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ON DOMAINS OF CONVERGENCE OF MULTIPLE RANDOM DIRICHLET SERIES

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We establish relations between domains of convergence and absolute convergence of the multiple random Dirichlet series.

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Устанавливаются соотношения для областей сходимости и абсолютной сходимости случайных кратных рядов Дирихле.

Let

$$(\lambda_k^{(i)}),\ 0=\lambda_0^{(i)}\le\lambda_1^{(i)}<\lambda_2^{(i)}<\dots\ (i\in\{1,\dots,p\}),$$

be a sequence of nonnegative numbers increasing to $+\infty$, and let (a_n) be a sequence of complex numbers,

$$n=(n_1,\dots,n_p),\ n_i\in\mathbb{Z}_+\overset{def}{=}\mathbb{N}\cup\{0\}\ (i\in\{1,\dots,p\}),$$
$$\|n\|=n_1+\dots+n_p,\ s=(s_1,s_2,\dots,s_p)\in\mathbb{C}^p,\ s_j=\sigma_j+it_j\ (j\in\{1,\dots,p\}),$$
$$\langle\lambda_n,s\rangle=\lambda_{n_1}^{(1)}s_1+\dots+\lambda_{n_p}^{(p)}s_p,\ \lambda_n=(\lambda_{n_1}^{(1)},\dots,\lambda_{n_p}^{(p)}).$$

Consider a multiple Dirichlet series of the form

$$F(s)=\sum_{\|n\|=0}^{+\infty}a_ne^{\langle\lambda_n,s\rangle}\tag{1}$$

where $\lambda_n=(\lambda_{n_1}^{(1)},\dots,\lambda_{n_p}^{(p)})$ for $n=(n_1,\dots,n_p)$.

Let $\tau\in\mathbb{R}_+^p$ be a system of positive numbers such that

$$\varlimsup_{\|n\|\to\infty}\frac{\ln n_1+\dots+\ln n_p}{\langle\tau,\lambda_n\rangle}\le 1.\tag{2}$$

The domains of convergence and absolutely convergence of series (1) are denoted by $\mathbf{G}_з(F)$ and $\mathbf{G}_a(F)$. Let

$$G_з(F)=\{x\in\mathbb{R}^p\colon x=\operatorname{Re}z,\ z\in\mathbf{G}_з(F)\},\ G_a(F)=\{x\in\mathbb{R}^p\colon x=\operatorname{Re}z,\ z\in\mathbf{G}_a(F)\}$$

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be their tracks in $\{x \in \mathbb{R}^p \colon x = \operatorname{Re} z,\ z \in \mathbb{C}^p\}$, and let G_μ be the domain of the existence of the maximal term

$$\mu(\sigma) = \max\left\{|a_n| \exp\{\langle \lambda_n, \sigma\rangle\} \colon n \in \mathbb{Z}_+^p\right\}$$

of series (1), i.e. the set of $\sigma \in \mathbb{R}^p$ such that $\mu(\sigma) < +\infty$.

The domains $G_з(F), G_a(F), G_\mu(F)$ are convex ([1]). It is easy to see, that $G_a \subset G_з \subset G_\mu$, and it is known ([1]), that $G_\mu \subset G_a + \tau$.

The coordinates of the points that on the boundary of the domains $G_з(F)$, $G_a(F)$, $G_\mu(F)$ are called the *conjugate abscissas of convergence*, *conjugate abscissas of absolutely convergence* and *conjugate abscissas of the existence of the maximal term*, respectively.

For example, the point $(\sigma_{c1}, ..., \sigma_{cp})$ is said to be the system of the conjugate abscissas of convergence of Dirichlet series (1), if this series converges on every domain of the form $\left\{s \colon \operatorname{Re} s_k < \sigma_{ck}, k \in \{1, ..., p\}\right\}$ and every domain of the form $\left\{s \colon \operatorname{Re} s_k < \sigma_k^0,\ k \in \{1, ..., p\}\right\}$, where $\sigma_i^0 > \sigma_{ci}, i \in I$ i $\sigma_j^0 \geq \sigma_{cj}, j \in J$ and $I \sqcup J = \{1, ..., p\}$, contains points of divergence.

