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UNIVERSAL NÖBELING SPACES AND PSEUDO-BOUNDARIES OF EUCLIDEAN SPACES

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We give a topological characterization of the n-dimensional pseudoboundary of the $(2n+1)$-dimensional Euclidean space.

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Дана топологическая характеризация n-мерной псевдограницы $(2n+1)$-мерного эвклидового пространства.

1. Introduction

In [12] Geoghegan and Summerhill constructed the n-dimensional universal pseudoboundary σ_n^k of the k-dimensional Euclidean space $\mathbb{R}^k$, $0 \leq n \leq k$, $k \geq 1$, as an $\mathcal{M}_n^k$-absorber of $\mathbb{R}^k$, where $\mathcal{M}_n^k$ denotes the collection of tame at most n-dimensional compacta in $\mathbb{R}^k$. In these notes we consider the space σ_n^{2n+1}. It has been remarked by several authors that from a certain point of view the space σ_n^{2n+1} can be considered as an n-dimensional counterpart of the pseudoboundaries σ and Σ of the Hilbert cube Q. Topological characterizations of the latter spaces have been obtained by Mogilski [14, 5]. As for the problem of topological characterization of σ_n^{2n+1} (see, for instance, [16, Problem #1017], [11, Problem #607], [9, Conjecture 4.10], [17, Question 3], [6, Conjecture 5.6.9]) we mention here the following two related results. First of all we note that according to [8] $\sigma_n^{2n+1} \cong \sigma_n^k$ for each $k \geq 2n+1$. Secondly $\sigma_n^{2n+1} \cong \Sigma^n$ (see [7, Theorem 7.4], [6, Theorem 5.6.10]), where Σ^n denotes the pseudo-boundary of the universal n-dimensional Menger compactum μ^n [4] constructed in [8]. Below (Corollary 2.8) we give a topological characterization of the space σ_n^{2n+1}.

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2. Topological characterization of finite-dimensional absorbing sets

2.1. Preliminaries. All spaces in these notes are assumed to be separable and metrizable. Maps are assumed to be continuous. By $\mathrm{cov}(X)$ we denote the family of all open covers of a space X.

Let $n \in \omega$. A subset A of a space X is said to be *locally connected in dimension n relative to X* (briefly $LC^n\mathrm{rel}.X$) if for each $k \leq n+1$, each $x \in X$ and each neighbourhood U of x in X there exists a neighbourhood V of x in X such that every map $f\colon \partial I^k \to V \cap A$ has an extension $F\colon I^k \to U \cap A$. A space X is said to be *locally connected in dimension n* (briefly LC^n) if X is $LC^n\mathrm{rel}.X$. Recall that class of $LC^{n-1} \cap C^{n-1}$-spaces coincides with the class $AE(n)$ of absolute extensors in dimension n. Discussion of basic properties of LC^n-spaces can be found in [4]. Recall that for an open cover $\mathcal{U} \in \mathrm{cov}(Y)$ of a space Y two maps $f, g\colon X \to Y$ are said to be *$\mathcal{U}$-close* if for each point $x \in X$ there exists an element $U \in \mathcal{U}$ such that $f(x), g(x) \in U$. A space X satisfies the *discrete n-cells property* if the set

$$\{f \in C(I^n \times \mathbb{N}, X) : \{f(I^n \times \{k\}) : k \in \mathbb{N}\} \text{ is discrete}\}$$

is dense in the space $C(I^n \times \mathbb{N}, X)$ equipped with the limitation topology. The latter topology on the space $C(X,Y)$ of all continuous maps of X into Y has a neighborhood base at a point $f \in C(X,Y)$ consisting of the sets

$$\{g \in C(X,Y) : g \text{ is } \mathcal{U}\text{-close to } f\}, \quad \mathcal{U} \in \mathrm{cov}(Y).$$

A closed subset A of a space X is said to be a *Z-set* (respectively a strong *Z-set*) in X if for every $\mathcal{U} \in \mathrm{cov}(X)$ there exists a map $f\colon X \to X$ which is $\mathcal{U}$-close to id_X and such that $f(X) \cap A = \varnothing$ (respectively $\overline{f(X)} \cap A = \varnothing$). See, e.g. [5] for properties of (strong) Z-sets.

Proof of our main result is based on the following two statements announced by S. Ageev[2] [1].

Theorem (Topological characterization of the Nöbeling space [1]). *Let $n \geq 0$. Then the following conditions are equivalent for any space X:*

1. *X is homeomorphic to the n-dimensional universal Nöbeling space N_n^{2n+1}.*
2. *X is a separable completely metrizable space satisfying the following properties:*
   (a) $\dim X = n$.
   (b) $X \in LC^{n-1} \cap C^{n-1}$.
   (c) $X \in n$-SDAP.

