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IS IT POSSIBLE TO POINT OUT A SIMPLE CRITERION OF MAXIMALITY FOR A NONNEGATIVE LINEAL IN THE SPACE WITH A REGULAR INDEFINITE METRIC?

O. G. Storozh. *Is it possible to point out a simple criterion of maximality for a nonnegative lineal in the space with a regular indefinite metric ?*, Matematychni Studii, **27** (2007) 220.

The problem indicated in the headline is formulated.

О. Г. Сторож. *Возможно ли сформулировать простой критерий максимальности для неотрицательного линеала в пространстве с регулярной индефинитной метрикой ?* // Математичні Студії. – 2007. – Т.27, №2. – С.220.

Сформулирована указанная в заглавии проблема.

Let H be a Hilbert space with inner product $(\cdot \mid \cdot)$. Suppose that G is a linear bounded operator on H such that zero lies in its resolvent set. Put for each x, y belonging to H $[x, y] := (Gx \mid y)$. Lineal $L \subset H$ is called nonnegative if $\forall\, x \in L\ [x, x] \geq 0$. In this case L is set to be maximal nonnegative if it is not contained in another nonnegative lineal.

Let us consider the situation when (H, G) is Krein space. It means that $G = J$ where

$$J = J^* = J^{-1}.$$

It is easy to see that there exists an orthogonal decomposition $H = H^+ \oplus H^-$ such that $J = P^+ - P^-$ where $P^\pm$ is the orthoprojector onto $H^\pm$.

Theorem ([1]). *Nonnegative lineal L is the maximal one in (H, J) iff $P^+L = H^+$.*

Problem. Can the mentioned theorem be carried over to general situation?

Remark. Applying expansion of identity of self-adjoint operator(specifically, using the existence of square root for self-adjoint operator) it may be shown that there exists a linear bounded operator C on H and operator J satisfying equalities $J = J^* = J^{-1}$ for which $G = CJC^*$. Thus, formally our problem is reduced to one in a Krein space. But finding of C is a very nontrivial thing. That is why it would be interesting to solve this problem *without use of spectrsl theorem and its corollaries.*

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