УДК 519.63 + 519.64

R. Chapko, N. Vintonyak

ON THE CONVERGENCE OF THE HYBRID METHOD FOR AN INVERSE BOUNDARY VALUE POTENTIAL PROBLEM IN A SEMI-INFINITE DOMAIN

R. Chapko, N. Vintonyak. *On the convergence of the hybrid method for an inverse boundary value potential problem in a semi-infinite domain*, Mat. Stud. **32** (2009), 45–52.

We consider a question related to the convergence analysis of the hybrid method used for an inverse potential problem in a semi-infinite region. The local convergence of this method when the data error tends to zero is proved.

Р. Хапко, Н. Винтоняк. *О сходимости гибридного метода для обратной задачи теории потенциала в полубесконечной области* // Мат. Студії. – 2009. – Т.32, №1. – C.45–52.

Рассматриваются вопросы анализа сходимости гибридного метода численного решения обратной граничной задачи теории потенциалов в частично неограниченной области. Доказана локальная сходимость метода в случае стремления к нулю ошибки входных данных.

1. Introduction. The mathematical modeling of thermal or electrostatic imaging methods in nondestructive testing and evaluation leads to models which are typically ill-posed in the sense of the instability. In practical applications we have no exact data, but only some data perturbed by noise, due to errors in the measurements, therefore algorithms developed for well-posed problems are not suitable.

In this paper we consider an inverse boundary value problem that consists in the identification of a some bounded inclusion in a semi-infinite region by measurement of the Cauchy data on the part of the boundary. We assume that $D_1 \subset \mathbb{R}^2$ be a semi-infinite region with the boundary Γ_1 and $D_0 \subset \mathbb{R}^2$ be a simply connected bounded domain with the boundary $\Gamma_0 \in C^2$ such that $\overline{D}_0 \subset D_1$. Denote $D := D_1 \setminus \overline{D}_0$. Let the bounded function $u \in C^2(D) \cap C(\overline{D})$ satisfy the Laplace equation

$$\Delta u = 0 \quad \text{in} \quad D \tag{1}$$

and the boundary value conditions

$$u = 0 \quad \text{on} \quad \Gamma_0, \tag{2}$$

$$u = f \quad \text{on} \quad \Gamma_1 \tag{3}$$

with a given bounded function f. We assume also that there exists Green's function for the domain D_1 with the Dirichlet boundary value condition on Γ_1.

2000 *Mathematics Subject Classification:* 65J22, 65N12, 65N21, 65R32.

The inverse problem consists in the following: under the assumption $f \neq 0$, to determine the boundary Γ_0 from the knowledge of the normal derivative

$$\frac{\partial u}{\partial \nu} = \psi \quad \text{on} \quad \Sigma. \tag{4}$$

Here ν is the outward unit normal on Γ_1, $\Sigma \subset \Gamma_1$ is a nonempty subset and ψ is a given function. We assume also that the function f has enough smoothness to ensure the existence of the normal derivative of the function u on Γ_1.

The inverse problem (1)–(4) is nonlinear and ill-posed in the sense that the solution is not continuously dependent on data perturbation. The results on the possibility to identify the unknown curve Γ_0 from the Cauchy data on Σ are analogous to the case of bounded domains described in [9].

The specific features of the domain lead to additional difficulties: it requires some information about approximate location of the bounded inclusion as we deal with semi-infinite regions. Thus, our inverse problem can be divided into the following parts: tracking the position of the inclusion, estimating of its size and recovering the shape of the inclusion. The first two problems can be interpreted as a problem of finding an initial guess in order to recover the shape of the boundary using some numerical methods. Some aspects of this problem are considered in [4, 11].

For the third problem of recovering of the boundary shape the various iterative algorithms with regularization technique can be used in order to obtain reliable reconstruction [9]. In this paper we concentrate our attention on the questions related to the convergence analysis of the method developed in [1, 10] that can be viewed as a hybrid of the decomposition method [8] and Newton's method. Note that this method has been well described for the inverse obstacle scattering problems in [13].

The investigation of the convergence for the hybrid method can be proved in two different directions. One approach uses the relation of the hybrid method to the corresponding minimization problem. Some steps in this context are developed in [1].