Let $(\Omega, \mathcal{A}, P)$ be a probability space. For real or complex random variable ξ let

$\mathbf{M}\xi = \int_\Omega \xi(\omega) P(d\omega)$ be its *mean value* and $\mathbf{D}\xi = \mathbf{M}(|\xi - M\xi|^2)$ be its *variance*.

Consider the multiple random Dirichlet series

$$F_\omega(s) = F_\omega(s, \omega) = \sum_{\|n\|=0}^{+\infty} \eta_n(\omega) a_n e^{\langle \lambda_n, s\rangle}, \tag{3}$$

where (a_n) are the coefficients of the above series (1) and (η_n) is a sequence of random variables on the probability space $(\Omega, \mathcal{A}, P)$.

The main result of this paper is contained in the following theorem.

Theorem 1. *Let (η_n) be a sequence of complex independent random variables such that $(\forall\ n \in \mathbb{Z}_+) \colon\ 0 < c_1 \leq |\eta_n(\omega)| \leq c_2 < +\infty$ almost surely (a.s.), and $\mathbf{M}\eta_n = 0$. Then for the random Dirichlet series $F_\omega(s)$*

$$G_a(F_\omega) \subset \frac{G_a(F_\omega) + G_\mu(F_\omega)}{2} \subset G_з(F_\omega) \subset \left(\left(G_a(F_\omega) + \frac{\tau}{2}\right) \cap G_\mu(F_\omega)\right) \tag{4}$$

holds (a.s.).

Note, that in [2, 3] and [1, 4] there were obtained main relations between abscissas of convergence and absolutely convergence of double and multiple Dirichlet series, respectively. In [5] a similar result to Theorem 1 is proved for the abscissa of convergence for random series of one variable, and in [6] the same theorem is proved for the case of double random Dirichlet series and symmetric real independent random variables.

To prove this theorem, we need some auxiliary results.

Let

$$F_2(s) = \sum_{\|n\|=0}^{+\infty} a_n^2 e^{2\langle \lambda_n, s\rangle}. \tag{5}$$

Lemma 1. *For the Dirichlet series (1) and (5)*

$$G_a(F) \subset \frac{G_a(F) + G_\mu(F)}{2} \subset G_a(F_2) \subset \left(\left(G_a(F) + \frac{\tau}{2}\right) \cap G_\mu(F)\right). \tag{6}$$

Proof of Lemma 1. To prove the first inclusion we observe that, by definition

$$G_a(F)+G_\mu(F)=\{\sigma_a+\sigma_\mu\colon \sigma_a\in G_a(F),\sigma_\mu\in G_\mu(F)\};$$

$$2G_a(F)=\{2\sigma_a\colon \sigma_a\in G_a(F)\}\text{ and }G_a(F)+G_a(F)=\{\sigma_a^*+\sigma_a^{**}\colon \sigma_a^*,\sigma_a^{**}\in G_a(F)\}.$$

So, we obtain

$$2G_a(F)\subset G_a(F)+G_a(F)\subset G_a(F)+G_\mu(F)$$

and that it was necessary to prove.

Now we prove the second inclusion. Let $\sigma\in\dfrac{G_a(F)+G_\mu(F)}{2}$. We choose $\sigma_a\in\partial G_a(F)$ and $\sigma_\mu\in\partial G_\mu(F)$ such that

$$\sigma_i<\frac{\sigma_{ai}+\sigma_{\mu i}}{2}\ (i\in\{1,...,p\}).$$

Denote $\varepsilon_i{:=}\dfrac{\sigma_{ai}+\sigma_{\mu i}-2\sigma_i}{2}$ $(i\in\{1,...,p\})$. Then $\varepsilon_i>0$ and $2\sigma_i-\sigma_{\mu i}+\varepsilon_i=\sigma_{ai}-\varepsilon_i<\sigma_{ai}$ $(i\in\{1,...,p\})$. So, the series (1) converges absolutely at the point $(2\sigma-\sigma_\mu+\varepsilon)$, and

$$\sum_{\|n\|=0}^{+\infty}|a_n|\exp\{\langle 2\sigma-\sigma_\mu+\varepsilon,\lambda_n\rangle\}<+\infty.$$

Moreover,

$$|a_n|\exp\{\langle\sigma_\mu-\varepsilon,\lambda_n\rangle\}\le 1,\ \ \|n\|\ge k_0.$$

