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ON A BANACH SPACE OF LAPLACE-STIELTJES INTEGRALS

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Let Ω be class of positive unbounded functions Φ on $(-\infty, +\infty)$ such that the derivative Φ' is positive, continuously differentiable and increasing to $+\infty$ on $(-\infty, +\infty)$, φ be the inverse function to Φ', and $\Psi(x) = x - \frac{\Phi(x)}{\Phi'(x)}$ be the function associated with Φ in the sense of Newton. Let F be nonnegative nondecreasing unbounded continuous on the right function on $[0, +\infty)$ and f be a real-value function on $[0, +\infty)$. By $LS_\Phi(F)$ we denote the class of integrals $I(\sigma) = \int_0^\infty f(x)e^{x\sigma}dF(x)$, convergent for all $\sigma \in \mathbb{R}$ such that $|f(x)| \exp\{x\Psi(\varphi(x))\} \to 0$ as $x \to +\infty$. Put $||I||_\Phi := \sup\{|f(x)| \exp\{x\Psi(\varphi(x))\} : x \geq 0\}$.

It is proved that if $\ln F(x) = o(x)$ as $x \to +\infty$ then $(LS_\Phi(F), ||\cdot||_\Phi)$ is a Banach space and it is studied its properties.

1. Introduction. Let V be the class of functions F on $[0, +\infty)$ nonnegative nondecreasing unbounded continuous on the right. We assume that a real-value function f on $[0, +\infty)$ is such that the Lebesgue-Stieltjes integral $\int_1^A f(x)e^{x\sigma}dF(x)$ exists for every $\sigma \in \mathbb{R}$ and $A \in (0, +\infty)$, and the integral

$$I(\sigma) = \int_0^\infty f(x)e^{x\sigma}dF(x), \quad \sigma \in \mathbb{R}, \tag{1}$$

is called of Laplace-Stieltjes ([1]). Let

$$M(\sigma) = M(\sigma, I) = \int_1^\infty |f(x)|e^{x\sigma}dF(x), \quad \sigma \in \mathbb{R}, \tag{2}$$

and

$$\mu(\sigma) = \mu(\sigma, I) = \sup\{|f(x)|e^{x\sigma} : x > 0\}, \quad \sigma \in \mathbb{R},$$

be the maximum of the integrand. It is clear, that if $f(x) \geq 0$ for all $x \geq 0$ then $M(\sigma, I) = I(\sigma)$, and asymptotic properties of integrals of such kind are studied in the monograph [1].

By σ_M we denote the abscissa of the convergence of integral (2), i. e. integral (2) converges for $\sigma < \sigma_M$ and diverges for $\sigma > \sigma_M$ (if integral (2) converges for all $\sigma \in \mathbb{R}$ then we put $\sigma_M = +\infty$). By analogy let σ_μ be the abscissa of the maximum of the integrand, i. e.

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$\mu(\sigma) < +\infty$ for all $\sigma < \sigma_\mu$ and $\mu(\sigma) = +\infty$ for all $\sigma > \sigma_\mu$ (if $\mu(\sigma) < +\infty$ for all $\sigma \in \mathbb{R}$ then we put $\sigma_\mu = +\infty$).

It is easy to show ([1, p. 8]) that

$$\sigma_\mu = \alpha := \varliminf_{x\to+\infty} \frac{1}{x} \ln \frac{1}{|f(x)|}, \tag{3}$$

and if $\ln F(x) = o(x)$ as $x \to +\infty$, then ([1, p. 13]) $\sigma_M \geq \sigma_\mu$. Hence it follows that if $\ln F(x) = o(x)$ and $\frac{1}{x} \ln \frac{1}{|f(x)|} \to +\infty$ as $x \to +\infty$ then $\sigma_M = +\infty$. The inequality $\sigma_M \leq \sigma_\mu$ is perhaps incorrect ([1, p. 21]).