Other possible way consists in the interpretation of the hybrid method as some kind of iterative Newton's method. The convergence of regularized Newton's schemes has been investigated in [5, 6]. As pointed out in [12] all general results for the convergence of regularized Newton-type methods require a condition on the nonlinearity of corresponding operator which could not be verified for the considered problems. The purpose of our paper is to prove the convergence of hybrid method without a need for the verification of nonlinearity condition using some ideas from [12].

The plan of the paper is as follows. In Section 2 we describe a scheme of the hybrid method. Section 3 is devoted to the convergence analysis for this method, where we establish local convergence results for the hybrid method both for exact and noisy data following the technique suggested in [12].

2. Hybrid method. Assume that the boundary curve Γ_0 is starlike and has the parametrization

$$\Gamma_0 = \{x(t) = (r(t)\cos t + d_1, r(t)\sin t + d_2) \colon 0 \le t \le 2\pi\}$$

with the center $(d_1,\, d_2)$ and unknown radial function $r \in C^2[0, 2\pi]$. The questions related to finding of parameters d_1 and d_2 are considered in [4]. The inverse problem (1)–(4) defines the nonlinear operator A that maps the boundary Γ_0 to the Dirichlet data u on Γ_0 for fixed

Γ_1 and given Cauchy data f and ϕ. Thus, the inverse problem can be written in the form of the nonlinear equation

$$A(r) = 0. \tag{5}$$

For the linearization of the mapping A we first note that the Fréchet derivative of the operator A exists and has the following representation [2].

Theorem 1. *The operator $A\colon C^2[0,2\pi] \to C[0,2\pi]$ is Fréchet differentiable and its derivative is given by*

$$A'(r)q = \frac{\partial u}{\partial \theta} q$$

with the derivative in the radial direction $\partial u/\partial\theta$ and some small perturbation $q \in C^2[0,\pi]$.

Newton linearization used for (5) gives the linear equation

$$A'(r)q + A(r) = 0 \tag{6}$$

with a correction q.

The numerical scheme consists in the iteration procedure

$$r_{n+1} = r_n - \left(u|_{\Gamma_{0,n}}\right) \Big/ \left(\frac{\partial u}{\partial \theta}\Big|_{\Gamma_{0,n}}\right). \tag{7}$$

Here the operator / denotes the quotient of the corresponding functions, and $\Gamma_{0,n}$ denotes current approximation of Γ_0 on the n-th iteration with the radial function r_n. Note that u is the exact solution of the corresponding direct Dirichlet problem. We assume that the condition

$$\frac{\partial u}{\partial \theta}\Big|_{\Gamma_0} \geq \alpha > 0$$

is satisfied. Due to the continuity of the solution of the direct boundary value problem (1)–(3) with respect to variations of the domain we conclude that the radial derivatives do not vanish in a small neighborhood U of Γ_0, that is

$$\frac{\partial u}{\partial \theta}\Big|_{\Gamma} \geq 4\epsilon > 0, \tag{8}$$

where the trace of the curve Γ belongs to U.

For the numerical calculation of the function u and the radial derivative $\partial u/\partial\theta$ on $\Gamma_{0,n}$ we use the potential approach with Green's function technique. Firstly we introduce the integral operators

$$(N\varphi)(x) = \int\limits_{\Gamma_{0,n}} \varphi(y) \frac{\partial G(x,y)}{\partial \nu(x)} ds(y), x \in \Gamma_1, (S\varphi)(x) = \int\limits_{\Gamma_{0,n}} \varphi(y) G(x,y) ds(y),\ x \in \Gamma_{0,n},$$

$$(K\varphi)(x) = \int\limits_{\Gamma_{0,n}} \varphi(y) \frac{\partial G(x,y)}{\partial \nu(x)} ds(y), \quad x \in \Gamma_{0,n}$$

and the function

$$w(x) = -\int\limits_{\Gamma_1} f(y) \frac{\partial G(x,y)}{\partial \nu(y)} ds(y), \quad x \in D. \tag{9}$$

