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FLUCTUATIONS OF THE EVOLUTION SYSTEM WITH MARKOV IMPULSIVE PERTURBATIONS

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In this paper we discuss asymptotic properties of evolution system fluctuations with Markov impulsive perturbations. Under the balance conditions, normalized fluctuation converges weakly to the diffusion process with Wiener perturbations. To achieve this we construct a generator for the limit process solving the singular perturbation problem for the original system and taking into account the asymptotic normality of impulsive perturbation.

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Исследованы асимптотические свойства флуктуаций эволюционных систем с марковскими импульсными возмущениями. При выполнении условий баланса установлена слабая сходимость нормированной флуктуации к диффузионному процессу с винеровскими возмущениями. Для решения проблемы сингулярного возмущения для исходной системы с учетом асимптотической нормальности импульсных возмущений построен генератор предельного процесса.

Introduction. A phase averaging algorithm for random evolutions is based on approximate equality of initial and averaged evolutional systems ([1]). Thus there arises a problem of studying such evolution fluctuations. In particular, in [2] the behavior of the diffusive evolutional system fluctuations with the Markov switching was investigated, where the function of speed has a singular perturbative element with a small parameter. Fluctuations behavior is very important during establishment of convergence speed of diffusive approximation of the evolutional stochastic systems in the scheme of averaging and with diffusive perturbation in the scheme of series ([3]), and also at establishment of asymptotic normality of stochastic approximation procedure ([4]).

1. Problem statement and notations. Consider the evolution system

$$du^\varepsilon(t) = C(u^\varepsilon(t), x(t/\varepsilon^4))dt + \varepsilon d\eta^\varepsilon(t), \quad u^\varepsilon(t) \in R, \tag{1}$$

where a Markov process $x(t), t \geq 0$, in the standard phase space $(X, \boldsymbol{X})$ is defined by the generator

$$Q\varphi(x) = q(x) \int_X P(x, dy) \left[\varphi(y) - \varphi(x)\right], \quad \varphi \in \mathbf{B}(X),$$

here $\mathbf{B}(X)$ is the Banach space of real bounded functions with supremum-norm $\|\varphi\| = \max_{x\in X} |\varphi(x)|$. A uniformly ergodic embedded Markov chain $x_n = x(\tau_n)$, $n \geq 0$, with stationary distribution $\rho(B), B \in \boldsymbol{X}$, is defined by the stochastic kernel $P(x, B), x \in X, B \in \boldsymbol{X}$.

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A stationary distribution $\pi(B), B \in \mathbf{X}$, of the Markov process $x(t), t \geq 0$, is defined by the representation

$$\pi(dx)q(x) = q\rho(dx),\ q = \int_X \pi(dx)q(x).$$

Denote by R_0 potential operator of the generator $Q([1])$: $R_0 = \Pi - (\Pi + Q)^{-1}$, where $\Pi\varphi(x) = \int_X \pi(dy)\varphi(y)$ is the projector on the zeros subspace $N_Q = \{\varphi \colon Q\varphi = 0\}$ of the operator Q.

An impulsive perturbation process (IPP) $\eta^\varepsilon(t), t \geq 0$, is defined by the representation

$$\eta^\varepsilon(t) = \int_0^t \eta^\varepsilon(ds; x(t/\varepsilon^4)); \tag{2}$$

where the family of independed increment processes $\eta^\varepsilon(t,x), t \geq 0, x \in X$, is defined by the generators

$$\Gamma^\varepsilon(x)\varphi(w) = \varepsilon^{-4} \int_R [\varphi(w + \varepsilon^2 v) - \varphi(w)]\Gamma(dv; x), x \in X. \tag{3}$$

Let also the following balance conditions take place:

$$\Pi b_1(x) = \int_X \pi(dx)b_1(x) = 0; \quad b_1(x) = \int_R v\Gamma(dv; x). \tag{4}$$

2. Impulse process characteristics.

Theorem 1. *Under the balance condition (4) the weak convergence takes place $\eta^\varepsilon(t) \to \eta^0(t), \varepsilon \to 0$. The limit process $\eta^0(t)$ is defined by the generator $\Gamma\varphi(w) = \frac{1}{2}B\varphi''(w)$, where*

$$B = B_1 + B_2;\ B_1 = 2\Pi b_1(x)R_0 b_1(x) = 2\int_X \pi(dx)b_1(x)R_0 b_1(x);$$

$$B_2 = \Pi b_2(x) = \int_X \pi(dx)b_2(x); \quad b_2(x) = \int_R v^2\Gamma(dv; x).$$

The limit process $\eta^0(t)$ can be written in the exact form $\eta^0(t) = \sigma W(t)$, where $\sigma^2 = B$ and $W(t)$ is the Wiener process.