Here we consider the case $\sigma_M = \sigma_\mu = +\infty$ and by Ω denote the class of positive functions Φ unbounded on $(-\infty, +\infty)$ such that the derivative Φ' is positive continuously differentiable and increasing to $+\infty$ on $(-\infty, +\infty)$. It is clear that the function $\Phi \in \Omega$ is convex on $(-\infty, +\infty)$, $\Phi(x) \to c \geq 0$ $(x \to -\infty)$ and $\Phi'(x) \to 0$ $(x \to -\infty)$. From now on, we denote by φ the inverse function to Φ', and let $\Psi(x) = x - \frac{\Phi(x)}{\Phi'(x)}$ be the function associated with Φ in the sense of Newton. It is clear that the function φ is continuously differentiable and increasing to $+\infty$ on $(0, +\infty)$. The function Ψ is ([1, p. 30]) continuously differentiable and increasing to $+\infty$ on $(-\infty, +\infty)$. The following lemma is true ([1, p. 30]).

Lemma 1. *Let $\Phi \in \Omega$. In order that $\ln \mu(\sigma) \leq \Phi(\sigma)$ for all $\sigma \in (\sigma_0, +\infty)$, it is necessary and sufficient that $\ln f(x) \leq -x\Psi(\varphi(x))$ for all $x \geq x_0$.*

2. Banach space of Laplace-Stiltijes integrals. Let $\Phi \in \Omega$ and

$$J(\sigma) = \int_0^\infty \exp\{-x\Psi(\varphi(x))\} e^{x\sigma} dF(x). \tag{4}$$

Since

$$\frac{1}{x} \ln \frac{1}{\exp\{-x\Psi(\varphi(x))\}} = \Psi(\varphi(x)) \to +\infty, \quad x \to +\infty,$$

integral (4) converges for all $\sigma \in (-\infty, +\infty)$. By $LS_\Phi(F)$ we denote the class of integrals (1) with real-valued functions f such that

$$|f(x)| \exp\{x\Psi(\varphi(x))\} \to 0, \quad x \to +\infty. \tag{5}$$

On $LS_\Phi(F)$ we define operations

$$(I_1 + I_2)(\sigma) = \int_0^\infty (f_1(x) + f_1(x)) e^{x\sigma} dF(x), \quad (\lambda I)(\sigma) = \int_0^\infty \lambda f(x) e^{x\sigma} dF(x),$$

where

$$I_p(\sigma) = \int_0^\infty f_p(x) e^{x\sigma} dF(x), \tag{6}$$

where $p = 1, 2$ and λ is a real number, and let

$$\|I\|_\Phi := \sup\{|f(x)| \exp\{x\Psi(\varphi(x))\} : x \geq 0\}.$$

Under these operations $LS_\Phi(F)$ becomes a normed linear space.

Theorem 1. *If $F \in V$ and $\ln F(x) = o(x)$ as $x \to +\infty$ then $(LS_\Phi(F), \|\cdot\|_\Phi)$ is non-uniformly convex Banach space.*

Proof. Let (I_p) be a Cauchy sequence in $LS_\Phi(F)$ such that $I_p(\sigma)$ is defined by (6). Then $|f_p(x)|\exp\{x\Psi(\varphi(x))\} \to 0$ as $x \to +\infty$ for each p. As (I_p) is Cauchy sequence so for a given $\varepsilon > 0$ there exits $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that $\|I_p - I_q\|_\Phi < \varepsilon$ for all $p \geq n_0$ and $q \geq n_0$, i. e.

$$\sup\{|f_p(x) - f_q(x)|\exp\{x\Psi(\varphi(x))\} : x \geq 0\} < \varepsilon$$

for all $p \geq n_0$ and $q \geq n_0$, and, thus,

$$|f_p(x)\exp\{x\Psi(\varphi(x))\} - f_q(x)\exp\{x\Psi(\varphi(x))\}| < \varepsilon$$

for all $x \geq 0$, $p \geq n_0$ and $q \geq n_0$. This shows that $(f_p(x)\exp\{x\Psi(\varphi(x))\})$ is Cauchy sequence in $\mathbb{R}$, so converges to $f_0(x)\exp\{x\Psi(\varphi(x))\}$ (say) as $p \to \infty$. Since

