УДК 517.524

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TRUNCATION ERROR BOUNDS FOR A TWO-DIMENSIONAL CONTINUED g-FRACTION

Kh. Yo. Kuchmins'ka, S. M. Vozna. *Truncation error bounds for a two-dimensional continued g-fraction*, Matematychni Studii, **24** (2005) 120–126.

Truncation error bounds for the two-dimensional continued g-fraction have been established in terms of a two-dimensional continued π-fraction which is an extension of the two-dimensional continued g-fraction.

К. И. Кучминская, С. М. Возная. *Оценки ошибок приближения для двумерной непрерывной g-дроби* // Математичні Студії. – 2005. – Т.24, №2. – С.120–126.

Оценки ошибок приближения для двумерной непрерывной g-дроби получены путем использования свойств двумерной непрерывной π-дроби, являющейся растяжением двумерной непрерывной g-дроби.

1. Introduction. Among different types of functional continued fractions the most studied one is a fraction of the form:

$$\frac{s_0}{1} + \frac{g_1 z}{1} + \frac{g_2(1-g_1)z}{1} + \frac{g_3(1-g_2)z}{1} + \dots = \frac{s_0}{1 + \mathop{\mathrm{D}}\limits_{n=1}^{\infty} \frac{g_n(1-g_{n-1})z}{1}}, \tag{1}$$

where $s_0 > 0$, $0 < g_n < 1$, $g_0 = 0$, $n \geq 1$, $z \in \mathbb{C}$.

J. Sleszyński was the first who investigated the fraction (1) with $z = 1$ under the condition $\lim_{n\to\infty} g_n(1-g_{n-1}) = 0$ ([5]). Later E. Van Vleck, O. Perron, W. Scott, H. S. Wall investigated this special type of the regular C-fraction, so called "g-fraction" ([4, 12]). This type of continued fractions is the important one because of its applications. In particular, H.S.Wall characterized the class of functions $f(z)$ $(\mathbf{W})$, holomorphic in the cut plane $|\arg(1+z)| < \pi$ with $\mathrm{Re}\,(\sqrt{1+z}/f(z)) > 0$ in terms of g-fraction (1) ([12]). These fractions were used for analytic continuation of functions, finding of zeros and poles and univalence domains of some analytic and meromorphic functions ([9, 10]), solving the power moment problems ([12]), the Feigenbaum-Cvitanović functional equation ([11]). Due to W.B.Gragg ([7]) we have the following: if a g-fraction converges in some domain to a holomorphic function $f(z)$ then a priori bounds for the n-th approximant $g_n(z)$ of the g-fraction (1) are valid:

$$|f(z) - g_n(z)| \leq \max\left\{1, \mathrm{tg}\left|\frac{\arg z}{2}\right|\right\} \frac{s_0}{\mathrm{Re}\sqrt{1+z}} \left|\sqrt{1+z} - \frac{1}{\sqrt{1+z}}\right| \left|\frac{1-\sqrt{1+z}}{1+\sqrt{1+z}}\right|^{n-1}, \quad n \geq 2.$$

2000 *Mathematics Subject Classification:* 11J70, 30B70.

Multidimensional generalizations of g-fractions were considered in [1, 2, 6].

2. Investigation purpose. The convergence of a multidimensional g-fraction (i.e. a branched continued g-fraction (BCF)) was investigated in [1, 6]. It is natural to continue previous investigations using the different type of generalization of a continued fraction, namely a two-dimensional continued fraction (TDCF) ([8]), which can not be obtained from the BCF.