Here G is Green's function of the Dirichlet boundary value problem for Laplace's equation in the domain D_1. Due to the given flux ψ on Γ_1 we can find the unknown density φ from the integral equation of the first kind

$$(N\varphi)(x) = \psi(x) - \frac{\partial w(x)}{\partial \nu(x)}, \quad x \in \Sigma, \tag{10}$$

where w is defined in (9). Now we can calculate the track of the solution u on $\Gamma_{0,n}$ by the representation

$$u(x) = (S\varphi)(x) + w(x), \quad x \in \Gamma_{0,n}. \tag{11}$$

For the radial derivative the following relation holds

$$\frac{\partial u}{\partial \theta} = \langle \theta, \nu \rangle \frac{\partial u}{\partial \nu} + \langle \theta, \vartheta \rangle \frac{\partial u}{\partial \vartheta}, \quad \text{on } \Gamma_{0,n},$$

where ϑ is the unit tangential vector on $\Gamma_{0,n}$, and by $\langle \cdot, \cdot \rangle$ we denote the scalar product in $\mathbb{R}^2$.

Therefore we have

$$\frac{\partial u}{\partial \theta} = L\varphi + \langle \theta, \nu \rangle \frac{\partial w}{\partial \nu} + \langle \theta, \vartheta \rangle \frac{\partial w}{\partial \vartheta}, \quad \text{on } \Gamma_{0,n}, \tag{12}$$

where $L\varphi := \langle \theta, \nu \rangle \left(\frac{1}{2}\varphi + K\varphi \right) + \langle \theta, \vartheta \rangle \frac{\partial}{\partial \vartheta}(S\varphi) \quad \text{on } \Gamma_{0,n}$.

The kernel of the operator $N\colon L^2(\Gamma_{0,n}) \to L^2(\Sigma)$ is continuous. As a result it is compact and some kind of regularization for solving (10) is needed.

Theorem 2. *The operator $N\colon L^2(\Gamma_{0,n}) \to L^2(\Gamma_1)$ is injective and has dense range.*

Proof. The injectivity follows from the following arguments. Let $\varphi \in L^2(\Gamma_{0,n})$ satisfy $N\varphi = 0$. We seek the solution of the direct boundary value problem in the form of logarithmic single-layer potential. Since the Cauchy data $u = 0$ and $\partial u/\partial \nu = 0$ on Σ, by Holmgren's theorem we have that $u = 0$ in D. From the maximum-minimum principle for harmonic functions we conclude that $u = 0$ in $D_{0,n}$, where $D_{0,n}$ is the bounded domain with the boundary $\Gamma_{0,n}$. Then the jump relations for the normal derivative of the single-layer potential in the L^2 sense imply that $\varphi = 0$.

To establish the dense range for N we show that the adjoint operator $N^*\colon L^2(\Gamma_1) \to L^2(\Gamma_{0,n})$ is injective. Let $\psi \in L^2(\Gamma_1)$ satisfy $N^*\psi = 0$. Then we consider the function

$$v(x) = \int_{\Gamma_1} \psi(y) \frac{\partial G(x,y)}{\partial \nu(y)} ds(y).$$

Clearly, v is harmonic in $D_{0,n}$ and $v|_{\Gamma_{0,n}} = 0$. Then again from the maximum-minimum principle $v = 0$ in $D_{0,n}$. Thus, $v = 0$ in D_1, hence $v|_{\Gamma_1} = 0$. Since G is Green's function, we have $\psi = 0$. □

Thus, as follows from [9] we can apply Tikhonov regularization method for (10). For given flux ψ^δ with noise level $\delta > 0$ we have

$$\varphi^\alpha = R_\alpha \left[\psi^\delta - \frac{\partial w}{\partial \nu} \right],$$

where R_α is a regularization scheme for the inverse operator N^{-1} with a regularization parameter $\alpha > 0$,

$$R_\alpha = (\alpha I + N^* N)^{-1} N^*.$$

We can find the regularized approximations of functions u and of $\partial u/\partial\theta$:

$$u^\alpha|_{\Gamma_{0,n}} = SR_\alpha \left[\psi^\delta - \frac{\partial w}{\partial \nu}\right] + w|_{\Gamma_{0,n}}, \tag{13}$$

$$\frac{\partial u^\alpha}{\partial \theta}|_{\Gamma_{0,n}} = LR_\alpha \left[\psi^\delta - \frac{\partial w}{\partial \nu}\right] + \langle \theta, \nu \rangle \frac{\partial w}{\partial \nu}\Big|_{\Gamma_{0,n}} + \langle \theta, \vartheta \rangle \frac{\partial w}{\partial \vartheta}\Big|_{\Gamma_{0,n}}. \tag{14}$$