Using the last two inequalities, we obtain

$$\sum_{\|n\|=k_0}^{\infty}|a_n|^2e^{2\langle\sigma,\lambda_n\rangle}=\sum_{\|n\|=k_0}^{\infty}|a_n|\exp\{\langle\sigma_\mu-\varepsilon,\lambda_n\rangle\}\cdot|a_n|\exp\{\langle 2\sigma-\sigma_\mu+\varepsilon,\lambda_n\rangle\}\le$$

$$\le\sum_{\|n\|=k_0}^{\infty}|a_n|\exp\{\langle 2\sigma-\sigma_\mu+\varepsilon,\lambda_n\rangle\}<\infty,$$

so, (5) converges absolutely at the point σ, i.e. $\sigma\in G_a(F_2)$, and this is what we need to prove.

Now we prove the last inclusion, using a similar method as in the case of $p=2$ ([6]).

Let $\gamma=(\gamma_1,...,\gamma_p)\in G_a(F_2)$, i.e. the series (5) converges absolutely at the point γ. It is easy to see that $\gamma\in G_\mu(F)$. So we need to prove that $\gamma\in\overline{G_a(F)}+\frac{\tau}{2}$. Since $G_a(F_2),G_a(F)$ are open, we immediately obtain that $G_a(F_2)\subset G_a(F)+\frac{\tau}{2}$.

Assume on the contrary, that $\gamma\notin\overline{G_a(F)}+\frac{\tau}{2}$. Then for $\gamma=(\gamma_1,...,\gamma_p)$ there exists a point $\sigma_a=(\sigma_{a1},...,\sigma_{ap})\in\partial G_a(F)$ such that $\sigma_a+\frac{\tau}{2}=(\sigma_{a1}+\frac{\tau_1}{2},...,\sigma_{ap}+\frac{\tau_p}{2})\in\partial(G_a(F)+\frac{\tau}{2})$ and $\sigma_{ai}+\frac{\tau_i}{2}<\gamma_i,\ \forall i\in\{1,...,p\}$. Let

$$\delta=(\delta_1,\dots,\delta_p),\ \delta_i{:=}\frac{1}{3}\Big(\gamma_i-\sigma_{ai}-\frac{\tau_i}{2}\Big)>0,\ i\in\{1,...,p\}.$$

It is easy to see, that $\sigma_a+\delta\notin\overline{G_a(F)}$. Condition (2) implies that $\sum_{i=1}^p\ln n_i\le\langle\tau+\delta,\lambda_n\rangle$ $(\|n\|\ge n_0)$. Then

$$\sum_{\|n\|=n_0}^{\infty}\exp\{-\langle\tau+2\delta,\lambda_n\rangle\}=\sum_{\|n\|=n_0}^{\infty}\exp\Big\{-\Big\langle\frac{\tau+4\delta}{\tau+\delta}(\tau+\delta),\lambda_n\Big\rangle\Big\}<$$

$$< \sum_{\|n\|=n_0}^{\infty} (n_1 \cdot \ldots \cdot n_p)^{-A} < \infty, \tag{7}$$

where $A = \min\{\frac{\tau_i+4\delta_i}{\tau_i+\delta_i} : i \in \{1, ..., p\}\} > 1$. Furthermore,

$$\sum_{\|n\|=0}^{\infty} |a_n|^2 \exp\{2\langle\gamma, \lambda_n\rangle\} < \infty, \tag{8}$$

because the series in (5) absolutely converges at the point γ. Thus, considering (7), (8) and Cauchy-Bunyakovsky's inequality, we obtain

$$\sum_{\|n\|=0}^{\infty} |a_n| \exp\{\langle\sigma_a + \delta, \lambda_n\rangle\} = \sum_{\|n\|=0}^{\infty} |a_n| \exp\{\langle\gamma, \lambda_n\rangle\} \exp\left\{-\left\langle\frac{\tau + 4\delta}{2}, \lambda_n\right\rangle\right\} \le$$

$$\le \left(\sum_{\|n\|=0}^{\infty} |a_n|^2 \exp\{2\langle\gamma, \lambda_n\rangle\}\right)^{\frac{1}{2}} \times \left(\sum_{\|n\|=0}^{\infty} \exp\{-\langle\tau + 4\delta, \lambda_n\rangle\}\right)^{\frac{1}{2}} < \infty.$$

So, the series in (1) converges at the point $(\sigma_a + \delta)$, but $(\sigma_a + \delta) \notin \overline{G_a(F)}$. We get a contradiction.

