УДК 517.56

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ON THE MAGNITUDES OF DEVIATIONS OF MEROMORPHIC AND HOLOMORPHIC FUNCTIONS IN THE DISK

L. V. Kaluzhynova, I. I. Marchenko. *On the magnitudes of deviations of meromorphic and holomorphic functions in the disk*, Matematychni Studii, **26** (2006) 131–139.

It was obtained an upper bound of $\sum_{a\in\overline{\mathbb{C}}}\widehat{\beta}(a,f)$ for both meromorphic and holomorphic in the unit disk functions such that $\varliminf\limits_{r\to 1}\frac{T(r,f)}{\log\frac{1}{1-r}}=L<\infty$.

Л. В. Калужинова, И. И. Марченко. *О величинах отклонений мероморфных и голоморфных функций в круге* // Математичні Студії. – 2006. – Т.26, №2. – С.131–139.

Получены оценки $\sum_{a\in\overline{\mathbb{C}}}\widehat{\beta}(a,f)$ для мероморфных и голоморфных в круге функций таких, что $\varliminf\limits_{r\to 1}\frac{T(r,f)}{\log\frac{1}{1-r}}=L<\infty$.

We will use the standard notations of the Nevanlinna theory of value distribution of meromorphic functions: $m(r,a,f), N(r,a,f), T(r,f), \delta(a,f), \Delta(a,f)$ (see [1] and [2]). We now state the main result on deficient values obtained by R. Nevanlinna.

Theorem A [1]. *Let $f(z)$ be a meromorphic function in the region $|z|<R\le\infty$, and for $R<\infty$*

$$L=\varliminf_{r\to R}\frac{T(r,f)}{\log(R-r)}.$$

Then

a) $0\le\delta(a,f)\le 1$ *for each fixed* $a\in\overline{\mathbb{C}}$;

b) $\sum_{a\in\mathbb{C}}\delta(a,f)\le\begin{cases}2+\frac{1}{L}, & \text{if } R<\infty,\\ 2, & \text{if } R=\infty.\end{cases}$

Since 1969 (Petrenko [3]) the following problem was considered. How Nevanlinna's theory varies if we consider an approximation of meromorphic functions to the value a in a metrics different from L^1. Petrenko introduced the approximation functions

$$\mathcal{L}(r,a,f)=\max_{|z|=r}\log^+\frac{1}{|f(z)-a|},\quad \mathcal{L}(r,\infty,f)=\max_{|z|=r}\log^+|f(z)|.$$

The quantity

$$\beta(a,f)=\varliminf_{r\to\infty}\frac{\mathcal{L}(r,a,f)}{T(r,f)}$$

2000 *Mathematics Subject Classification:* 30B10, 30D35.

is called the quantity of deviation of a meromorphic function $f(z)$ at a point a.

The quantity $\beta(a,f)$ characterizes the speed of approach of the function $f(z)$ to the value a in a stronger metric than $\delta(a,f)$ does. Petrenko obtained the sharp upper estimate of $\beta(a,f)$ for meromorphic functions of finite lower order

$$\lambda := \varliminf_{r\to\infty} \frac{\log T(r,f)}{\log r}.$$

He also obtained some estimates for $\sum_{a\in\overline{\mathbb{C}}} \beta(a,f)$.

Theorem B. *For a meromorphic function $f(z)$ of finite lower order λ the inequalities*

$$\sum_{a\in\overline{\mathbb{C}}} \beta(a,f) \le 816\pi(\lambda+1)^2, \quad \beta(a,f) \le \begin{cases} \frac{\pi\lambda}{\sin\pi\lambda}, & \text{if} \quad \lambda \le 0.5, \\ \pi\lambda, & \text{if} \quad \lambda > 0.5 \end{cases}$$

hold.

The precise estimate of the sum $\sum_{a\in\overline{\mathbb{C}}} \beta(a,f)$ was obtained by Marchenko and Shcherba in 1990 ([4]).

Theorem C [4]. *If $f(z)$ is a meromorphic function of finite lower order λ, then*

$$\sum_{a\in\overline{\mathbb{C}}} \beta(a,f) \le \begin{cases} \frac{2\pi\lambda}{\sin\pi\lambda}, & \text{if} \quad \lambda \le 0.5, \\ 2\pi\lambda, & \text{if} \quad \lambda > 0.5. \end{cases}$$

The estimate of the value of deviation through $\Delta(a,f)$ was obtained by Shea (see [3,5]).

