene are a sun-like Klein and an angelic-looking Althoff (see Section 2.2), depicted as bestowing blessings upon the congregants. The cartoon is dated at 1908 in also [22], [23], [44].

Mehrtens’ purpose in reproducing the cartoon seems to be to compare it to the logo of the *Göttinger Vereinigung* which depicts a allegorical scene alluding to academic-industrial interactions. Thus, Mehrtens declares the cartoon to be a ‘more accurate’ (“genauer”) representation of the activities of the *Göttinger Vereinigung* than the official logo (see [39, p. 382]).<sup>42</sup> Mehrtens’ own attitude toward such activities is spelled out clearly enough in the pages leading up to the cartoon:

The disposition over technical knowledge is being regulated via legal norms, by patent laws, and by laws for the protection of various secrets, in such a way that said knowledge is *co-opted* for use within the capitalist market, and for use within the *military state*.<sup>43</sup> [39, pp. 378–379] (emphasis added)

Mehrtens’ skillful insinuation of a connection between Klein and the *military state* finds its full expression in Mehrtens’ treatment of Theodor Vahlen; see Section 4.6.

<sup>40</sup>In the original: “Im gleichen Jahr betonte Klein vor dem Förderverein ))die Bedeutung des mathematisch-naturwissenschaftlichen Studiums im Konkurrenzkampf der Nationen und die allgemeine Kulturbedeutung der höheren Schule und eines leistungsfähigen, vom öffentlichen Vertrauen getragenen Standes der Studierende.(( Wenn das höhere Ganze die Kultur ist, dann ist das konkrete Ganze die Nation im imperialistischen Konkurrenzkampf. . . . Es kam darauf an, eine *moderne* Kultur so zu definieren und zur Staatsangelegenheit zu machen, daß die beteiligten Professionellen aus Natur- und Sozialwissenschaft, Wirtschaft und Bürokratie gemeinsam in den Rang einer staatlichen Elite aufrückten.”

<sup>41</sup>Mehrtens describes the cartoon as follows: “A contemporary caricature is more precise [than the logo] in that regard (Figure 5). Therein, at the signpost pointing towards the ‘Göttinger Vereinigung’, professors, clad in cap and gown, and with a large and a small book tucked under their arms, meet with the capitalists, unmistakable with their bellies and top hats, who carry a large and a small moneybag. After exchanging the small book for the small moneybag, they push along in pairs, engrossed in conversation” (translation ours). Mehrtens’ sarcastic tone indicates that he views the cartoon as a satirical caricature; see note 42.

<sup>42</sup>While Mehrtens interprets the cartoon as a satirical caricature of the goings-on at the *Göttinger Vereinigung*, Hermann and Schönbeck note in [23, Caption on p. 355] that the cartoon was sent out (as part of the invitation letter) by the organizers of the conference themselves (apparently in self-deprecating humor). Ohse et al. [44, p. 373] note that the cartoon displays the following text: “Gruss vom Festkommers zur Feier des 10-jährigen Bestehens der Göttinger Vereinigung Göttingen, 22. Febr. 1908” (translation: “Greetings from the *Commercium* on the occasion of the 10th anniversary of the existence of the Göttinger Vereinigung Göttingen, 22 february 1908”). The source for the cartoon is given as follows: Archiv der Aerodynamischen Versuchsanstalt, Prandtl Zimmer, Ordner ))Göttinger Vereinigung 1905-1919((.

<sup>43</sup>In the original: “Mit Patentrecht und Geheimnisschutz wird die Verfügung über technisches Wissen durch Rechtsnormen so reguliert, daß es für den kapitalistischen Markt und den *Militärstaat* *zugerichtet* wird” (emphasis added).

**4.8. Modernism in America.** According to Mehrtens, after the war modernism triumphed and culminated in the works of Nicolas Bourbaki [39, p. 320]. However, this is a simplistic view of mathematical modernism. Thus, Mehrtens makes no mention of the debate over modernist ideas in America. Notably, Marshall Stone claimed that

while several important changes have taken place in our conception of mathematics or in our points of view concerning it, the one which truly involves a revolution in ideas is the discovery that mathematics is entirely independent of the physical world (except that thinking takes place in the brain). (Stone [58, p. 716])

Stone's idea of mathematics as independent of the physical world was eventually picked up by Quinn; see Bair et al. [2].