The lemma is proved. □

We need also the following theorem.

Theorem (J.-P. Kahane [7]). *Let (θ_n) be a sequence of independent random variables and*

$$\theta_n'(\omega) = \begin{cases} \theta_n(\omega), & |\theta_n(\omega)| \le 1, \\ \dfrac{\theta_n(\omega)}{|\theta_n(\omega)|}, & |\theta_n(\omega)| > 1. \end{cases}$$

The series $\sum\limits_{\|n\|=0}^{\infty} \theta_n$ converges a.s. if and only if the series $\sum\limits_{\|n\|=0}^{\infty} \mathbf{D}\theta_n'$ and $\sum\limits_{\|n\|=0}^{\infty} \mathbf{M}\theta_n'$ converge.

Now let us prove an auxiliary lemma.

Lemma 2. *Let $(\forall\ n \in \mathbb{Z}_+)$: $0 < c_1 \le |\eta_n(\omega)| \le c_2 < +\infty$ (a.s.). If $s \in G_3(F_\omega)$ (a.s.), then $\sum\limits_{\|n\|=0}^{\infty} a_n e^{\langle s,\lambda_n\rangle} \mathbf{M}\eta_n$ converges.*

Proof of Lemma 2. Let $s \in G_3(F_\omega)$ (a.s.) $(\omega \in B,\ P(B) = 1)$. Since $|\eta_n| \ge c_1 > 0$ (a.s.), one has that $s \in G_\mu(F)$, and

$$(\exists k_0)\,(\forall n, \|n\| \ge k_0) : |a_n e^{\langle s,\lambda_n\rangle}| < \varepsilon.$$

We denote $A_n = \{\omega\colon |\eta_n a_n e^{\langle s,\lambda_n\rangle}| \le 1\}$. We take $\varepsilon = \frac{1}{c_2}$. Then $|a_n e^{\langle s,\lambda_n\rangle}| < \frac{1}{c_2}$, as $\|n\| \ge k_0$ so

$$|\eta_n a_n e^{\langle s,\lambda_n\rangle}| \le c_2 |a_n e^{\langle s,\lambda_n\rangle}| \le 1 \text{ (a.s.)}.$$

Then as $\|n\| \ge k_0$: $P(A_n) = 1,\ P(\overline{A_n}) = 0$.

We denote $\theta_n = \eta_n a_n e^{\langle s,\lambda_n\rangle}$ and consider

$$\mathbf{M}\theta_n' = a_n e^{\langle s,\lambda_n\rangle} \int_{A_n} \eta_n P(d\omega) + \frac{a_n e^{\langle s,\lambda_n\rangle}}{|a_n e^{\langle s,\lambda_n\rangle}|} \int_{\overline{A_n}} \frac{\eta_n}{|\eta_n|} P(d\omega) =$$

$$= a_n e^{\langle s,\lambda_n\rangle} \int_{\Omega} \eta_n P(d\omega) = a_n e^{\langle s,\lambda_n\rangle} \mathbf{M}\eta_n.$$

Since $s \in G_3(F_\omega)$ (a.s.), the series $\sum_{\|n\|=0}^{\infty} \mathbf{M}\theta_n'$ converges by Kahane's theorem and the series $\sum_{\|n\|=0}^{\infty} a_n e^{\langle s,\lambda_n\rangle} \mathbf{M}\eta_n$ converges as well. □

Now we prove Theorem 1.

Proof of Theorem 1. We first prove, that $G_3(F_\omega) = G_a(F_2)$ (a.s.). Let $s \in G_3(F_\omega)$ (a.s.). Then, using the fact $\mathbf{M}\eta_n = 0$ and Lemma 2 we obtain that $\mathbf{M}\theta_n' = 0$ and

$$\mathbf{D}\theta_n' = \mathbf{M}|\theta_n'|^2 = |a_n e^{\langle s,\lambda_n\rangle}|^2 \mathbf{M}|\eta_n|^2 \geq c_1^2 |a_n e^{\langle s,\lambda_n\rangle}|^2.$$

By Kahane's theorem, we have $\sum_{\|n\|=0}^{\infty} \mathbf{D}\theta_n' < +\infty$ and

$$\sum_{\|n\|=0}^{\infty} |a_n|^2 e^{2\langle s,\lambda_n\rangle} \leq \frac{1}{c_1} \sum_{\|n\|=0}^{\infty} \mathbf{D}\theta_n' < +\infty,$$

so $s \in G_a(F_2)$.