Theorem D. *Let $f(z)$ be a meromorphic function of finite lower order λ. Then*

$$\beta(a,f) \le B(\lambda,\Delta) = \begin{cases} \frac{\pi\lambda}{\sin\pi\lambda}(1-(1-\Delta))\cos\pi\lambda, & \text{if} \quad \arccos(1-\Delta) \ge \pi\lambda \\ \pi\lambda\sqrt{\Delta(2-\Delta)}, & \text{if} \quad \arccos(1-\Delta) < \pi\lambda, \end{cases}$$

where

$$\Delta = \Delta(a,f) = \varlimsup_{r\to\infty} \frac{m(r,a,f)}{T(r,f)}.$$

The estimate of the sum $\sum_{a\neq\infty} \beta(a,f)$ via $\Delta(0,f')$ was shown in [6].

Theorem E. *Let $f(z)$ be a meromorphic function of finite lower order λ. Then*

$$\sum_{a\neq\infty} \beta(a,f) \le 2B(\lambda,\Delta(0,f')).$$

In the case of functions of finite lower order meromorphic in the disk, the magnitude of deviation $\beta(a,f)$ can be equal to $+\infty$. Therefore in 1981 Krytov [7] introduced the quantity $\widehat{\beta}(a,f)$:

$$\widehat{\beta}(\infty,f) = \varliminf_{r\to 1}(1-r)\frac{\max\limits_{|z|=r}\log^+|f(z)|}{T(r,f)}, \ \widehat{\beta}(a,f) = \widehat{\beta}\left(\infty, \frac{1}{f-a}\right).$$

The precise estimate of the quantity $\widehat{\beta}(a,f)$ and an analog of the relations of deficient values for these quantities were presented in [4].

Theorem F. *Let $f(z)$ be a function of finite lower order λ meromorphic in the unit disk. Then for each $a \in \overline{\mathbb{C}}$*

$$\widehat{\beta}(a, f) \le \pi\lambda \cos^{-1-\lambda} \frac{\pi}{2(1+\lambda)}.$$

If the condition

$$\varliminf_{r\to 1} \frac{T(r, f)}{\log \frac{1}{1-r}} = \infty \tag{1}$$

holds, then

$$\sum_{a\in\overline{\mathbb{C}}} \widehat{\beta}(a, f) \le 2\pi\lambda \cos^{-1-\lambda} \frac{\pi}{2(1+\lambda)}.$$

The estimate of the sum of the quantities $\sum_{a\neq\infty} \widehat{\beta}(a, f)$ via $\Delta(0, f')$ was obtained in 2004 (see [8]).

Theorem G. *Let $f(z)$ be a function of finite lower order λ meromorphic in the unit disk, which satisfies (1). Then*

$$\sum_{a\neq\infty} \widehat{\beta}(a, f) \le \begin{cases} \frac{2\pi\lambda}{\sin\lambda x_0}\{\cos^{-\lambda} x_0 - (1-\Delta(0, f'))\cos\lambda x_0\}, & \text{if} \quad \lambda > 0, \\ 4\Delta(0, f'), & \text{if} \quad \lambda = 0, \end{cases}$$

where x_0 is the smallest positive root of the equation

$$\cos(\lambda+1)x = (1-\Delta(0, f'))\cos^{\lambda+1} x.$$

In this paper we present an upper bound of $\sum_{a\in\overline{\mathbb{C}}} \widehat{\beta}(a, f)$ and $\sum_{a\neq\infty} \widehat{\beta}(a, f)$ for both meromorphic and holomorphic in the unit disk functions such that

$$\varliminf_{r\to 1} \frac{T(r, f)}{\log \frac{1}{1-r}} = L < \infty. \tag{2}$$

Theorem 1. *Let $f(z)$ be a meromorphic in the unit disk function for which (2) holds. Then*

$$\sum_{a\in\overline{\mathbb{C}}} \widehat{\beta}(a, f) \le 4(1 + 2/L).$$

Theorem 2. *Let $f(z)$ be a meromorphic in the unit disk function for which (2) holds. Then*