We will investigate a TDCF of the form:

$$\cfrac{s_0}{1+\Phi_0(\mathbf{z})+\cfrac{g_{11}z_1z_2}{1+\Phi_1(\mathbf{z})+\underset{j=2}{\overset{\infty}{\mathrm{D}}}\cfrac{g_{j-1,j-1}g_{jj}z_1z_2}{1+\Phi_j(\mathbf{z})}}}, \tag{2}$$

where $s_0>0$,

$$\Phi_k(\mathbf{z})=\underset{j=1}{\overset{\infty}{\mathrm{D}}}\frac{(1-g_{j+k-1,k})g_{j+k,k}z_1}{1}+\underset{j=1}{\overset{\infty}{\mathrm{D}}}\frac{(1-g_{k,j+k-1})g_{k,j+k}z_2}{1},$$

$k\geq 0,\ g_{00}=0,\ 0<g_{kj}<1,\ k\geq 0,\ j\geq 0,\ k+j\geq 1,\ \mathbf{z}=(z_1,z_2)\in\mathbb{C}^2$
with the approximants

$$g_1(\mathbf{z})=s_0,\quad g_2(\mathbf{z})=\frac{s_0}{1+g_{10}z_1+g_{01}z_2+g_{11}z_1z_2},$$

$$g_n(\mathbf{z})=\cfrac{s_0}{1+\Phi_0^{(n-1)}(\mathbf{z})+\cfrac{g_{11}z_1z_2}{1+\Phi_1^{(n-2)}(\mathbf{z})+\underset{j=2}{\overset{n-1}{\mathrm{D}}}\cfrac{g_{j-1,j-1}g_{jj}z_1z_2}{1+\Phi_j^{(n-1-j)}(\mathbf{z})}}},\quad n\geq 3, \tag{3}$$

where $\Phi_j^{(0)}(\mathbf{z})=0$, and at $0\leq k\leq n-2$

$$\Phi_k^{(n-1-k)}(\mathbf{z})=\underset{j=1}{\overset{n-1-k}{\mathrm{D}}}\frac{(1-g_{j+k-1,k})g_{j+k,k}z_1}{1}+\underset{j=1}{\overset{n-1-k}{\mathrm{D}}}\frac{(1-g_{k,j+k-1})g_{k,j+k}z_2}{1}.$$

We are going to get a priori bounds for its n-th approximant.

3. Main results. TDCF (2) is called a two-dimensional continued g-fraction (shortly, TDCg-F). It was introduced in [3] where one can find the following theorems concerning the convergence problem.

Theorem 1. *TDCg-F* (2) *converges in the domain*

$$Q=\{\mathbf{z}:\ |z_1|+|z_2|+2|z_1z_2|<1\}.$$

Theorem 2. *TDCg-F* (2) *converges to a holomorphic function in the domain*

$$D=\bigcup_{\alpha\in(-\pi/2;\pi/2)}P_\alpha,$$

where

$$P_\alpha=\{\mathbf{z}:\ |z_1|+|z_2|+2|z_1z_2|-\operatorname{Re}\left((z_1+z_2+2z_1z_2)e^{-2i\alpha}\right)<2\cos^2\alpha\}, \tag{4}$$

moreover, it converges uniformly on every compact subset of this domain.

By analogy with W.B.Gragg ([7]) truncation error bounds for TDCg-F (2) will be given in terms of two-dimensional continued π-fractions (TDCπ-Fs):

$$\cfrac{\pi_0}{1+z_1+z_2+\Psi_0(\mathbf{z})+\displaystyle\mathop{\mathrm{D}}_{j=1}^{\infty}\frac{\pi_{j-1,j-1}\pi_{jj}z_1z_2}{1+\pi_{jj}+z_1+z_2+\Psi_j(\mathbf{z})}}, \tag{5}$$

where $\pi_0>0$,

$$\Psi_k(\mathbf{z})=-\cfrac{z_1}{1+\cfrac{\pi_{k+1,k}}{1+z_1-\cfrac{z_1}{1+\cfrac{\pi_{k+2,k}}{1+z_1-\ddots}}}}-\cfrac{z_2}{1+\cfrac{\pi_{k,k+1}}{1+z_2-\cfrac{z_2}{1+\cfrac{\pi_{k,k+2}}{1+z_2-\ddots}}}},$$

$k\ge 0,\ \pi_{00}=1,\ \pi_{kj}>0,\ k\ge 0,\ j\ge 0,\ k+j\ge 1,\ \mathbf{z}\in\mathbb{C}^2.$