Thus, the iterative method (7) is replaced by

$$r_{n+1} = r_n - \left(u^\alpha|_{\Gamma_{0,n}}\right) / \left(\frac{\partial u^\alpha}{\partial \theta}\Big|_{\Gamma_{0,n}}\right). \tag{15}$$

Note that due to ill-posedness of our inverse problem the linearized operator equations on every Newton's iteration are still ill-posed. Therefore, it is necessary to apply some regularization techniques in (15). It can be, for example, minimum norm solution, Tikhonov regularization, quasi solution concept and others. The full discretization variant of this method with various numerical experiments is presented in [2].

3. Convergence analysis. Let u^* be the solution of the direct problem (1)–(3) with exact Γ_0 and u the solution of the direct problem problem (1)–(3) with current approximation of $\Gamma_{0,n}$.

We consider the linear operator $T\colon u^*|_{\Gamma_{0,n}} \to u|_{\Gamma_{0,n}}$ and the nonlinear operator $B\colon \Gamma_{0,n} \to u^*|_{\Gamma_{0,n}}$. Clearly we have $A = TB$.

Equation (6) can be rewritten in the equivalent form

$$T(B'(r) + B(r))q = 0.$$

Since $T' = T$, the iteration procedure in contrast with (7) is given by

$$r_{n+1} = r_n - \left(u^*|_{\Gamma_{0,n}}\right) / \left(\frac{\partial u^*}{\partial \theta}|_{\Gamma_{0,n}}\right). \tag{16}$$

Note that for a given analytical initial guess r_0, on each iteration step the updated approximation r_n obtained by (16) is an analytic function. Since, $u^*|_{\Gamma_0} = 0$ we obtaine the estimate

$$\|u^*|_{\Gamma_0} - u^*|_{\Gamma_{0,n}} - \frac{\partial u^*}{\partial \theta}\Big|_{\Gamma_{0,n}} (r^* - r_n)\|_{C([0,2\pi])} = O(\|r_n - r^*\|^2_{C([0,2\pi])}).$$

From

$$r_{n+1} - r^* = r_n - \left(u^*|_{\Gamma_{0,n}}\right) / \left(\frac{\partial u^*}{\partial \theta}\Big|_{\Gamma_{0,n}}\right) - r^* = \left((r_n - r^*)\frac{\partial u^*}{\partial \theta}\Big|_{\Gamma_{0,n}} - u^*|_{\Gamma_{0,n}}\right) / \left(\frac{\partial u^*}{\partial \theta}\Big|_{\Gamma_{0,n}}\right)$$

we obtaine the convergence rate

$$\|r_{n+1} - r^*\|_{C([0,2\pi])} = O(\|r_n - r^*\|^2_{C([0,2\pi])}).$$

Here r^* is the radial function of the exact boundary Γ_0.

Thus, we have the following lemma:

Lemma 1. *There is a neighborhood of* Γ_0 *such that the Newton scheme for the solution* (5) *is given by* (16) *and it converges quadratically.*

We have remarked that the perfect Newton scheme (16) is ill-posed and does not converge without regularization. On the other hand we can observe that the smoothness of the derivative $\partial u/\partial\theta$ is, in general, one order less than the smoothness of the boundary. Thus, according to (16) we have lost one order of smoothness on each Newton step. It means that when the initial guess is sufficiently smooth, the C^2-norm may become very large. Therefore, we consider the regularized version of the scheme (16) to control the C^2-norm of the boundary. We use the conception of the quasi-solution introduced in [7] for it.