$$\sum_{a\neq\infty} \widehat{\beta}(a, f) \le 4(1 + 1/L)\Delta(0, f') + 4/L.$$

Theorem 3. *Let $f(z)$ be a holomorphic in the unit disk function for which (2) holds. Then*

$$\sum_{a\in\overline{\mathbb{C}}} \widehat{\beta}(a, f) \le 2(1 + 4/L).$$

Theorem 4. *Let $f(z)$ be a holomorphic in the unit disk function for which (2) holds. Then*

$$\sum_{a\neq\infty} \widehat{\beta}(a, f) \le 2(1 + 2/L)\Delta(0, f') + 4/L.$$

In order to prove the above theorems we need the following results.

Lemma A [4]. *For each fixed number $B>1$ there exist two sequences of positive numbers v_k and R_k ($v_k=v_k(B)$, $R_k=R_k(B)$) such that $\lim_{k\to\infty} v_k=\lim_{k\to\infty} R_k=\lim_{k\to\infty} R_k/v_k=0$ and for each $\varepsilon>0$ there exists a number $k_0(\varepsilon)$ such that, for $k>k_0(\varepsilon)$,*

$$T(1-R_k/B)R_k^{\lambda}+T(1-v_k/B)v_k^{\lambda}<\varepsilon\int\limits_{R_k}^{v_k} T(1-r)r^{\lambda-1}\,dr.$$

Let, for $q\ge 0$, $\{a_k\}_1^q$ be a collection of distinct complex numbers such that for each $1\le k\le q<\infty$, there is $\widehat{\beta}(a_k,f)>0$, and let $c=\min\{|a_i-a_j|:i\ne j\}$. For each fixed $\varepsilon\in(0,1)$ we denote [4]

$$G_k=\left\{z\in K\left(1-2v_k,1-\frac{2R_k}{B}\right):\log|f'(z)|<-2\varepsilon T\left(1-\frac{1-|z|}{8},f\right)\right\},$$

where $\{R_k\}$, $\{v_k\}$ and B are the sequences defined in Lemma A.

Let G_{kj} be a set consisting of those connected components of G_k which contain a point z_1 such that $|f(z_1)-a_j|<c/4$. Applying one of Weitsman's methods [9], it can be shown that the sets G_{kj} and G_{ki} do not intersect for $i\ne j$ [4]. Let [4]

$$u_{kj}(z)=\begin{cases}\max\left\{\log\frac{1}{|f'(z)|},\,4\varepsilon T\left(1-\frac{1-|z|}{8},f\right)\right), & \text{if}\quad z\in G_{kj},\\ 4\varepsilon T\left(1-\frac{1-|z|}{8},f\right), & \text{if}\quad z\notin G_{kj}.\end{cases}$$

Lemma B [4]. *Let $f(z)$ be a meromorphic in the unit disk function of lower order $\lambda=0$, and let ε be a constant such that $\varepsilon\in(0,\pi/2)$. Then for each $\alpha:0<\alpha<\frac{\pi}{2}-\varepsilon$ and $\mu:0<\mu<1/2$, the inequality*

$$\int\limits_{R_k}^{v_k}\left\{\frac{R}{1-R}\frac{\sin\mu\alpha}{\pi}\sum_{j=1}^{q}\max_{|z|=1-R}u_{kj}(z)+\mu N\left(1-R,\infty,\frac{1}{f'}\right)\cos\mu\alpha-\right.$$

$$\left.\mu T(1-R,1/f')\right\}\frac{dR}{R}<\varepsilon\int\limits_{R_k}^{v_k}T(1-R,f)\frac{dR}{R}$$

holds.

Lemma 1. *Let $f(z)$ be a meromorphic in the unit disk function such that (2) holds. Then for each $\varepsilon>0$ and $k>k_0(\varepsilon)$*

$$\int\limits_{R_k}^{v_k}\max_{|z|=1-R}\log^{+}\left|\frac{f'(z)}{f(z)}\right|\,dR<(\varepsilon+4/L)\int\limits_{R_k}^{v_k}T(1-R,f)\,\frac{dR}{R},$$

where v_k and R_k are the sequences defined in Lemma A.