Finite TDCπ-Fs $f_1(\mathbf{z})=\pi_0/(1+z_1+z_2),\ f_2(\mathbf{z})=\pi_0,$

$$f_n(\mathbf{z})=\cfrac{\pi_0}{1+z_1+z_2+\Psi_0^{(n-1)}(\mathbf{z})+\displaystyle\mathop{\mathrm{D}}_{j=1}^{[(n-1)/2]}\frac{\pi_{j-1,j-1}\pi_{jj}z_1z_2}{1+\pi_{jj}+z_1+z_2+\Psi_j^{(n-1-2j)}(\mathbf{z})}},\quad n\ge 3,$$

where $\Psi_j^{(0)}(\mathbf{z})=0$, $\Psi_j^{(1)}(\mathbf{z})=-z_1-z_2$, and at $0\le k\le[(n-1)/2]-1$

$$\Psi_k^{(n-1-2k)}(\mathbf{z})=-\cfrac{z_1}{1+\cfrac{\pi_{k+1,k}}{1+z_1-\ddots-\cfrac{z_1}{1+\cfrac{\pi_{[(n-1)/2],k}}{1+z_1}}}}-\cfrac{z_2}{1+\cfrac{\pi_{k,k+1}}{1+z_2-\ddots-\cfrac{z_2}{1+\cfrac{\pi_{k,[(n-1)/2]}}{1+z_2}}}}$$

for even n,

$$\Psi_k^{(n-1-2k)}(\mathbf{z})=-\cfrac{z_1}{1+\cfrac{\pi_{k+1,k}}{1+z_1-\ddots-\cfrac{z_1}{1+\pi_{[(n-1)/2],k}}}}-\cfrac{z_2}{1+\cfrac{\pi_{k,k+1}}{1+z_2-\ddots-\cfrac{z_2}{1+\pi_{k,[(n-1)/2]}}}}$$

for odd n, are called n-th approximants of fraction (5).

Let us introduce notations for the s-th approximant tails of TDCπ-F (5):

$$G_{[(s-1)/2]}^{(0)}(\mathbf{z})=1+\pi_{[(s-1)/2],[(s-1)/2]}+z_1+z_2,\quad G_0^{(0)}(\mathbf{z})=1+z_1+z_2,\quad G_0^{(1)}(\mathbf{z})=1,$$

$$G_{[(s-1)/2]}^{(1)}(\mathbf{z})=1+\pi_{[(s-1)/2],[(s-1)/2]},\qquad G_j^{(s-1-2j)}(\mathbf{z})=$$

$$=1+\pi_{jj}(1-\sigma_{0j})+z_1+z_2+\Psi_j^{(s-1-2j)}(\mathbf{z})+\mathop{\mathrm{D}}_{r=j+1}^{[(s-1)/2]}\frac{\pi_{r-1,r-1}\pi_{rr}z_1z_2}{1+\pi_{rr}+z_1+z_2+\Psi_r^{(s-1-2r)}(\mathbf{z})},$$

where $s \geq 3$, $0 \leq j \leq [(s-1)/2]-1$, σ_{0j} is Kronecker's symbol.

$$G^{(s-1-2j)}_{[(s-1)/2],j}(z_1)=1, \quad G^{(s-1-2j)}_{j+k,j}(z_1)=1+\cfrac{\pi_{j+k+1,j}}{1+z_1-\ddots-\cfrac{z_1}{1+\pi_{[(s-1)/2],j}}},$$

$$G^{(s-1-2j)}_{j,[(s-1)/2]}(z_2)=1, \quad G^{(s-1-2j)}_{j,j+k}(z_2)=1+\cfrac{\pi_{j,j+k+1}}{1+z_2-\ddots-\cfrac{z_2}{1+\pi_{j,[(s-1)/2]}}}$$

where $s=2r,\ r\geq 1$, $0\leq j\leq [(s-1)/2]-1$, $0\leq k\leq [(s-1)/2]-j-1$, where $s=2r,\ r\geq 1$, $0\leq j\leq [(s-1)/2]-1$, $0\leq k\leq [(s-1)/2]-j-1$,