Lemma 1. *In an asymptotic representation of the family of independent increment processes $\eta^\varepsilon(t,x)$, $t \geq 0$, $x \in X$, the generator on test functions $\varphi(w) \in C^3(R)$ has the form*

$$\Gamma^\varepsilon(x)\varphi(w) = \varepsilon^{-2}\Gamma_1(x)\varphi(w) + \Gamma_2(x)\varphi(w) + \gamma^\varepsilon(x)\varphi(w), \tag{5}$$

where

$$\Gamma_1(x)\varphi(w) = b_1(x)\varphi'(w); \Gamma_2(x)\varphi(w) = \tfrac{1}{2}b_2(x)\varphi''(w),$$

and the remainder term has the form $\|\gamma^\varepsilon(x)\varphi(w)\| \to 0$ while $\varepsilon \to 0$; $\varphi(w) \in C^3(R)$.

Proof. Let us transform the generator (3) using the Taylor decomposition of the function $\varphi(w)$:

$$\Gamma^\varepsilon(x)\varphi(w) = \varepsilon^{-4}\int_R [\varphi(w + \varepsilon^2 v) - \varphi(w)]\Gamma(dv; x) =$$

$$= \varepsilon^{-4}\int_R [\varphi(w + \varepsilon^2 v) - \varphi(w) - \varepsilon^2 v\varphi'(w) - \tfrac{1}{2}\varepsilon^4 v^2\varphi''(w)]\Gamma(dv; x) +$$

$$+\varepsilon^{-2}b_1(x)\varphi'(w) + \tfrac{1}{2}b_2(x)\varphi''(w) = \varepsilon^{-2}b_1(x)\varphi'(w) + \tfrac{1}{2}b_2(x)\varphi''(w) + \gamma^\varepsilon(x)\varphi(w).$$

Taking into account that $\gamma^\varepsilon(x)\varphi(w) = O(\varepsilon^2)$, $\varphi(w) \in C^3(R)$ we obtain representation (5). □

Lemma 2. *A generator of the two-component Markov process $\eta^\varepsilon(t)$, $x(t/\varepsilon^4)$, $t \geq 0$, has the form*

$$\widehat{\Gamma}^\varepsilon(x)\varphi(w,x) = \varepsilon^{-4}Q\varphi(w,x) + \varepsilon^{-2}\Gamma_1(x)\varphi(w,x) + \Gamma_2(x)\varphi(w,x) + \gamma^\varepsilon(x)\varphi(w,x), \qquad (6)$$

where the operators $\Gamma_1(x), \Gamma_2(x)$ are defined in Lemma 1, and the remainder term is such that $\|\gamma^\varepsilon(x)\varphi(w,x)\| \to 0$ as $\varepsilon \to 0$, $\varphi(w,\cdot) \in C^3(R)$.

Proof. The proof is constructed using the generator definition of generator and the processes $\eta^\varepsilon(t)$ and representation $x(t/\varepsilon^4)$ of generator. □

The truncated generator has the form

$$\Gamma_0^\varepsilon(x)\varphi(w) = \varepsilon^{-4}Q\varphi(w,x) + \varepsilon^{-2}\Gamma_1(x)\varphi(w,x) + \Gamma_2(x)\varphi(w,x). \qquad (7)$$

Lemma 3. *Under the balance condition* (4)*, the singular perturbation problem for the truncated operator (7) on the test functions $\varphi^\varepsilon(w,x) = \varphi(w) + \varepsilon^2\varphi_2(w,x) + \varepsilon^4\varphi_0(w,x)$ has the solution of the form*

$$\Gamma_0^\varepsilon(x)\varphi^\varepsilon(w,x) = \Gamma\varphi(w) + \varepsilon^2\theta_\eta^\varepsilon(x)\varphi(w), \qquad (8)$$

where the remainder term $\theta_\eta^\varepsilon(x)\varphi(w)$ is such that $|\theta_\eta^\varepsilon(x)\varphi(w)| \leq C < \infty$, $\varphi(w) \in C^3(R)$.