$$\begin{gathered}|f_0(x)|\exp\{x\Psi(\varphi(x))\} = \\ = |f_0(x)\exp\{x\Psi(\varphi(x))\} - f_p(x)\exp\{x\Psi(\varphi(x))\} + f_p(x)\exp\{x\Psi(\varphi(x))\}| \leq \\ \leq |f_0(x)\exp\{x\Psi(\varphi(x))\} - f_p(x)\exp\{x\Psi(\varphi(x))\}| + |f_p(x)|\exp\{x\Psi(\varphi(x))\} \to 0, \quad x \to +\infty,\end{gathered}$$

the integral

$$I_0(\sigma) = \int_0^\infty f_0(x)e^{x\sigma}dF(x)$$

belongs to $LS_\Phi(F)$. Further,

$$\|I_p - I_0\|_\Phi = \sup\{|f_p(x) - f_0(x)|\exp\{x\Psi(\varphi(x))\} : x \geq 0\} \to 0, \quad p \to \infty,$$

i. e. the $(LS_\Phi(F), \|I\|_\Phi)$ is complete and, therefore, a Banach space.

Now we choose numbers $0 < a_n < b_n < c_n < +\infty$ such that

$$\int_0^{a_n} \exp\{-x\Psi(\varphi(x))\}e^{x\sigma}dF(x) \neq 0, \quad \int_{b_n}^{c_n} \exp\{-x\Psi(\varphi(x))\}e^{x\sigma}dF(x) \neq 0$$

and put

$$I_n(\sigma) = \int_0^{a_n} \exp\{-x\Psi(\varphi(x))\}e^{x\sigma}dF(x)$$

and

$$J_n(\sigma) = \int_0^{a_n} \exp\{-x\Psi(\varphi(x))\}e^{x\sigma}dF(x) + \int_{b_n}^{c_n} \exp\{-x\Psi(\varphi(x))\}e^{x\sigma}dF(x).$$

Then $I_n \in LS_\Phi(F)$, $J_n \in LS_\Phi(F)$, $\|I_n\| = 1, \|J_n\| = 1, \|J_n + I_n\| = 2$, but $\|J_n - I_n\| = 1 \not\to 0$, i. e. the space $(LS_\Phi(F), \|\cdot\|_\Phi)$ is non-uniformly convex (see. for example, [2, p. 183]). □

Remark 1. From (5) it follows that $\ln|f(x)| \leq -x\Psi(\varphi(x))$ for all $x \geq x_0$. Therefore, by Lemma 1 $\ln\mu(\sigma, I) \leq \Phi(\sigma)$ for all $\sigma \in (\sigma_0, +\infty)$ for every $I \in LS_\Phi(F)$. On the other hand, if $\ln F(x) = O(\Phi(\Psi(\varphi(x))))$ as $x \to +\infty$ and $\ln\mu(\sigma, I) \leq \Phi(\sigma)$ for all $\sigma \in (\sigma_0, +\infty)$ then [1, p. 103] $\ln I(\sigma) \leq (1 + o(1))\Phi(\sigma)$ as $\sigma \to +\infty$, i. e. $\overline{\lim\limits_{\sigma\to+\infty}} \frac{\ln I(\sigma)}{\Phi(\sigma)} \leq 1$ for each $I \in LS_\Phi(F)$.

The following statement concerns for uniform convergence.

Proposition 1. *Let $F \in V$ and $\ln F(x) = o(x)$ as $x \to +\infty$. If $(I_m) \subset LS_\Phi(F)$ converges to $I \in LS_\Phi(F)$ by $\|\cdot\|_\Phi$, then $I_m(\sigma)$ converges uniformly to $I(\sigma)$ over compact subset of $\mathbb{R}$.*