Definition 1. Let X, Y be normed spaces, $A\colon X \to Y$ be a bounded injective linear operator and $\rho > 0$. For a given $f \in Y$ an element $\varphi_0 \in X$ is called a *quasi-solution of operator equation* $A\varphi = f$ *with constraint* ρ if $\varphi_0 \le \rho$ and

$$\|A\varphi_0 - f\| = \inf\{\|A\varphi - f\|\colon \ \|\varphi\| \le \rho\}.$$

Note that φ_0 is a quasi-solution of $A\varphi = f$ with constraint ρ if and only if $A\varphi_0$ is the best approximation for f (see [9]).

The regularized Newton scheme has the form

$$\widetilde{r}_{n+1} = r_n - \left(u^*|_{\Gamma_{0,n}}\right) / \left(\frac{\partial u^*}{\partial\theta}\Big|_{\Gamma_{0,n}}\right), \quad r_{n+1} = Q(\widetilde{r}_{n+1}), \tag{17}$$

where the operator $Q\colon C([0,2\pi]) \to C^3([0,2\pi])$ maps the function $\widetilde{r}_{n+1}$ onto the quasi-solution r_{n+1} with constraint C_0, i.e.

$$\|\widetilde{r}_{n+1} - r_{n+1}\|_{C([0,2\pi])} \le \|\widetilde{r}_{n+1} - r\|_{C([0,2\pi])} \tag{18}$$

for all $r \in C^3([0,2\pi])$ with $\|r\|_{C^3([0,2\pi])} < C_0$. We choose the constant C_0 by some a priori information about the boundary Γ_0 such that $\|r^*\|_{C([0,2\pi])} \le C_0$.

Lemma 2. *There is a neighborhood of* Γ_0 *such that the regularized Newton scheme* (17) *for the solution* (5) *converges quadratically.*

Proof. Using results of Lemma 1 and a property of quasi-solution (18) we have the estimate

$$\|r_{n+1} - r^*\|_{C([0,2\pi])} \le \|r_{n+1} - \widetilde{r}_{n+1}\|_{C([0,2\pi])} + \|\widetilde{r}_{n+1} - r^*\|_{C([0,2\pi])} \le 2\|\widetilde{r}_{n+1} - r^*\|_{C([0,2\pi])} = \\ = O(\|r_n - r^*\|^2_{C([0,2\pi])}).$$

□

Let us consider the case where the given flux ψ^δ includes noise with the level $\delta > 0$ and use the regularization of the equation (10). The full regularized scheme can be defined by

$$\widetilde{r}_{n+1} = r_n - \left(u^\alpha|_{\Gamma_{0,n}}\right) / \left(\frac{\partial u^\alpha}{\partial\theta}\Big|_{\Gamma_{0,n}}\right), \quad r_{n+1} = Q(\widetilde{r}_{n+1}). \tag{19}$$

The regularized scheme is locally convergent if there exists a neighbourhood U of the true solution r^* such that the regularized solution $r^\delta \to r^*$ when the data error $\delta \to 0$. The next lemma is related to the Tikhonov regularization of the integral equation (10).

Lemma 3. *For the regularized solution* u^α *and radial derivative* $\partial u^\alpha/\partial\theta$ *obtained by Tikhonov regularization with choosing the regularization parameter* $\alpha = \alpha(\delta)$ *such that*

$$\frac{\delta^2}{\alpha(\delta)} \to 0 \quad \text{as } \delta \to 0,$$

the following estimates hold

$$\|u^\alpha - u^*\|_{C(\Gamma_{0,n})} \le \varepsilon_1(\delta) \quad \text{and} \quad \left\|\frac{\partial u^\alpha}{\partial\theta} - \frac{\partial u^*}{\partial\theta}\right\|_{C(\Gamma_{0,n})} \le \varepsilon_2(\delta),$$

where ε_ℓ, $\ell \in \{1,2\}$, *are monotonously decreasing and* $\varepsilon_\ell(\delta) \to 0$ *as* $\delta \to 0$.