The limit generator Γ is defined by the formula

$$\Gamma\Pi = \Pi\Gamma_2(x)\Pi + \Pi\Gamma_1(x)R_0\Gamma_1(x)\Pi. \qquad (9)$$

Proof. Let us collect the similar terms with respect to ε in order to prove equality (8)

$$\Gamma_0^\varepsilon(x)\varphi^\varepsilon(w,x) = \varepsilon^{-4}Q\varphi(w) + \varepsilon^{-2}[Q\varphi_2(w,x) + \Gamma_1(x)\varphi(w)] + [Q\varphi_0(w,x) + \Gamma_1(x)\varphi_2(w,x)+ \\ +\Gamma_2(x)\varphi(w)] + \varepsilon^2[\Gamma_1(x)\varphi_0(w,x) + \Gamma_2(x)\varphi_2(w,x)] + \varepsilon^4\Gamma_2(x)\varphi_0(w,x).$$

Since $\varphi(w)$ does not depend on x, we see that $Q\varphi(w) = 0, \Leftrightarrow \varphi(w) \in N_Q$. The following equation can be solved under the balance condition: (4) $Q\varphi_2(w,x) + \Gamma_1(x)\varphi(w) = 0$. That is why

$$\varphi_2(w,x) = R_0\Gamma_1(x)\varphi(w). \qquad (10)$$

Using (10) we can bring the equation

$$Q\varphi_0(w,x) + \Gamma_1(x)\varphi_2(w,x) + \Gamma_2(x)\varphi(w) = \Gamma\varphi(w)$$

to the form

$$Q\varphi_0(w,x) + \Gamma_1(x)R_0\Gamma_1(x)\varphi(w) + \Gamma_2(x)\varphi(w) = \Gamma\varphi(w).$$

We can obtain the limit process Γ in the form (9) using the solution condition of the last equation. Then

$$\varphi_0(w,x) = R_0[\Gamma_1(x)R_0\Gamma_1(x) + \Gamma_2(x) - \Gamma]\varphi(w), \qquad (11)$$

and taking into account that $R_0\Gamma = 0$, we obtain $\varphi_0(w,x) = R_0[\Gamma_1(x)R_0\Gamma_1(x) + \Gamma_2(x)]\varphi(w)$. Using (10) and (11) we can bring all other terms to the form

$$\varepsilon^2[\Gamma_1(x)\varphi_0(w,x) + \Gamma_2(x)\varphi_2(w,x)] + \varepsilon^4\Gamma_2(x)\varphi_0(w,x) = \varepsilon^2[\Gamma_1(x)R_0[\Gamma_1(x)R_0\Gamma_1(x) + \Gamma_2(x)]+ \\ +\Gamma_2(x)R_0\Gamma_1(x)]\varphi(w) + \varepsilon^4\Gamma_2(x)R_0[\Gamma_1(x)R_0\Gamma_1(x) + \Gamma_2(x)]\varphi(w) = \varepsilon^2\theta_\eta^\varepsilon(x)\varphi(w).$$

We can prove that θ_η^ε is bounded on the functions $\varphi(w) \in C^3(R)$ using the form of operators $\Gamma_1(x)$, $\Gamma_2(x)$ and R_0. □

Proof. The proof uses Lemma 3 and Theorem 2.1 in [6]. □

3. Fluctuation of evolution system. It can be shown that the solution of the evolution system (1) converges to the equilibrium point $u_0 = 0$ of the average system ([6])

$$\frac{d\widehat{u}(t)}{dt} = \widehat{C}(\widehat{u}(t)), \quad \widehat{C}(u) = \int_X \pi(dx) C(u, x). \tag{12}$$

Such an equilibrium point exists under the balance condition

$$\Pi C(0, x) = \int_X \pi(dx) C(0, x) = 0. \tag{13}$$

Let us consider normed fluctuations of the initial system:

$$v^\varepsilon(t) = \varepsilon^{-1} \left[u^\varepsilon(t) - \varepsilon \eta^\varepsilon(t) \right]. \tag{14}$$