Proof of Lemma 1. We choose a number $\alpha : 0 < \alpha = \alpha(\varepsilon) \le \min\{\pi/2 - \varepsilon, \pi/2\mu\}$ for $0 < \mu < 1/2$ for a fixed number ε such that $\varepsilon : 0 < \varepsilon < \pi/2$.

In the same way as in Lemma 9 [4] we get following inequality

$$\int\limits_{R_k}^{v_k} \left\{ \frac{R}{1-R} \frac{\max\limits_{|z|=1-R} \log^+ \left|\frac{f'(z)}{f(z)}\right|}{\pi} - T(1-R,f)(1+\varepsilon)\left[\frac{\varepsilon + 2/L}{\sin\alpha\cos\alpha} + \mu(2+\varepsilon+2/L)\sin\mu\alpha + \right.\right.$$

$$\left.\left. + \mu(2+\varepsilon+2/L)\operatorname{tg}\alpha\right]\right\}\frac{dR}{R} < \varepsilon \int\limits_{R_k}^{v_k} T(1-R,f)\frac{dR}{R} \qquad (k \to \infty).$$

We put $\alpha = \pi/4$ in this inequality and passing to the limit with $\mu \to 0$, we get Lemma 1. □

Proof of Theorem 1. From Lemma B we have

$$\int\limits_{R_k}^{v_k} \left\{ \frac{R}{1-R} \frac{\sin\mu\alpha}{\pi} \sum_{j=1}^{q} \max_{|z|=1-R} u_{kj}(z) - \mu T(1-R, 1/f') \right\} \frac{dR}{R} < \varepsilon \int\limits_{R_k}^{v_k} T(1-R,f)\frac{dR}{R}.$$

Note that

$$T\left(r, \frac{1}{f'}\right) = T(r,f') + O(1) \le m\left(r, \frac{f'}{f}\right) + m(r,f) + N(r,f') + O(1) \le$$

$$\le m\left(r, \frac{f'}{f}\right) + 2T(r,f) + O(1), \qquad (r \to 1).$$

By the lemma on the logarithmic derivative [2] we have

$$\int\limits_{R_k}^{v_k} T(1-R, 1/f')\frac{dR}{R} < \left(2 + \frac{2}{L} + \varepsilon\right) \int\limits_{R_k}^{v_k} T(1-R,f)\frac{dR}{R}. \tag{3}$$

Thus

$$\int\limits_{R_k}^{v_k} \frac{R}{1-R} \frac{\sin\mu\alpha}{\pi} \sum_{j=1}^{q} \max_{|z|=1-R} u_{kj}(z)\frac{dR}{R} <$$

$$< \varepsilon \int\limits_{R_k}^{v_k} T(1-R,f)\frac{dR}{R} + \mu\left(2 + \varepsilon + \frac{2}{L}\right)\int\limits_{R_k}^{v_k} T(1-R,f)\frac{dR}{R}. \tag{4}$$

Note that

$$\max_{|z|=1-R} \log^+ \frac{1}{|f(z)-a_j|} < \max_{|z|=1-R} \log^+ \left|\frac{f'(z)}{f(z)-a_j}\right| +$$

$$+ \max_{|z|=1-R,\ z\in G_{kj}} \log^+ \frac{1}{|f'(z)|} + 2\varepsilon T\left(1 - \frac{R}{8}, f\right). \tag{5}$$

Moreover,

$$\max_{|z|=1-R} u_{kj}(z) = \max_{|z|=1-R,\ z\in G_{kj}} \log^+ \frac{1}{|f'(z)|}. \tag{6}$$

From Lemma A we have

$$\int\limits_{R_k}^{v_k} T\left(1-\frac{R}{8},f\right)\frac{dR}{R}=\int\limits_{R_k/8}^{v_k/8} T(1-t,f)\frac{dt}{t}\le\int\limits_{R_k}^{v_k} T(1-t,f)\frac{dt}{t}+\int\limits_{R_k/8}^{R_k} T(1-t,f)\frac{dt}{t}\le$$

$$\le\int\limits_{R_k}^{v_k} T(1-t,f)\frac{dt}{t}+3T\left(1-\frac{R_k}{8},f\right)<(1+\varepsilon)\int\limits_{R_k}^{v_k} T(1-t,f)\frac{dt}{t}. \tag{7}$$