In response to Stone's paper a symposium was held. Here Richard Courant, who was in many respects Klein's heir, had this to say:

Certainly mathematical thought operates by abstraction; mathematical ideas are in need of abstract progressive refinement, axiomatization, crystallization. It is true indeed that important simplification becomes possible when a higher plateau of structural insight is reached. . . . Yet, the life blood of our science rises through its *roots*; these roots reach down in endless ramification deep into what might be called reality, whether this "reality" is mechanics, physics, biological form, economic behavior, geodesy, or, for that matter, other mathematical substance already in the realm of the familiar. (Courant in Carrier et al. [9]; emphasis added)

Courant's *tree* metaphor, later elaborated by Kline, contrasted with the Bourbaki–Tucker *city* metaphor; see Phillips [45] for an analysis. Courant continued:

Abstraction and generalization is not more vital for mathematics than individuality of phenomena and, before all, not more than inductive intuition. Only the interplay between these forces and their synthesis can keep mathematics alive and prevent its drying out into a dead skeleton. . . . We must not accept the old blasphemous nonsense that the ultimate justification of mathematical science is 'the glory of the human mind'. (Courant in [9])

Similarly, a letter by Morris Kline and others claimed that the relevance of mathematics for students emerges from concrete situations, not formalisms; Kline was to argue his point most forcefully in [33]. The history of modernism in mathematics is more complex than Herbert Mehrtens is willing to grant.

**4.9. Klein's Evanston lectures.** Klein gave twelve lectures at Northwestern University at Evanston in 1893. The published version of the lectures occupies 98 pages. His sixth lecture contains the following passage:

[Alfred] Köpcke,<sup>44</sup> of Hamburg, has advanced the idea that our space-intuition is exact as far as it goes, but so limited as to make it impossible for us to picture to ourselves curves without tangents.

On one point Pasch does not agree with me, and that is as to the exact value of the axioms. He believes – and this is the traditional view – that it is possible finally to discard intuition entirely, basing the whole science on the axioms alone. I am of the opinion that, certainly, for the purposes of research it is always necessary

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<sup>44</sup>Mathematician, 1852–1927, doctoral student of Leo Königsberger 1875 at Heidelberg.

to combine the intuition with the axioms. I do not believe, for instance, that it would have been possible to derive the results discussed in my former lectures, the splendid researches of Lie, the continuity of the shape of algebraic curves and surfaces, or the most general forms of triangles, without the constant use of geometrical intuition.

Pasch's idea of building up the science purely on the basis of the axioms has since been carried still farther by Peano, in his logical calculus.

Finally, it must be said that the degree of exactness of the intuition of space may be different in different individuals, perhaps even in different races. It would seem as if a strong naive space-intuition were an attribute pre-eminently of the *Teutonic race*, while the critical, purely logical sense is more fully developed in the *Latin* and *Hebrew races*. A full investigation of this subject, somewhat on the lines suggested by Francis Galton in his researches on heredity, might be interesting. (Klein [29, p. 45–46]; emphasis added)

Klein's tentative comments on Teutonic, Hebrew, and Latin races, occupying a total of 6 lines in a 98-page text, were made in the context of

- a discussion of Köpcke's views concerning space-intuition;
- an appreciation of Pasch's accomplishment vis-a-vis the foundations;
- Klein's disagreement with Pasch with regard to the role of intuition in the creative process.