Theorem 2. *Under the balance conditions (4), (13) and convergence conditions of the initial system (1), weak convergence takes place* $(v^\varepsilon(t), \eta^\varepsilon(t)) \to (\zeta(t), \sigma W(t))$, $t > 0$, $\varepsilon \to 0$. *The limit process* $(\zeta(t), \sigma W(t))$ *is defined by the generator* $L\varphi(v, w) = c(v + w)\varphi'_v(v, w) + \frac{1}{2} B \varphi''_w(v, w)$, *where*

$$c = \int_X \pi(dx) C'(0, x);$$

and also $\sigma^2 = B$; $W(t)$ *denotes the Wiener process.*

The limit process $\zeta(t)$ satisfies the stochastic differential equation

$$d\zeta(t) = c \left[\zeta(t) + \sigma W(t) \right] dt. \tag{15}$$

Lemma 4. *Normed fluctuation (14) satisfies the differential equation*

$$dv^\varepsilon(t) = \mathbf{C}^\varepsilon(\varepsilon z, x(t/\varepsilon^4)) dt, \tag{16}$$

where $\mathbf{C}^\varepsilon(v, x) = \varepsilon^{-1} C(v, x)$, $z = v + w$, $v = v^\varepsilon(t)$, $w = \eta^\varepsilon(t)$.

Proof. To obtain (16) we need to differentiate (14) using (1). □

Remark 1. The normed fluctuation $v^\varepsilon(t)$ produces associated determinated evolutions $V_{w,x}(t)$: $dV_{w,x}(t) = \mathbf{C}^\varepsilon(\varepsilon(V_{w,x}(t) + w), x) dt$ with the semigroup generators

$$\mathbf{C}^{\varepsilon,V}(w, x)\varphi(v) = \varepsilon^{-1} C(w, x) \varphi'(v). \tag{17}$$

Lemma 5. *A generator of the three component Markov process*

$$v^\varepsilon(t), \ \eta^\varepsilon(t), \ x(t/\varepsilon^4) = x_t^\varepsilon, \ t \geq 0 \tag{18}$$

has the form

$$L^\varepsilon \varphi(v, w, x) = \left[\varepsilon^{-4} Q + \Gamma^\varepsilon(x) + \mathbf{C}^\varepsilon(\varepsilon z, x) \right] \varphi(v, w, x), \tag{19}$$

Proof. Let us denote $x(t/\varepsilon^4) = x_t$, $v^\varepsilon(t) = v_t$, $\eta^\varepsilon(t) = w_t$ and then calculate the conditional mathematical expectation:

$$E[\varphi(v^\varepsilon(t+\Delta), \eta^\varepsilon(t+\Delta), x((t+\Delta)/\varepsilon^4)) - \varphi(v^\varepsilon(t), \eta^\varepsilon(t), x(t/\varepsilon^4))|v^\varepsilon(t) = v, \eta^\varepsilon(t) = w,$$
$$x(t/\varepsilon^4) = x] = E[\varphi(v_{t+\Delta}, w_{t+\Delta}, x_{t+\Delta}) - \varphi(v, w, x)] = E[\varphi(v_{t+\Delta}, w_{t+\Delta}, x_{t+\Delta}) -$$
$$-\varphi(v, w_{t+\Delta}, x_{t+\Delta})] + E[\varphi(v, w_{t+\Delta}, x_{t+\Delta}) - \varphi(v, w, x)].$$

Then we can use the definition of the generator of Markov process (18)

$$L^\varepsilon \varphi(v, w, x) = \lim_{\Delta \to 0} \frac{1}{\Delta} E[\varphi(v^\varepsilon(t+\Delta), \eta^\varepsilon(t+\Delta), x((t+\Delta)/\varepsilon^4)) - \varphi(v^\varepsilon(t), \eta^\varepsilon(t), x(t/\varepsilon^4))|$$
$$v^\varepsilon(t) = v, \eta^\varepsilon(t) = w, x(t/\varepsilon^4) = x] = \lim_{\Delta \to 0} \frac{1}{\Delta} E[\varphi(v_{t+\Delta}, w_{t+\Delta}, x_{t+\Delta}) - \varphi(v, w, x))] =$$
$$= \lim_{\Delta \to 0} \frac{1}{\Delta} E[\varphi(v_{t+\Delta}, w_{t+\Delta}, x_{t+\Delta}) - \varphi(v, w_{t+\Delta}, x_{t+\Delta})] +$$
$$+ \lim_{\Delta \to 0} \frac{1}{\Delta} E[\varphi(v, w_{t+\Delta}, x_{t+\Delta}) - \varphi(v, w, x)]. \tag{20}$$