Therefore applying the inequalities (4)-(7), Lemma 1 and condition (2) we obtain that for each $k\ge k_0(\varepsilon)$ there exists a number $\widetilde{R}_k\in(R_k,v_k)$ such that

$$\frac{\widetilde{R}_k}{1-\widetilde{R}_k}\sum_{j=1}^{q}\max_{|z|=1-\widetilde{R}_k}\log^+\frac{1}{|f(z)-a_j|}<$$

$$<\left(\frac{\pi}{\sin\mu\alpha}\left(\mu\left(2+\frac{2}{L}\right)+\varepsilon\right)+(\varepsilon+4/L)\right)T(1-\widetilde{R}_k,f)+\frac{2\widetilde{R}_k\varepsilon q}{1-\widetilde{R}_k}T(1-\widetilde{R}_k,f).$$

Thus

$$\sum_{j=1}^{q}\widehat{\beta}(a_j,f)\le\varliminf_{\widetilde{R}_k\to0}\frac{\widetilde{R}_k\sum\limits_{j=1}^{q}\max\limits_{|z|=1-\widetilde{R}_k}\log^+\frac{1}{|f(z)-a_j|}}{T(1-\widetilde{R}_k,f)}<$$

$$<\varliminf_{\widetilde{R}_k\to0}\left(\frac{\pi}{\sin\mu\alpha}(\varepsilon+\mu(2+2/L))+\varepsilon+4/L+2q\widetilde{R}_k\varepsilon\right)=$$

$$=\frac{\pi\mu}{\sin\mu\alpha}(2+2/L+\varepsilon/\mu)+\varepsilon+4/L.$$

By the arbitrariness of the number ε, we obtain

$$\sum_{j=1}^{q}\widehat{\beta}(a_j,f)\le\frac{\pi\mu}{\sin\mu\alpha}\left(2+\frac{2}{L}\right)+4/L.$$

Choosing $\alpha=\frac{\pi}{2(1+\mu)}$ we get

$$\sum_{j=1}^{q}\widehat{\beta}(a_j,f)\le\frac{\pi\mu}{\sin\frac{\pi\mu}{2(1+\mu)}}(2+2/L)+4/L.$$

Passing to the limit as $\mu\to0$ we obtain $\sum_{a\in\overline{\mathbb{C}}}\widehat{\beta}(a,f)\le4(1+2/L)$. Hence, the set $\{a\in\mathbb{C}:\widehat{\beta}(a,f)\ge0\}$ is at most countable.

Theorem 1 is proven in the case of $\widehat{\beta}(\infty,f)=0$, as in the opposite case it is sufficient to consider the function $\widetilde{f}(z)=1/(f(z)-a)$ where $\widehat{\beta}(a,f)=0$. □

Proof of Theorem 2. Let $\{a_k\}_1^q$ be a collection of distinct complex numbers such that $\widehat{\beta}(a_k,f)>0,\ \ c=\min\{|a_i-a_j|:i\ne j\}\ \ (q\ge2)$.

From Lemma B we have

$$\int\limits_{R_k}^{v_k}\left\{\frac{R}{1-R}\frac{\sum\limits_{j=1}^{q}\max\limits_{|z|=1-R}u_{kj}(z)\sin\mu\alpha}{\pi}+\mu N\left(1-R,\infty,\frac{1}{f'}\right)\cos\mu\alpha-\right.$$

$$\left.-\mu T\left(1-R,\frac{1}{f'}\right)\right\}\frac{dR}{R}<\varepsilon\int\limits_{R_k}^{v_k}T(1-R,f)\frac{dR}{R}\qquad(k\to\infty).$$

Note that

$$N(1-R,\infty,1/f')>(1-\Delta(0,f')-\varepsilon)T(1-R,f')=$$
$$=(1-\Delta(0,f')-\varepsilon))T\left(1-R,\frac{1}{f'}\right)\qquad R\to 0.$$