$$F^{(s-1-2j)}_{[(s-1)/2],j}(z_1)=1+z_1, \quad F^{(s-1-2j)}_{j+k+1,j}(z_1)=1+z_1-\cfrac{z_1}{1+\cfrac{\pi_{j+k+2,j}}{1+z_1-\ddots-\cfrac{z_1}{1+\pi_{[(s-1)/2],j}}}},$$

$$F^{(s-1-2j)}_{j,[(s-1)/2]}(z_2)=1+z_2, \quad F^{(s-1-2j)}_{j,j+k+1}(z_2)=1+z_2-\cfrac{z_2}{1+\cfrac{\pi_{j,j+k+2}}{1+z_2-\ddots-\cfrac{z_2}{1+\pi_{j,[(s-1)/2]}}}},$$

where $s=2r,\ r\geq 3$, $0\leq j\leq [(s-1)/2]-2$, $0\leq k\leq [(s-1)/2]-j-2$.

Under these notations the following recurrent relations are valid:

$$G^{(s-1)}_0(\mathbf{z})=1+z_1+z_2+\Psi^{(s-1)}_0(\mathbf{z})+\frac{\pi_{11}z_1z_2}{G^{(s-3)}_1(\mathbf{z})} \quad (s\geq 3),$$

$$G^{(s-1-2j)}_j(\mathbf{z})=1+\pi_{jj}+z_1+z_2+\Psi^{(s-1-2j)}_j(\mathbf{z})+\frac{\pi_{jj}\pi_{j+1,j+1}z_1z_2}{G^{(s-3-2j)}_{j+1}(\mathbf{z})}$$

($s\geq 5$, $1\leq j\leq [(s-1)/2]-1$),

$$G^{(s-1-2j)}_{j+k,j}(z_1)=1+\frac{\pi_{j+k+1,j}}{F^{(s-1-2j)}_{j+k+1,j}(z_1)}, \quad G^{(s-1-2j)}_{j,j+k}(z_2)=1+\frac{\pi_{j,j+k+1}}{F^{(s-1-2j)}_{j,j+k+1}(z_2)} \tag{6}$$

($s=2r,\ r\geq 1$, $0\leq j\leq [(s-1)/2]-1$, $0\leq k\leq [(s-1)/2]-j-1$),

$$F^{(s-1-2j)}_{j+k+1,j}(z_1)=1+z_1-\frac{z_1}{G^{(s-1-2j)}_{j+k,j}(z_1)}, \quad F^{(s-1-2j)}_{j,j+k+1}(z_2)=1+z_2-\frac{z_2}{G^{(s-1-2j)}_{j,j+k}(z_2)} \tag{7}$$

($s=2r,\ r\geq 3$, $0\leq j\leq [(s-1)/2]-2$, $0\leq k\leq [(s-1)/2]-j-2$).

Theorem 3. *Let $f_n=f_n(z)$ denote the n-th approximant of TDCπ-F* (5), and let $g_n=g_n(z)$ denote the n-th approximant of TDCg-F (2). Then:

a) $g_n(z)=f_{2n}(z),\ n\in\{1,2,\ldots\}$.

b) If TDCg-F (2) converges in the domain

$$D=\bigcup_{\alpha\in(-\pi/2,\pi/2)}(P_\alpha\cap G_\alpha),$$

to a holomorphic function $g(z)$ where P_α is defined by formula (4),

$$G_\alpha = \{\mathbf{z}: \ \sqrt{|z_1 z_2|}\cos\alpha < \cos^2\alpha - (|z_1 z_2| - \mathrm{Re}\,(z_1 z_2 e^{-2i\alpha})\},$$