Since

$$\lim_{\Delta \to 0} \frac{1}{\Delta} E[\varphi(v_{t+\Delta}, w_{t+\Delta}, x_{t+\Delta}) - \varphi(v, w_{t+\Delta}, x_{t+\Delta})] = \mathbf{C}^{\varepsilon, V}(\varepsilon z, x)\varphi(v, w, x),$$

$$\lim_{\Delta \to 0} \frac{1}{\Delta} E[\varphi(v, w_{t+\Delta}, x_{t+\Delta}) - \varphi(v, w, x)] = \varepsilon^{-4} Q\varphi(v, w, x) + \Gamma^\varepsilon(x)\varphi(v, w, x),$$

from (20) we obtain (19). □

Lemma 6. *An asymptotic decomposition of the generator L^ε on the test functions $\varphi(v, w, \cdot) \in C^{2,3}(R \times R)$ has the following form:*

$$L^\varepsilon \varphi(v, w, x) = \left[\varepsilon^{-4} Q + \varepsilon^{-2}\Gamma_1(x) + \varepsilon^{-1}\mathbf{C}(x) + \mathbf{C}_1(x) + \Gamma_2(x) + \theta_L^\varepsilon(x)\right]\varphi(v, w, x), \tag{21}$$

where

$$\mathbf{C}(x)\varphi(v, w, x) = C(0, x)\varphi'_v(v, w, x);\ \mathbf{C}_1(x)\varphi(v, w, x) = zC'(0, x)\varphi'_v(v, w, x),\ z = v + w, \tag{22}$$

and the remainder term is such that $\|\theta_D^\varepsilon(x)\varphi(v, w, x)\| \to 0$ while $\varepsilon \to 0$, $\varphi(v, w, \cdot) \in C^{2,3}(R \times R)$.

Proof. We obtain (21) after Taylor decomposition of the function $C(u, x)$ on the first variable for the generator (17). □

The truncated operator has the form

$$L_0^\varepsilon = \varepsilon^{-4} Q + \varepsilon^{-2}\Gamma_1(x) + \varepsilon^{-1}\mathbf{C}(x) + \mathbf{C}_1(x) + \Gamma_2(x). \tag{23}$$

Let us solve the singular perturbation problem for the generator (23) on the test functions

$$\varphi^\varepsilon(v, w, x) = \varphi(v, w) + \varepsilon^2\varphi_2(v, w, x) + \varepsilon^3\varphi_1(v, w, x) + \varepsilon^4\varphi_0(v, w, x).$$

Lemma 7. *Under the conditions of the theorem, the singular perturbation problem for the truncated operator (23) has the solution in the form*

$$L_0^\varepsilon \varphi^\varepsilon(v,w,x) = L\varphi(v,w) + \varepsilon\theta_0^\varepsilon(x)\varphi(v,w), \tag{24}$$

where the remaining term $\theta_0^\varepsilon(x)$ *is such that* $|\theta_0^\varepsilon(x)\varphi(w)| \leq C < \infty$, $\varphi(w) \in C^3(R)$.

The limit generator L is defined by the equality

$$L\Pi = \Pi\mathbf{C}_1(x)\Pi + \Pi\Gamma_0(x)\Pi + \Pi\Gamma_1(x)R_0\Gamma_1(x)\Pi. \tag{25}$$

Proof. Let us collect the similar terms with respect to ε in order to prove equality (24):

$$\begin{gathered} L_0^\varepsilon\varphi^\varepsilon(v,w,x) = \varepsilon^{-4}Q\varphi(v,w) + \varepsilon^{-2}[Q\varphi_2(v,w,x) + \Gamma_1(x)\varphi(v,w)] + \\ +\varepsilon^{-1}[Q\varphi_1(v,w,x) + \mathbf{C}(x)\varphi(v,w)] + [Q\varphi_0(v,w,x) + \Gamma_1(x)\varphi_2(v,w,x) + \\ +(\mathbf{C}_1(x) + \Gamma_2(x))\varphi(v,w)] + \varepsilon[\Gamma_1(x)\varphi_1(v,w,x) + \mathbf{C}(x)\varphi_2(v,w,x)] + \\ +\varepsilon^2[\Gamma_1(x)\varphi_0(v,w,x) + \mathbf{C}(x)\varphi_1(v,w,x) + (\mathbf{C}_1(x) + \Gamma_2(x))\varphi_2(v,w,x)] \\ +\varepsilon^3[\mathbf{C}(x)\varphi_0(v,w,x) + (\mathbf{C}_1(x) + \Gamma_2(x))\varphi_1(v,w,x)] + \varepsilon^4[(\mathbf{C}_1(x) + \Gamma_2(x))\varphi_0(v,w,x)]. \end{gathered}$$