We propose our own tentative interpretation of the “Teutonic, Latin, Hebrew” (T, L, H) comment. Confronted with the striking difference in mathematical style between Köpcke (T) of Hamburg and Pasch (H), Klein is led to speculate on possible ethnic origins of such differences, with the remark about the Latin races added because of Klein's awareness of the axiomatic/foundational contribution of the Italian mathematician Peano (L). Historian Sanford Segal reacted to Klein's statements as follows:

For Klein, though certainly a conservative nationalist, was also certainly no anti-Semite—he had helped bring first Hermann Minkowski and then Edmund Landau as well as Karl Schwarzschild to Göttingen, and spoke favorably of the emancipation of the Jews in Prussia in 1812 (by Napoleon Bonaparte): “With this action a large new reservoir of mathematical talent was opened up for our country, the powers of which, coupled with the increase therein achieved by the French emigrants, very soon proved itself fruitful in our science.” He also had good relations with a number of Jewish mathematicians and had an extensive correspondence with his good friend Max Noether (Emmy's father): eighty-nine letters from Noether and 129 from Klein, often addressed “Lieber Noether”. (Segal [57, p. 270])

We will analyze Mehrtens' rather different reaction in Section 4.10 immediately following.

**4.10. Mehrtens on Klein's 6-line comment.** Historian Sanford Segal's relation to Klein's statements appears above (Section 4.9). The reaction to the same statements by Mehrtens is on record:

It was less antisemitism or racist nationalism which found its expression in [Klein's] statements, rather it was the imperialist racism of the dominating civilized peoples; this means that Klein gave an internationalist argumentation, by implicitly mentioning the limits of the Internationalism of his time. That [Klein's comments] had

the potential to enable others to articulate possible conflict between those ‘civilized nations’ biologically, is, with hindsight, all too clear.<sup>45</sup> (Mehrtens [39, p. 345])

It appears that *antisemitism*, *racism*, *imperialism*, *domination*, and (especially the sinister) *biologism* (see Section 4.4) are the first things that come to Mehrtens’ mind when reading Klein’s tentative comments on space-intuition and possible differences among ethnicities in intellectual outlook. As we already noted in Section 3.4, Mehrtens’ *racism* slur against Klein is based on equivocation.

Mehrtens’ diatribe against Klein rings particularly hollow to a reader aware of Mehrtens’ cynical exploitation of the infamous *yellow star* badge in his comments on Hilbert:

Hilbert’s [set-theoretic] paradise is a dictatorship. . . The yellow star is, when viewed mathematically, pure set-building.<sup>46</sup> (Mehrtens [39, p. 460])

Mehrtens’ flippant “set-building” comment comes disturbingly close to an odious trivialisation of the *yellow star* badge, a tragic symbol of the Holocaust.

**4.11. Mehrtens and Gray on Hilbert.** Mehrtens presents Hilbert as the arch-modernist (“Generaldirektor der Moderne”). In Gray’s words:

Hilbert in particular has a major role, and his work is presented ironclad as a program to make all of mathematics abstract, axiomatic and internally self-consistent. (Gray [18, p. 10])

However, such a view of Hilbert’s posture is based on a very selective reading of his works, taking into account only the *Grundlagen der Geometrie* and his work on the logical foundations of mathematics in the 1920s.

On Hilbert’s *Zahlbericht*, Corry writes: “The general idea of algebra as the discipline dealing with algebraic structures is still absent from Hilbert’s work on algebraic number theory” (Corry [12, p. 154]).

The same can be said concerning Hilbert’s papers on integral equations, which were written between 1904 and 1910 in the language of classical analysis, and are far from the foundational work on abstract function spaces that was done at the same time by Fréchet, Riesz and Schmidt. Hilbert never ventured into a study of such spaces.

Corry quotes a passage from a 1905 lecture, which illustrates Hilbert’s rather *informal* approach to axiomatics:

The edifice of science is not raised like a dwelling, in which the foundations are first firmly laid and only then one proceeds to construct and to enlarge the rooms. Science prefers to secure as soon as possible comfortable spaces to wander around and only subsequently, when signs appear here and there that the loose foundations are not able to sustain the expansion of the rooms, it sets about supporting and fortifying them. This is not a weakness, but rather the right and healthy path of development. (Hilbert quoted in Corry [14])

<sup>45</sup>In the original: “Weniger Antisemitismus oder rassistischer Nationalismus kam hier zum Ausdruck als eher der imperialistische Rassismus der dominierenden ‘Kulturvölker’; das heißt, Klein argumentierte internationalistisch, indem er implizit die Grenzen des zeitgenössischen Internationalismus mit ansprach. Daß darin das Potential lag, den möglichen Konflikt unter jenen ‘Kulturnationen’ biologisch zu artikulieren, ist im nachhinein allzu deutlich” (an excerpt of this quotation already appeared in Section 4.4).