then for each $n \in \{3, 4, \ldots\}$ and α, $-\pi/2 < \alpha < \pi/2$,

$$|g(\mathbf{z}) - g_n(\mathbf{z})| \le \frac{s_0}{(1-\omega(z_1)-\omega(z_2)-2\omega(z_1\cdot z_2))^2\cos^2\alpha} \times$$
$$\times\left(L_{n0}(\mathbf{z}) + \sum_{j=1}^{n-3}\frac{L_{nj}(\mathbf{z})|z_1z_2|^j}{((1-2\omega(z_1\cdot z_2))\cos\alpha)^{2j}} + \frac{(|z_1|^2+|z_2|^2)|z_1z_2|^{n-2}}{((1-2\omega(z_1\cdot z_2))\cos\alpha)^{2n-4}\cos^2\alpha} + \right.$$
$$\left. + \frac{(|z_1|+|z_2|)|z_1z_2|^{n-1}}{((1-2\omega(z_1\cdot z_2))\cos\alpha)^{2n-2}\cos\alpha} + \frac{|z_1z_2|^n}{((1-2\omega(z_1\cdot z_2))\cos\alpha)^{2n-1}}\right), \tag{8}$$

where

$$\omega(t) = \frac{|t| - \mathrm{Re}\,(te^{-2i\alpha})}{2\cos^2\alpha},$$

$$L_{nk}(\mathbf{z}) = \sum_{j=1}^{2} L_j(z_j)\left|\frac{1-\sqrt{1+z_j}}{1+\sqrt{1+z_j}}\right|^{n-2-k}, \quad 0 \le k \le n-3,$$

$$L_j(z_j) = \max\left\{1, \mathrm{tg}\frac{|\arg(1+z_j)|}{2}\right\}\frac{|z_j|(\cos\alpha+|z_j|)}{\mathrm{Re}\sqrt{1+z_j}\cos^2\alpha}\left|\sqrt{1+z_j} - \frac{1}{\sqrt{1+z_j}}\right|.$$

The principal branch of the square root is chosen in all expressions $L_{nk}(\mathbf{z})$.

Proof. As in [2], using the properties of two-dimensional linear fractional transformations, one easily obtains that the even part of TDCπ-F (5), where $z_p \ne -1$, $p = 1, 2$, $z_1 + z_2 \ne -1$, $1 + \pi_{jj} + z_1 + z_2 \ne -1$, $j \ge 1$, is TDCg-F (2), where $s_0 = \pi_0$, $\pi_{00} = 1$ $g_{kl} = \pi_{kl}/(1+\pi_{kl})$, $k \ge 0$, $l \ge 0$, $k + l \ge 1$, with approximants (3).

The convergence of TDCg-F (2) to the holomorphic function $g(z)$ in D (part b)) follows from Theorem 2 ([3]).

In order to get the truncation error bound we have to estimate $|f_{2m}(\mathbf{z}) - f_{2n}(\mathbf{z})|$ for $m > n \ge 3$. It follows from (4) that

$$|z_j| - \mathrm{Re}\,(z_j e^{-2i\alpha}) < 2\cos^2\alpha, \quad j \in \{1, 2\}, \tag{9}$$
$$|z_1| + |z_2| - \mathrm{Re}\,\left((z_1+z_2)e^{-2i\alpha}\right) < 2\cos^2\alpha, \quad |z_1z_2| - \mathrm{Re}\,(z_1z_2e^{-2i\alpha}) < \cos^2\alpha.$$

In addition, using the proof of Lemma 4.41 ([4]) we have

$$\min_{-\infty<y<+\infty} \mathrm{Re}\,\frac{u+iv}{x+iy} = -\frac{\sqrt{u^2+v^2}-u}{2x}. \tag{10}$$

for $x \ge c > 0$ and $v^2 \le 4u + 4$. Let us prove the following inequalities:

$$\mathrm{Re}\,(F_{j+k,j}^{(s-1-2j)}(z_1)e^{-i\alpha}) \ge (1-\omega(z_1))\cos\alpha, \tag{11}$$
$$\mathrm{Re}\,(F_{j,j+k}^{(s-1-2j)}(z_2)e^{-i\alpha}) \ge (1-\omega(z_2))\cos\alpha, \tag{12}$$