Since $\varphi(u,w)$ does not depend on x, we obtain $Q\varphi(v,w) = 0, \Leftrightarrow \varphi(v,w) \in N_Q$. The following equation can be solved under the balance condition (4): $Q\varphi_2(v,w,x) + \Gamma_1(x)\varphi(v,w) = 0$. That is why

$$\varphi_2(v,w,x) = R_0\Gamma_1(x)\varphi(v,w). \tag{26}$$

The following equation can be solved under the balance condition (13)

$$Q\varphi_1(v,w,x) + \mathbf{C}(x)\varphi(v,w) = 0.$$

Thus

$$\varphi_1(v,w,x) = R_0\mathbf{C}(x)\varphi(v,w). \tag{27}$$

Using (26) and (27) we can bring the equation

$$Q\varphi_0(v,w,x) + \Gamma_1(x)\varphi_2(v,w,x) + (\mathbf{C}_1(x) + \Gamma_2(x))\varphi(v,w) = L\varphi(v,w),$$

to the form

$$Q\varphi_0(v,w,x) + [\mathbf{C}_1(x) + \Gamma_2(x) + \Gamma_1(x)R_0\Gamma_1(x)]\varphi(v,w) = L\varphi(v,w).$$

We can obtain the limit process L in the form (25) using the solution condition of the last equation. Then

$$\varphi_0(v,w,x) = R_0[\mathbf{C}_1(x) + \Gamma_2(x) + \Gamma_1(x)R_0\Gamma_1(x) - L]\varphi(v,w) \tag{28}$$

and taking into account that $R_0L = 0$, we obtain

$$\varphi_0(v,w,x) = R_0[\mathbf{C}_1(x) + \Gamma_2(x) + \Gamma_1(x)R_0\Gamma_1(x)]\varphi(v,w) = \widetilde{L}\varphi(v,w).$$

Using (26)–(28), we can bring all the other terms to the form:

$$\begin{gathered} \varepsilon[\Gamma_1(x)R_0\mathbf{C}(x) + \mathbf{C}(x)R_0\Gamma_1(x)]\varphi(v,w) + \\ \varepsilon^2[\Gamma_1(x)\widetilde{L} + \mathbf{C}(x)R_0\mathbf{C}(x) + (\mathbf{C}_1(x) + \Gamma_2(x))R_0\Gamma_1(x)]\varphi(v,w) \\ \varepsilon^3[\mathbf{C}(x)\widetilde{L} + (\mathbf{C}_1(x) + \Gamma_2(x))R_0\mathbf{C}(x)]\varphi(v,w) + \varepsilon^4[(\mathbf{C}_1(x) + \Gamma_2(x))\widetilde{L}]\varphi(v,w) = \varepsilon\theta_0^\varepsilon(x)\varphi(v,w). \end{gathered}$$

We can show that the operator $\theta_0^\varepsilon(x)$ on the functions $\varphi(v,w) \in C^3(R)$ is bounded using the form of the operators $\Gamma_1(x)$, $\Gamma_2(x)$, $\mathbf{C}(x)$, $\mathbf{C}_1(x)$ and R_0. □

Proof of Theorem. Let us calculate the right part of (25) using (22). In this case, we obtain

$$L\varphi(v,w) = [\Pi\mathbf{C}_1(x) + \Pi\Gamma_2(x) + \Pi\Gamma_1(x)R_0\Gamma_1(x)]\varphi(v,w) = \int\limits_X \pi(dx)(v+w)C'(0,x)$$

$$\varphi'_v(v,w) + \frac{1}{2}\int\limits_X \pi(dx)b_2(x)\varphi''_w(v,w) + \int\limits_X \pi(dx)b_1(x)R_0b_1(x)\varphi''_w(v,w).$$

And in the final result we obtain $L\varphi(v,w) = c(v+w)\varphi'_v(v,w) + \frac{1}{2}B\varphi''_w(v,w)$. To complete the proof use Theorem 2.1 in [6]. □

4. Conclusions. Fluctuations similar to [2] of the evolution system with Markov asymptotically normal perturbations are described by the solution of the evolution equation (15), where the perturbation is defined by the Wiener process with defined dispersion. Fluctuations of the evolution system decays with growth of time under the stability conditions of the average system (12) and additional condition $c < 0$.

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