<sup>46</sup>In the original: “Das Hilbertsche Paradies ist eine Diktatur. . . Der gelbe Stern ist, mathematisch gesehen, reine Mengenbildung.”

The following quotation from the foreword to the translation of Hilbert and Cohn-Vossen's *Anschauliche (!) Geometrie* [25] shows Hilbert in agreement with Klein with regard to the interplay between abstraction and intuition:

In mathematics, as in any scientific research, we find two tendencies present. On the one hand, the tendency toward abstraction seeks to crystallize the logical relations inherent in the maze of material that is being studied, and to correlate the material in a systematic and orderly manner. On the other hand, the tendency toward intuitive understanding fosters a more immediate grasp of the objects one studies, a live rapport with them, so to speak, which stresses the concrete meaning of their relations.

As to geometry, in particular, the abstract tendency has here led to the magnificent systematic theories of Algebraic Geometry, of Riemannian Geometry, and of Topology; these theories make extensive use of abstract reasoning and symbolic calculation in the sense of algebra. Notwithstanding this, it is still as true today as it ever was that intuitive understanding plays a major role in geometry. And such concrete intuition is of great value not only for the research worker, but also for anyone who wishes to study and appreciate the results of research in geometry.

Here Hilbert can hardly be said to be involved in an ironclad program of making all of mathematics abstract and axiomatic. The empiricist inclinations in Hilbert's work in geometry are studied by Corry [13]. Notes Rowe:

As Michael Toepell has convincingly shown, *Anschauung* held an important place in Hilbert's geometrical work, and he was by no means convinced that one could ultimately dispense with it altogether. (Rowe [53, p. 197])

As far as the relationship between Klein and Hilbert is concerned, Mehrtens claims that Klein, described as the jupiter of Göttingen, "tolerated Hilbert's formalism":

It pertains to the [concept of] *Unordnung* [disorder] that Felix Klein, the Jupiter of Göttingen, and the defender of *Anschauung*, became an advocate of university education for women and tolerated Hilbert's formalism, and is not so disorderly after all, if one takes into consideration Liebermann, the Secession, and the male self-construction in the portrait [of Klein by M. Liebermann] (cf. 7.2).<sup>47</sup> (Mehrtens [39, p. 577])

Since the bulk of Hilbert's work with Bernays on Formalism and foundations started at about the time of Klein's death, Mehrtens' comment is an *ahistorical collage* in addition to involving an unhelpful stereotype.

**5. Conclusion: Mehrtens' tools.** In sum, this particular marxist historian has exploited a variety of tools in his analysis of Klein that ranges from massaging the evidence (Sections 1.7, 4.5) and character smear (Sections 3.4, 4.6) to assigning guilt by association (Sections 4.3, 4.6) and ahistorical collage (Section 4.11).

On occasion, Mehrtens sheds the mask of a marxist historian and engages in what is discernibly bourgeois yellow journalism, as in Mehrtens' tale concerning the dress embroidered

<sup>47</sup>In the original: "Daß Felix Klein, der Jupiter Göttingens und Verteidiger der Anschauung, für das Frauenstudium eintrat und den Formalismus Hilberts tolerierte, gehört zur Unordnung und ist im übrigen so ordnungslos nicht, denkt man an Liebermann, die Secession und die männliche Selbstkonstruktion im Porträt (vgl. 7.2)."

with images of analytic curves with which Felix Klein allegedly “covered the body” of his bride, the tale in question being further embroidered by Mehrtens’ tasteless comments at [39, p. 214].

Mehrtens’ claim that Hilbert was interested in production rather than meaning applies only to the claim’s author. Mehrtens’ portrayal of Klein as countermodern is contrary to much historical evidence and must be rejected.

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