Proof. If $\|I_p - I\|_\Phi < \varepsilon$ for every $\varepsilon > 0$ and all $p \geq p_0(\varepsilon)$ then

$$\sup\{|f_p(x) - f(x)| \exp\{x\Psi(\varphi(x))\} : x \geq 0\} < \varepsilon$$

and, thus, $|f_p(x) - f(x)| \exp\{x\Psi(\varphi(x))\} < \varepsilon$ for every $\varepsilon > 0$, all $p \geq p_0(\varepsilon)$ and all $x \geq 0$. Therefore, if $p \geq p_0(\varepsilon)$ and $\sigma \in [\sigma_1, \sigma_2]$ then in view of condition $\ln F(x) = o(x)$ as $x \to +\infty$ we have

$$|I_p(\sigma) - I(\sigma)| < \int_0^{+\infty} |f_p(x) - f(x)| \exp\{x\Psi(\varphi(x))\} \exp\{-x\Psi(\varphi(x))\} e^{x\sigma} dF(x) <$$

$$< \varepsilon \int_0^{+\infty} \exp\{-x\Psi(\varphi(x))\} e^{x\sigma_2} dF(x) = \varepsilon \int_0^{+\infty} e^{-x(\Psi(\varphi(x))-\sigma_2)} dF(x) \leq$$

$$\leq \varepsilon \int_0^{+\infty} F(x) de^{-x(\Psi(\varphi(x))-\sigma_2)} \leq \varepsilon \int_0^{+\infty} e^{K_1 x} de^{-x(\Psi(\varphi(x))-\sigma_2)} \leq K_2\varepsilon, \quad K = \text{const}.$$

Therefore, $(I_p(\sigma))$ converges uniformly to $I(\sigma)$ on $[\sigma_1, \sigma_2]$. □

Remark 2. The converse statement to Proposition 1 is false. Indeed, let for every $m \in \mathbb{Z}_+$ and $n \in \mathbb{N}$

$$F(x) = \begin{cases} 0, \ 1 \leq x < 2, \\ n, \ 2n \leq x < 2(n+1), \end{cases} \qquad f_m(x) = \begin{cases} \alpha_{m,n} > 0, \ x = 2n-1, \\ 0, \ x \neq 2n-1. \end{cases}$$

Then for all $m \in \mathbb{Z}_+$

$$I_m(\sigma) = \int_0^{\infty} f_m(x) e^{x\sigma} dF(x) = \sum_n f_m(2n) e^{2n\sigma} = 0,$$

i. e. $I_m(\sigma) \to I_0(\sigma)$ for all $\sigma \in [\sigma_1, \sigma_2]$. On the other hand,

$$\|I_m - I_0\|_\Phi = \sup\{|f_m(x) - f_0(x)| \exp\{-x\Psi(\varphi(x))\} : x \geq 0\} \geq$$
$$\geq |f_m(1) - f_0(1)| \exp\{\Psi(\varphi(1))\} = |\alpha_{m,1} - \alpha_{0,1}| \exp\{\Psi(\varphi(1))\} \geq h_1 > 0$$

provided $\alpha_{m,1} - \alpha_{0,1} \geq \eta_1 > 0$ for all $m \in \mathbb{N}$.

For $(LS_\Phi(F), \|\cdot\|_\Phi)$ by $LS_\Phi(F)^*$ we denote the dual space, i. e. $LS_\Phi(F)^*$ is the family of all continuous linear functionals on $(LS_\Phi(F), \|\cdot\|_\Phi)$.

Let $\Lambda(I) = \int_1^\infty f(x)g(x)dF(x)$, where g is a real-valued function on $(1, +\infty)$ such that $\int_1^\infty |f(x)g(x)|dF(x) < +\infty$. Then Λ is a linear functional and the following proposition is true.

Proposition 2. *In order that $\Lambda \in LS_{\Phi}(F)^*$ it is sufficient that*

$$\int\limits_{1}^{\infty} |g(x)| \exp\{-x\Psi(\varphi(x))\} dF(x) < +\infty. \tag{7}$$