for $s=2r,\ r\geq 2,\ 0\leq j\leq[(s-1)/2]-1,\ 1\leq k\leq[(s-1)/2]-j$,

$$\operatorname{Re}(G_0^{(s-1)}(\mathbf{z})e^{-i\alpha})\geq(1-\omega(z_1)-\omega(z_2)-2\omega(z_1\cdot z_2))\cos\alpha, \tag{13}$$

for $s\geq 2j,\ j\geq 2$, and

$$\operatorname{Re}(G_j^{(s-1-2j)}(\mathbf{z})e^{-i\alpha})\geq\pi_{jj}\,(1-2\omega(z_1\cdot z_2))\cos\alpha, \tag{14}$$

for $s=2r,\ r\geq 3,\ 1\leq j\leq[(s-1)/2]$.

For $k=[(s-1)/2]-j$ inequalities (11) are obvious. Assume that (11) hold for $k=p+1<[(s-1)/2]-j$. Then for $k=p$ we have

$$F_{j+p,j}^{(s-1-2j)}(z_1)e^{-i\alpha}=(1+z_1)e^{-i\alpha}-\frac{z_1e^{-i\alpha}}{G_{j+p+1,j}^{(s-1-2j)}(z_1)}=e^{-i\alpha}+\frac{\pi_{j+p+1,j}z_1e^{-2i\alpha}}{\left(\pi_{j+p+1,j}+F_{j+p+1,j}^{(s-1-2j)}(z_1)\right)e^{-i\alpha}}.$$

Using relations (9) and (10) we obtain

$$\operatorname{Re}(F_{j+p,j}^{(s-1-2j)}(z_1)e^{-i\alpha})\geq$$
$$\geq\cos\alpha-\frac{\pi_{j+p+1,r}\,(|z_1|-\operatorname{Re}(z_1e^{-2i\alpha}))}{2\left(\pi_{j+p+1,j}\cos\alpha+\operatorname{Re}(F_{j+p+1,j}^{(s-1-2j)}(z_1)e^{-i\alpha})\right)}>(1-\omega(z_1))\cos\alpha.$$

Similarly, one can prove the validity of inequalities (12)-(14). From (6), (7), (11)-(14) we obtain that all tails of TDCπ-F (5) are not equal zero. Using the difference formula for approximants of TDCπ-F (5) ([8]), taking into account inequalities for $\left|\Psi_k^{(2m-1-2k)}(\mathbf{z})-\Psi_k^{(2n-1-2k)}(\mathbf{z})\right|$ [7] and (13), (14) for $m>n\geq 3$ we have

$$|f_{2m}(\mathbf{z})-f_{2n}(\mathbf{z})|\leq$$
$$\leq\frac{\pi_0}{(1-\omega(z_1)-\omega(z_2)-2\omega(z_1\cdot z_2))^2\cos^2\alpha}\left(L_{n0}(\mathbf{z})+\sum_{j=1}^{n-3}\frac{L_{nj}(\mathbf{z})|z_1z_2|^j}{((1-2\omega(z_1\cdot z_2))\cos\alpha)^{2j}}+\right.$$
$$+\frac{(|z_1|^2+|z_2|^2)|z_1z_2|^{n-2}}{((1-2\omega(z_1\cdot z_2))\cos\alpha)^{2n-4}\cos^2\alpha}+$$
$$\left.+\frac{(|z_1|+|z_2|)|z_1z_2|^{n-1}}{((1-2\omega(z_1\cdot z_2))\cos\alpha)^{2n-2}\cos\alpha}+\frac{|z_1z_2|^n}{((1-2\omega(z_1\cdot z_2))\cos\alpha)^{2n-1}}\right).$$

Passing to the limit as $m\to\infty$, we obtain (8). □

4. Conclusion. Two-dimensional continued π- and g-fractions can be used for investigation of meromorphic and holomorphic functions of two variables. The convergence problem for TDCπ-F is still open.

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Received 21.09.2004
Revised 14.06.2005