Therefore we obtain

$$\int\limits_{R_k}^{v_k}\frac{R}{1-R}\frac{\sum\limits_{j=1}^{q}\max\limits_{|z|=1-R}u_{kj}(z)\sin\mu\alpha}{\pi}\frac{dR}{R}<$$

$$<\int\limits_{R_k}^{v_k}\left\{\mu(\Delta(0,f')+\varepsilon-1)T\left(1-R,\frac{1}{f'}\right)\cos\mu\alpha+\mu T\left(1-R,\frac{1}{f'}\right)+\ \varepsilon T(1-R,f)\right\}\frac{dR}{R}.$$

Thus for each $k\geq k_0(\varepsilon)$ there exists a number $R'_k\in(R_k,v_k)$ such that

$$\frac{R'_k}{1-R'_k}\frac{\sum\limits_{j=1}^{q}\max\limits_{|z|=1-R'_k}u_{kj}(z)\sin\mu\alpha}{\pi}<$$

$$<\left(\mu\left(2+\frac{2}{L}+\varepsilon\right)(1+(\Delta(0,f')-1)\cos\mu\alpha)+\varepsilon\right)T\left(1-R'_k,f\right).\qquad(8)$$

Moreover,

$$\max_{|z|=1-R}\log^+\frac{1}{|f(z)-a_j|}<\max_{|z|=1-R}\log^+\left|\frac{f'(z)}{f(z)-a_j}\right|+$$
$$+\max_{|z|=1-R,\ z\in G_{kj}}\log^+\frac{1}{|f'(z)|}+2\varepsilon T\left(1-\frac{R}{8},f\right).\qquad(9)$$

Note that

$$\max_{|z|=1-R}u_{kj}(z)=\max_{|z|=1-R,\ z\in G_{kj}}\log^+\frac{1}{|f'(z)|}.\qquad(10)$$

From inequalities (7)-(10), Lemma 1 and condition (2) we obtain

$$\sum_{j=1}^{q}\widehat{\beta}(a_j,f)\leq\frac{\pi\mu}{\sin\mu\alpha}\left(1+(\Delta(0,f')-1)\cos\mu\alpha\right)\left(2+\frac{2}{L}\right)+4/L.$$

Passing to the limit as $\mu\to 0$ we obtain

$$\sum_{j=1}^{q}\widehat{\beta}(a_j,f)\leq\frac{\pi}{\alpha}(2+2/L)\Delta(0,f')+4/L.$$

This inequality holds for all $\alpha : 0 < \alpha < \frac{\pi}{2} - \varepsilon$. Take the limit as $\alpha \to \pi/2 - \varepsilon$:

$$\sum_{j=1}^{q} \widehat{\beta}(a_j, f) \leq \frac{4\pi}{\pi - \varepsilon}(1 + 1/L)\Delta(0, f') + 4/L.$$

By the arbitrariness of the number $\varepsilon > 0$ we obtain Theorem 2. □

Proof of Theorem 3. It is clear that for a holomorphic function

$$T(r, f') = m(r, f') \leq m\left(r, \frac{f'}{f}\right) + m(r, f) = m\left(r, \frac{f'}{f}\right) + T(r, f), \qquad (0 < r < 1).$$

By the lemma of the logarithmic derivative [2] we have

$$\int_{R_k}^{v_k} T(1 - R, 1/f')\frac{dR}{R} < \left(1 + \frac{1}{L} + \varepsilon\right) \int_{R_k}^{v_k} T(1 - R, f)\frac{dR}{R}. \tag{11}$$

Hence from (4) we have

$$\int_{R_k}^{v_k} \frac{R}{1 - R} \frac{\sin \mu\alpha}{\pi} \sum_{j=1}^{q} \max_{|z|=1-R} u_{kj}(z)\frac{dR}{R} <$$

$$< \varepsilon \int_{R_k}^{v_k} T(1 - R, f)\frac{dR}{R} + \mu \left(1 + \varepsilon + \frac{2}{L}\right) \int_{R_k}^{v_k} T(1 - R, f)\frac{dR}{R}.$$

Next, following the same way as in the proof of Theorem 1, we get that

$$\sum_{j=1}^{q} \widehat{\beta}(a_j, f) \leq 2(1 + 4/L).$$

□

Applying (11), the proof of Theorem 4 can be conducted similarly to that of Theorem 3.

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Received 10.08.2006