Proof. By definition we have

$$\|\Lambda(I)\|_{\Phi} = \sup\{|\Lambda(I)| : \|I\|_{\Phi} \le 1\} =$$

$$= \sup\left\{ \int\limits_{0}^{\infty} |f(x)| e^{x\Psi(\varphi(x))} |g(x)| e^{-x\Psi(\varphi(x))} dF(x) : \sup_{x>0} |f(x) e^{x\Psi(\varphi(x))} \le 1 \right\} \le$$

$$\le \int\limits_{1}^{\infty} |g(x)| \exp\{-x\Psi(\varphi(x))\} dF(x) < +\infty,$$

i. e. $\Lambda \in LS_{\Phi}(F)^*$. Proposition 2 is proved. □

Conjecture. *Every bounded linear functional Λ defined for $I \in (LS_{\Phi}(F), \|\cdot\|_{\Phi})$ is of the form*

$$\Lambda(I) = \int\limits_{1}^{\infty} f(x) g(x) dF(x), \quad I(\sigma) = \int\limits_{0}^{\infty} f(x) e^{x\sigma} dF(x),$$

where $g(x)$ satisfies (7).

Below we prove this conjecture for entire Dirichlet series.

3. Banach space of Dirichlet series. Let $0 < \lambda_n \uparrow \infty$ and $n(x) = \sum_{\lambda_n \le x} 1$ be a counting function of the sequence $\lambda = (\lambda_n)$. If we choose $f(x) = a_n$ for $x = \lambda_n$ and $f(x) = 0$ for $x \ne \lambda_n$ and $F(x) = n(x)$ then from (1) we obtain a Dirichlet series

$$D(\sigma) = \sum_{n=1}^{\infty} a_n e^{\lambda_n \sigma}. \tag{8}$$

By $S(\lambda, \Phi)$ we denote a class of entire Dirichlet series (8) with real-valued coefficients a_n such that

$$|a_n| \exp\{\lambda_n \Psi(\varphi(\lambda_n))\} \to 0, \quad n \to +\infty, \tag{9}$$

and we put

$$\|D\|_{\Phi} := \sup\{|a_n| \exp\{\lambda_n \Psi(\varphi(\lambda_n))\} : n \ge 1\}.$$

Then from Theorem 1 we obtain the following statement.

Corollary 1. *If $\ln n(x) = o(x)$ as $x \to +\infty$ then $(S(\lambda, \Phi), \|\cdot\|_{\Phi})$ is non-uniformly convex Banach space.*

The following theorem completes Proposition 1.

Theorem 2. *If $\ln n(x) = o(x)$ as $x \to +\infty$ then in order that $(D_m) \subset S(\lambda, \Phi)$ converges to $D \in S(\lambda, \Phi)$ by $\|\cdot\|_{\Phi}$ it is necessary and sufficient that $D_m(\sigma)$ converges uniformly to $D(\sigma)$ over each compact subset of $\mathbb{R}$.*

Proof. The necessity follows from Proposition 1. Conversely, let $|D_m(\sigma) - D(\sigma)| < \varepsilon$ for all $\sigma \in B = [\sigma_1, \sigma_1]$ and all $m \geq m_0 = m_0(\varepsilon, B)$, where $D_m(\sigma) = \sum_{n=1}^{\infty} a_{m,n} e^{\lambda_n \sigma}$. Then $|a_{m,n} - a_n| e^{\lambda_n \sigma_1} < \varepsilon$ for all $n \geq 1$ and all $m \geq m_0 = m_0(\varepsilon, \sigma_1)$, whence

$$|a_{m,n} - a_n| \exp\{\lambda_n \Psi(\varphi(\lambda_n))\} \leq \varepsilon \exp\{-(\sigma_1 - \lambda_n \Psi(\varphi(\lambda_n)))\}.$$

Choosing $\sigma_1 = \lambda_n \Psi(\varphi(\lambda_n))$ hence we obtain that $|a_{m,n} - a_n| \exp\{\lambda_n \Psi(\varphi(\lambda_n))\} \leq \varepsilon$ for every $\varepsilon > 0$, all $n \geq 1$ and all $m \geq m_0 = m_0(\varepsilon, n)$, i. e. $\|D_m - D\|_\Phi \to 0$ as $m \to \infty$. □

For $(S(\lambda, \Phi), \|\cdot\|_\Phi)$ by $S(\lambda, \Phi)^*$ we denote the dual space and put $\Lambda(D) = \sum_{n=1}^{\infty} a_n g_n$, where (g_n) is a real-valued sequence such that $\sum_{n=1}^{\infty} |g_n| \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\} < +\infty$. Then Λ is a linear functional and the following proposition is true.

