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Corrections to the paper "Sharp estimates of the growth of the Poisson-Stieltjes integral in the polydisc" by I.E.Chyzhykov, O.A.Zolota

The proof of the sufficiency of the theorem in [1] contains a gap, because the estimate of the Poisson kernel on the set $Q_k = E_k \setminus E_{k-1}$ (p.194) does not hold. The authors do not know whether the sufficiency statement is true in the general case, but here we give the proof under the stronger assumption $0 < \beta_j < 2$, $j \in \{1, \ldots, n\}$ instead of $\frac{1}{\beta_1} + \cdots + \frac{1}{\beta_n} > \frac{1}{2}$ as it was stated in [1].

Theorem. *Let μ be a finite Borel measure on $T^n, n \in \mathbb{N}$, $0 < \beta_j < 2$, $1 \le j \le n$. In order that*

$$\Big| \int_{T^n} \mathcal{P}(z, w)\, d\mu(w) \Big| \le C\delta^{1-\frac{1}{\beta_*}}, \quad 0 < \delta < 1,\ |z_j| = 1 - \delta^{\frac{1}{\beta_j}}, \tag{1}$$

hold for some positive constant C, it is sufficient, and for nonnegative μ it is necessary that $\mu \in H^{(\beta_1, \ldots, \beta_n)}$.

Proof. Sufficiency. For fixed $\delta \in (0, 1)$ we set $r_j = 1 - \delta^{\frac{1}{\beta_j}}$, and let $z_j = r_j e^{i\varphi_j}$, $1 \le j \le n$. For $1 \le j \le n$ we denote

$$E_k = \left\{ \left(e^{i\theta_1}, \ldots, e^{i\theta_n}\right) : |\theta_j - \varphi_j| \le \left(2^k \cdot \delta\right)^{\frac{1}{\beta_j}}, 1 \le j \le n \right\},\ 0 \le k \le N_j,$$

where $N_j = \left[\log_2 \frac{\pi^{\beta_j}}{\delta}\right] + 1$, and

$$R_{m_1 \ldots m_n} = \left\{ \begin{array}{ll} \left(e^{i\theta_1}, \ldots, e^{i\theta_n}\right) \in T^n : \ (2^{m_j-1}\delta)^{\frac{1}{\beta_j}} \le |\theta_j - \varphi_j| \le (2^{m_j}\delta)^{\frac{1}{\beta_j}}, & \text{if } m_j > 0, \\ |\theta_j - \varphi_j| \le \delta^{\frac{1}{\beta_j}}, & \text{if } m_j = 0, \end{array} \right\}.$$

Then $T^n = \bigcup\limits_{m_1=0}^{N_1} \ldots \bigcup\limits_{m_n=0}^{N_n} R_{m_1 \ldots m_n}$. As in [2] (proof of Theorem 3.1) we have for fixed k that:

$$\Big| \int_{R_{m_1 \ldots m_n}} \mathcal{P}(z, w)\, d\mu(w) \Big| \le |\mu|\left(R_{m_1 \ldots m_n}\right) \cdot \prod_{j=1}^{n} \mathcal{P}(|z|, \tilde{w}),$$

where $z \in U^n$, $\tilde{w} = (\tilde{w}_1, \ldots, \tilde{w}_n)$, $\tilde{w}_j = e^{i\tilde{\theta}_j}$, $\tilde{\theta}_j = \left(2^{k-1} \cdot \delta\right)^{\frac{1}{\beta_j}}, 1 \le j \le n$.

The following estimates are known ([2])

$$P_0\left(re^{i\varphi}, e^{i\theta}\right) \le \frac{\pi^2}{(\varphi - \theta)^2}(1 - r), \quad P_0\left(re^{i\varphi}, e^{i\theta}\right) \le \frac{2}{1 - r}. \tag{2}$$

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Applying the first estimate, we get

$$P_0\left(|z_j|, \tilde{w}_j\right) \le \frac{\pi^2}{\tilde{\theta}_j^2}\left(1-r_j\right) = \frac{\pi^2}{\left(2^{k-1}\cdot\delta\right)^{\frac{2}{\beta_j}}}\left(1-r_j\right), k \ge 1.$$

Using the second inequality in (2), we obtain

$$\Big|\int_{R_{0\dots0}} \mathcal{P}\left(z,w\right)d\mu\left(w\right)\Big| \le |\mu|\left(R_{0\dots0}\right)\cdot\frac{2^n}{(1-r_1)\cdot\ldots\cdot(1-r_n)}.$$

Using the definition of the class $H^{(\beta_1,\dots,\beta_n)}$ we have that $|\mu|\left(R_{0\dots0}\right) \le C\delta$. The equalities $1-r_j = \delta^{\frac{1}{\beta_j}},\ 1 \le j \le n$ imply

$$\Big|\int_{R_{0\dots0}} \mathcal{P}\left(z,w\right)d\mu\left(w\right)\Big| \le C\delta^{1-\left(\frac{1}{\beta_1}+\ldots+\frac{1}{\beta_n}\right)}.$$

Then we estimate the integral over the sets $R_{m_1\dots m_n}$. Since, $R_{m_1\dots m_n} \subset E_{\max\{m_1,\dots,m_n\}}$, we get that $|\mu|\left(R_{m_1\dots m_n}\right) \le C\cdot 2^{m_1+\ldots+m_n}\delta$.

Therefore, for $N = \max N_j,\ 1 \le j \le n$

$$\Big|\sum_{m_1=1}^{N}\cdots\sum_{m_n=1}^{N}\int_{R_{m_1\dots m_n}} \mathcal{P}\left(z,w\right)d\mu\left(w\right)\Big| \le \sum_{m_1=1}^{N}\cdots\sum_{m_n=1}^{N}|\mu|\left(R_{m_1\dots m_n}\right)\cdot\prod_{j=1}^{n}\frac{\pi^2\left(1-r_j\right)}{\left(2^{m_j-1}\delta\right)^{\frac{2}{\beta_j}}} \le$$

$$\le C\sum_{m_1=1}^{N}\cdots\sum_{m_n=1}^{N}2^{m_1}\cdot\ldots\cdot 2^{m_n}\cdot\delta\cdot\prod_{j=1}^{n}\frac{\delta^{\frac{1}{\beta_j}}}{\left(2^{\frac{2m_j}{\beta_j}}\cdot\delta^{\frac{2}{\beta_j}}\right)} \le$$

$$\le C\delta^{1-\left(\frac{1}{\beta_1}+\ldots+\frac{1}{\beta_n}\right)}\sum_{m_1=1}^{N}\left(2^{1-\frac{2}{\beta_1}}\right)^{m_1}\cdots\sum_{m_n=1}^{N}\left(2^{1-\frac{2}{\beta_n}}\right)^{m_n} < C\delta^{1-\frac{1}{\beta^*}}. \qquad \square$$

REFERENCES

1. Chyzhykov I.E., Zolota O.A. *Sharp estimates of the growth of the Poisson-Stieltjes integral in the polydisc*// Mat. Stud. – 2010. – V.34, №2. – P. 193–196.
2. Rudin W. Function theory in polydiscs. – New York-Amsterdam: Math. Lecture Note Series, 1969. – 162p.

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