Now let $s \in G_a(F_2)$. Then $s \in G_\mu(F_2) = G_\mu(F)$. We get as well as in Lemma 2 that $P(A_n) = 1$ and $\mathbf{M}\theta_n' = 0$ for $\|n\| \geq k_0$. Consider

$$\mathbf{D}\theta_n' = \mathbf{M}|\theta_n'|^2 = |a_n e^{\langle s,\lambda_n\rangle}|^2 \mathbf{M}|\eta_n|^2 \leq c_2^2 |a_n|^2 |e^{2\langle s,\lambda_n\rangle}|.$$

Since $s \in G_a(F_2)$, we have that

$$\sum_{\|n\|=0}^{\infty} \mathbf{D}\theta_n' < +\infty,$$

and $s \in G_3(F_\omega)$ (a.s.) by Kahane's theorem. Thus, $s \in G_3(F_\omega)$ (a.s.) if and only if $s \in G_a(F_2)$. Note that

$$G_\mu(F) = G_\mu(F_\omega) \text{ (a.s.) and } G_a(F) = G_a(F_\omega) \text{ (a.s.)}.$$

as soon as $(\forall\, n \in \mathbb{Z}_+)\colon\ 0 < c_1 \leq |\eta_n(\omega)| \leq c_2 < +\infty$ (a.s.). Finally we get

$$G_3(F_\omega) = G_a(F_2) \subset \Big(\Big(G_a(F) + \frac{\tau}{2}\Big) \cap G_\mu(F)\Big) = \Big(\Big(G_a(F_\omega) + \frac{\tau}{2}\Big) \cap G_\mu(F_\omega)\Big) \text{ (a.s.)}.$$

The theorem is proved. □

Consider a random Dirichlet series of the form

$$\widetilde{F}_\omega(s) = \sum_{\|n\|=0}^{\infty} \xi_n(\omega) \exp\{\langle s, \lambda_n\rangle\}, \tag{9}$$

where (ξ_n) be a sequence of random variables in a probability space $(\Omega, \mathcal{A}, P)$.

Corollary 1. *Let (ξ_n) be a sequence of independent symmetric random variables in a probability space $(\Omega, \mathcal{A}, P)$. Then for the random Dirichlet series defined by (9)*

$$G_a(\widetilde{F}_\omega) \subset \frac{G_a(\widetilde{F}_\omega) + G_\mu(\widetilde{F}_\omega)}{2} \subset G_з(\widetilde{F}_\omega) \subset \Big(\Big(G_a(\widetilde{F}_\omega) + \frac{\tau}{2}\Big) \cap G_\mu(\widetilde{F}_\omega)\Big) \quad \text{(a.s.)} \tag{10}$$

holds.

Proof of Corollary 1. Let (ε_n) be a Rademacher sequence in the Steinhaus probability space $(\Omega', \mathcal{A}', P')$. Consider the series (9) and the series

$$F_{(\omega',\ \omega)}(s) = \sum_{\|n\|=0}^{\infty} \varepsilon_n \xi_n \exp\{\langle s, \lambda_n \rangle\}$$

in the probability space $(\Omega' \times \Omega, \mathcal{A}' \times \mathcal{A}, P' \times P)$. Note that $G_\mu(F_{(\omega',\ \omega)}) = G_\mu(\widetilde{F}_\omega)$ and $G_a(F_{(\omega',\ \omega)}) = G_a(\widetilde{F}_\omega)$ (a.s.) by Fubini's theorem, and $G_з(F_{(\omega',\ \omega)}) = G_з(\widetilde{F}_\omega)$ (a.s.) by the reduction principle ([2, p.20]). For every fixed $\omega \in \Omega$ and for almost all $\omega' \in \Omega'$, by Lemma 1,