Theorem 3. *Every bounded linear functional Λ defined for $(S(\lambda, \Phi), \|\cdot\|_\Phi)$ is of the form*

$$\Lambda(D) = \sum_{n=1}^{\infty} a_n g_n, \quad D(\sigma) = \sum_{n=1}^{\infty} a_n e^{\lambda_n \sigma}, \tag{10}$$

where (g_n) is a real-valued sequence such that $\sum_{n=1}^{\infty} |g_n \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\}| < +\infty$.

Proof. We use a method from [3]. As in the proof of Proposition 2 now we have

$$\sum_{n=1}^{\infty} |a_n g_n| \leq \sup\{|a_n| \exp\{\lambda_n \Psi(\varphi(\lambda_n))\} \sum_{n=1}^{\infty} |g_n| \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\} =$$

$$= \|D\| \sum_{n=1}^{\infty} |g_n| \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\} < +\infty,$$

i. e. Λ is well defined functional on $(S(\lambda, \Phi), \|\cdot\|_\Phi)$. Moreover,

$$|\Lambda(D)| \leq \|D\| \sum_{n=1}^{\infty} |g_n| \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\},$$

whence

$$\|\Lambda\| \leq \sum_{n=1}^{\infty} |g_n| \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\}. \tag{11}$$

Conversely, first we remark that if $D \in S(\lambda, \Phi), \|\cdot\|_\Phi)$ and $D_m(\sigma) = \sum_{n=1}^{m} a_n e^{\lambda_n \sigma}$ then

$$\|D_m - D\|_\Phi = \sup_{n>m}\{|a_n| \exp\{\lambda_n \Psi(\varphi(\lambda_n))\}\} \to 0$$

as $m \to \infty$ and by Theorem 2 $D_m(\sigma)$ converges uniformly to $D(\sigma)$ over each compact subset of $\mathbb{R}$. Therefore, if $\Lambda \in S(\lambda, \Phi)^*$ and be defined as $\Lambda(e^{\sigma \lambda_n}) = g_n$ for each n then

$$\Lambda(D) = \Lambda\left(\lim_{m\to\infty} \sum_{n=1}^{m} a_n e^{\lambda_n \sigma}\right) = \lim_{m\to\infty} \sum_{n=1}^{m} a_n L(e^{\lambda_n \sigma}) = \sum_{n=1}^{\infty} |a_n g_n|.$$

Now we show that $\sum\limits_{n=1}^{\infty} |a_n g_n| \leq \|\Lambda\||$ so that $\sum\limits_{n=1}^{\infty} |a_n g_n| < +\infty$. For it take any $p \geq 1$ and let $p \in \mathbb{N}$ and

$$a_n = \begin{cases} \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\} \mathrm{sgn}(g_n), & 1 \leq n \leq p; \\ 0, & n > p. \end{cases}$$

If we define $D(\sigma) = \sum\limits_{n=1}^{\infty} a_n \exp\{\sigma\lambda_n\}$ then obviously $D \in S(\lambda, \Phi)$ and $\|D\|_\Phi = 1$. Hence

$$|\Lambda(D)| = \left|\sum_{n=1}^{p} \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\}\mathrm{sgn}(g_n)\Lambda(e^{\sigma\lambda_n})\right| = \sum_{n=1}^{p} \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\}|g_n|$$

and $|\Lambda(D)| \leq \|\Lambda\|\|D\| = \|\Lambda\|$, so that $\sum\limits_{n=1}^{p} \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\}|g_n| \leq \|\Lambda\|$ and

$$\sum_{n=1}^{\infty} \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\}|g_n| \leq \sup_p \sum_{n=1}^{p} \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\}|g_n| \leq \sup_p \|\Lambda\| = \|\Lambda\|. \qquad (12)$$

Inequalities (11) and (12) together show that $\sum\limits_{n=1}^{\infty} \exp\{-\lambda_n \Psi(\varphi(\lambda_n))\}|g_n| = \|\Lambda\|$ and this completes the proof of the theorem. □

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