Proof. According to (11) and (13) we have

$$\|u^\alpha - u^*\|_{C(\Gamma_{0,n})} = \|SR_\alpha\left[\psi^\delta - \frac{\partial w}{\partial\nu}\right] - S\varphi\|_{C(\Gamma_{0,n})} \le \|S\|\|R_\alpha\left[\psi^\delta - \frac{\partial w}{\partial\nu}\right] - \varphi\|_{C(\Gamma_{0,n})} \le$$
$$\le C\left(\|R_\alpha\|\delta + \|R_\alpha N\varphi - \varphi\|_{C(\Gamma_{0,n})}\right)$$

and analogously from (12) and (14)

$$\left\|\frac{\partial u^\alpha}{\partial\theta} - \frac{\partial u^*}{\partial\theta}\right\|_{C(\Gamma_{0,n})} \le \widetilde{C}\left(\|R_\alpha\|\delta + \|R_\alpha N\varphi - \varphi\|_{C(\Gamma_{0,n})}\right).$$

Here $C > 0$, $\widetilde{C} > 0$. We have used the boundedness of the operators S and L in $C(\Gamma_{0,n})$ [9]. Now the statement of the theorem follows from the result about convergence of the classical Tikhonov regularization (see [5]). □

Note that if (8) holds than for sufficiently small neighborhood U of Γ_0 and sufficiently small δ we have

$$\left|\frac{\partial u^\alpha}{\partial\theta}\Big|_\Gamma\right| > 2\epsilon > 0. \tag{20}$$

Definition 2. For the reconstruction of the unknown boundary Γ_0 by a regularized iterative scheme (19) we *stop the iteration* if for two successive approximations we observe

$$\|r_{n+1} - r_n\|_{C([0,2\pi])} \le C_1(\delta), \quad \text{where } C_1(\delta) := \frac{2\varepsilon_1(\delta)}{\epsilon - 2\varepsilon_2(\delta)}. \tag{21}$$

Theorem 3. *Assume that* $\Gamma_0 \in C^\infty$ *and that* (20) *holds. Then the completely regularized iterative scheme* (19) *with the stopping rule* (21) *is locally convergent, i.e.*

$$\|r^\delta - r^*\|_{C([0,2\pi])} \to 0 \quad \text{for } \delta \to 0.$$

Proof. The elementary calculations give us

$$\widetilde{r}_{n+1} - r^* = \Big(- u^*|_{\Gamma_{0,n}} - (r^* - r_n)\frac{\partial u^*}{\partial\theta}\Big|_{\Gamma_{0,n}} + (u^* - u^\alpha)|_{\Gamma_{0,n}} +$$
$$+ (r_n - r^*)\Big(\frac{\partial u^\alpha}{\partial\theta}\Big|_{\Gamma_{0,n}} - \frac{\partial u^*}{\partial\theta}\Big|_{\Gamma_{0,n}}\Big)\Big)/\Big(\frac{\partial u^\alpha}{\partial\theta}\Big|_{\Gamma_{0,n}}\Big).$$

Therefore, we have the estimate

$$\|r_{n+1} - r^*\|_{C([0,2\pi])} \le \frac{C}{\epsilon}\|r_n - r^*\|^2_{C([0,2\pi])} + \frac{\varepsilon_2(\delta)}{\epsilon}\|r_n - r^*\|_{C([0,2\pi])} + \frac{\varepsilon_1(\delta)}{\epsilon}.$$

Here $C > 0$ is depended on u^α. Thus, we need to find the conditions when

$$\frac{C}{\epsilon}\|r_n - r^*\|^2_{C([0,2\pi])} + \frac{\varepsilon_2(\delta)}{\epsilon}\|r_n - r^*\|_{C([0,2\pi])} + \frac{\varepsilon_1(\delta)}{\epsilon} < \frac{\|r_n - r^*\|_{C([0,2\pi])}}{2}.$$

We obtain

$$\frac{-\varepsilon_2(\delta) + \frac{\epsilon}{2} - \epsilon\sqrt{D}}{2C} < \|r_n - r^*\|_{C[0,2\pi]} < \frac{-\varepsilon_2(\delta) + \frac{\epsilon}{2} + \epsilon\sqrt{D}}{2C} \quad \text{with} \quad D = \left(\frac{\varepsilon_2(\delta)}{\epsilon} - \frac{1}{2}\right)^2 - 4C\frac{\varepsilon_1(\delta)}{\epsilon^2}.$$