$$G_a(F_{(\omega',\ \omega)}) \subset \frac{G_a(F_{(\omega',\ \omega)}) + G_\mu(F_{(\omega',\ \omega)})}{2} \subset G_з(F_{(\omega',\ \omega)}) \subset$$
$$\subset \Big(\Big(G_a(F_{(\omega',\ \omega)}) + \frac{\tau}{2}\Big) \cap G_\mu(F_{(\omega',\ \omega)})\Big) \tag{11}$$

and this implies (10). □

Corollary 2. *Let (ξ_n) be a sequence of independent random variables in a probability space $(\Omega, \mathcal{A}, P)$. Then for the random Dirichlet series (9), either inclusions (4) are valid or there exists a usual Dirichlet series F of the form (1) with $G_з(F) = G_з(\widetilde{F}_\omega)$, $G_a(F) = G_a(\widetilde{F}_\omega), G_\mu(F) = G_\mu(\widetilde{F}_\omega)$ (a.s.) such that $F^*_\omega = \widetilde{F} - F$ (a.s.) and*

$$G_a(F^*_\omega) \subset \frac{G_a(F^*_\omega) + G_\mu(F^*_\omega)}{2} \subset G_з(F^*_\omega) \subset \Big(\Big(G_a(F^*_\omega) + \frac{\tau}{2}\Big) \cap G_\mu(F^*_\omega)\Big) \ \text{(a.s.)}$$

holds.

Proof of Corollary 2. Let $\eta_n(\omega',\ \omega) = \xi_n(\omega) - \xi_n(\omega')$ for every $\|n\| \geq 0$ and $(\omega',\ \omega) \in \Omega \times \Omega$. Then (η_n) is a sequence of independent symmetric random variables in the probability space $(\Omega \times \Omega, \mathcal{A} \times \mathcal{A}, P \times P)$. Now we consider the series

$$F_{(\omega',\ \omega)}(s) = F_\omega(s) - F_{\omega'}(s) = \sum_{\|n\|=0}^{\infty} (\xi_n(\omega) - \xi_n(\omega')) \exp\{\langle s, \lambda_n \rangle\}.$$

As well as in Corollary 1 inclusions (11) hold (a.s.) for this series in $\Omega \times \Omega$. By the converse Fubini theorem, there exists a fixed point $\omega' \in \Omega'$ such that for almost all $\omega \in \Omega$ and for the series $F(s) = F_{\omega'}(s)$

$$G_з(F) = G_з(\widetilde{F}_\omega),\ G_a(F) = G_a(\widetilde{F}_\omega),\ G_\mu(F) = G_\mu(\widetilde{F}_\omega)$$

and (11) are valid. This completes the proof of Corollary 2. □

We remark that the inclusions in Lemma 1 and Theorem 1 are exact. Consider the series

$$F(z) = \sum_{\|n\|=0}^{+\infty} (-1)^{\|n\|} e^{z_1 \ln(n_1+1)+\ldots+z_p \ln(n_p+1)}. \tag{1'}$$

We can easy see that the domain of the absolutely convergence is the following set

$$G_a(F) = \{\sigma \in \mathbb{R}^p \colon (\forall j \in \{1, ..., p\})\ [\sigma_j < -1]\},$$

and the domain of the existence of the maximal term of this series is the set

$$G_\mu(F) = \{\sigma \in \mathbb{R}^p \colon (\forall j \in \{1, ..., p\})\ [\sigma_j < 0]\},$$

Let the system τ be as follows: $\tau = (1, ..., 1) \in \mathbb{R}^p$. Consider the series

$$F_2(s) = \sum_{\|n\|=0}^{+\infty} e^{2(z_1 \ln(n_1+1)+\ldots+z_p \ln(n_p+1))}. \tag{5'}$$

We see that the domain of absolutely convergence is equal to

$$G_a(F_2) = \{\sigma \in \mathbb{R}^p \colon (\forall j \in \{1, ..., p\})\ [\sigma_j < -1/2]\}.$$

Thus,

$$\frac{G_a(F) + G_\mu(F)}{2} = G_a(F_2) = \left(\left(G_a(F) + \frac{\tau}{2}\right) \cap G_\mu(F)\right),$$

and from Theorem 1 follows that (a.s.)

$$G_a(F_\omega) = \frac{G_a(F_\omega) + G_\mu(F_\omega)}{2} = G_з(F_\omega) = G_a(F_2) = \left(G_a(F_\omega) + \frac{\tau}{2}\right) \cap G_\mu(F_\omega).$$

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