From these estimates for a sufficiently small fixed δ we obtain

$$\|r_{n+1} - r^*\|_{C([0,2\pi])} \leq \frac{\|r_n - r^*\|_{C([0,2\pi])}}{2} \tag{22}$$

if

$$C_1(\delta) = \frac{4\varepsilon_1(\delta)}{\epsilon - 2\varepsilon_2(\delta)} < \|r_n - r^*\|_{C([0,2\pi])} < \frac{\epsilon}{2C}. \tag{23}$$

Assume that conditions (23) are satisfied. Using (22) we have

$$\|r_{n+1} - r_n\|_{C([0,2\pi])} \geq \|r_n - r^*\|_{C([0,2\pi])} - \|r_{n+1} - r^*\|_{C([0,2\pi])} \geq \frac{1}{2}\|r_n - r^*\|_{C([0,2\pi])}.$$

Thus, for $\|r_{n+1}-r_n\|_{C([0,2\pi])} \leq C_1(\delta)$ we obtain $\|r_{n+1}-r^*\|_{C([0,2\pi])} \leq C_1(\delta)$. Then we establish $r^\delta := r_{n+1}$ and, therefore, we get $\|r^\delta - r^*\|_{C([0,2\pi])} \leq C_1(\delta) \to 0 \quad \text{for } \delta \to 0.$ □

The given proof of the local convergence can be modified for a general-shaped and sufficiently smooth boundary Γ_0.

REFERENCES

1. R. Chapko, R. Kress, *A hybrid method for inverse boundary value problems in potential theory,* Journal of Ill-Posed and Inverse Problems **13** (2005), 27—40.
2. R. Chapko, N. Vintonyak, *A hybrid method for inverse boundary value problems for an inclusion in semi-infinite two-dimensional domains,* Journal of Integral Equations and Applications **19** (2007), 311—333.
3. R. Chapko, N. Vintonyak, *A hybrid method for the solution of inverse potential boundary value problems in semi-infinite domains with crack,* Reports of the National Academy of Sciences of Ukraine **7** (2007), 42—48 (in Ukrainian).
4. R. Chapko, N. Vintonyak, *Location search technique and size estimation of the bounded inclusion for some inverse boundary value problems,* Visnyk Khmelnytskogo universytetu, **2(93)** (2007), №3, 186–191.
5. H.W. Engl, M. Hanke, A. Neubauer, Regularization of inverse problems, Kluwer Academic Publishers, London, 1996.
6. T. Hohage, *Logarithmic convergence rates of the iteratively regularized Gauss-Newton method for an inverse potential and an inverse scattering problem,* Inverse Problems **13** (1997), 1279–1299.
7. V.K. Ivanov, *Integral equations of the first kind and an approximate solution for the inverse problem of potential,* Soviet Math. Doklady **3** (1962), 210–212.
8. A. Kirsch, R. Kress, *An optimization method in inverse acoustic scattering* Boundary Element IX, Springer-Verlag, Berlin, **3** (1987), 3–18.
9. R. Kress, Linear Integral Equations, 2nd. ed. Springer-Verlag, Berlin, 1999.
10. R. Kress, *Newton's method for inverse obstacle scattering meets the method of least squares,* Inverse Problems **19** (2003), 91–104.
11. R. Kress, N. Vintonyak, *Iterative methods for planar crack reconstruction in semi-infinite domains,* Journal of Inverse and Ill-Posed Problems **16** (2008), 743–761.
12. R. Potthast, *On the convergence of a new Newton-type method in inverse scattering,* Inverse Problems **17** (2001), 1419–1434.
13. P. Serranho, A hybrid method for inverse obstacle scattering problems. Dissertation.- Georg-August-Universitaet Goettingen, 2007.

Ivan Franko National University of Lviv
Faculty of Applied Mathematics and Computer Science

Received 12.01.2008
Revised 20.04.2009