Theorem (Z-set unknotting [1]). *Let $n \geq 0$ and Z_1, Z_2 be Z-sets in N_n^{2n+1}. Then for each open cover $\mathcal{U} \in \mathrm{cov}(N_n^{2n+1})$ there exists an open cover $\mathcal{V} \in \mathrm{cov}(N_n^{2n+1})$ such that the following property is satisfied:*

- *Every homeomorphism $h\colon Z_1 \to Z_2$ between Z-sets of N_n^{2n+1} which is $\mathcal{V}$-close to the inclusion $Z_1 \hookrightarrow N_n^{2n+1}$ can be extended to a homeomorphism $H\colon N_n^{2n+1} \to N_n^{2n+1}$ which is $\mathcal{U}$-close to the identity $\mathrm{id}_{N_n^{2n+1}}$.*

[2] The first named author recalls with satisfaction series of very interesting lectures given by S. Ageev during his stay at the University of Saskatchewan in May-June of 1999. See also the remarks at the end of the paper.

2.2. Uniqueness of finite-dimensional absorbing sets. In this section we prove that any two "absorbing sets" for a class of finite-dimensional spaces are homeomorphic.

Let $\mathcal{K}$ be a class of spaces that is topological, finitely additive and hereditary with respect to closed subspaces. A space X is *strongly $\mathcal{K}$-universal* if, for every map $f\colon C \to X$ from a space $C \in \mathcal{K}$ into X, for every closed subspace $D \subseteq C$ such that $f/D\colon D \to X$ is a Z-embedding and for every open cover $\mathcal{U} \in \operatorname{cov}(X)$, there exists a Z-embedding $g\colon C \to X$ such that $g|D = f|D$ and g is $\mathcal{U}$-close to f.

The class consisting of countable unions of closed sets that are members of $\mathcal{K}$ is denoted by $\mathcal{K}_\sigma$.

Let $n \in \omega \cup \{\infty\}$. An n-dimensional separable metrizable space X is a *$\mathcal{K}$-absorbing set* if:

(a) $X \in AE(n)$.

(b) X is a countable union of strong Z-sets.

(c) $X \in \mathcal{K}_\sigma$.

(d) X is strongly $\mathcal{K}$-universal.

Several examples of $\mathcal{K}$-absorbing sets (for various classes $\mathcal{K}$) can be found in [11] and [3].

First we show that spaces we are interested in can be nicely embedded into the Nöbeling space of the same dimension. Recall that a map is called a *near-homeomorphism* if it can be approximated by homeomorphisms in the limitation topology.

Proposition 2.1. *Let $n \geq 0$ and X be a separable metrizable $LC^{n-1}\&C^{n-1}$-space satisfying the discrete n-cells property. Then X can be embedded into a copy M of the universal n-dimensional Nöbeling space N_n^{2n+1} so that the set $\{f \in C(I^n, M)\colon f(I^n) \subseteq X\}$ is dense in the space $C(I^n, M)$. In particular, the following properties are satisfied:*

(a) Every F_σ-subset F of M such that $F \cap X = \varnothing$ is a Z_σ-set in M.

(b) Every G_δ-subspace of M, containing X, is homeomorphic to N_n^{2n+1}.

(c) If A and B are G_δ-subsets of M such that $X \subseteq A \subseteq B$, then $B - A$ is a Z_σ-subset in B.

(d) If A and B are G_δ-subsets of M such that $X \subseteq A \subseteq B$, then the inclusion $A \hookrightarrow B$ is a near-homeomorphism.

Proof. Let $\widetilde{X}$ be an n-dimensional metrizable compactification of X. By [2, Theorem 2], there exists a G_δ-set $M \subseteq \widetilde{X}$, containing X, so that

(1) X is LC^{n-1} rel. M;

(2) $M \in \mathrm{LC}^{n-1}$;

(3) For every at most n-dimensional Polish space Y the set of all closed embeddings is dense in $C(Y, M)$.

Let us show that $M \in C^{n-1}$. Indeed, let $f\colon \partial I^k \to M$ be a map defined on the boundary ∂I^k of the k-dimensional disk I^k, $k \leq n$. According to [6, Proposition 4.1.7], there exists an open cover $\mathcal{V} \in \operatorname{cov}(M)$ such that the following condition is satisfied:

$(*)_{n-1}$ If a $\mathcal{V}$-close to f map $g\colon \partial I^k \to M$, $k \leq n$, has an extension $G\colon I^k \to M$, then f also has an extension $F\colon I^k \to M$.

Since X is LC^{n-1} rel. M, it follows by [6, Theorem 2.8] that $M - X$ is locally n-negligible in M. According to [6, Theorem 2.3] we can find a map $g\colon \partial I^k \to X$ which is $\mathcal{V}$-close to f. Since $X \in C^{n-1}$, there exists an extension $G\colon I^k \to X$ of g. The above stated property $(*)_{n-1}$ of the cover $\mathcal{V}$ guarantees that f also has an extension $F\colon I^k \to M$. This shows that $M \in C^{n-1}$. Therefore M is an n-dimensional, separable, completely metrizable $LC^{n-1}\&C^{n-1}$-space satisfying property (3). Topological characterization of the Nöbeling space (see Section 1) implies that M is homeomorphic to N_n^{2n+1}. The fact that the set $\{f \in C(I^n, M)\colon f(I^n) \subseteq X\}$ is dense in the space $C(I^n, M)$ follows from [6, Theorem 2.8].

Let F be an F_σ-subset of M such that $F \cap X = \varnothing$. Since

$$\{f \in C(I^n, M)\colon f(I^n) \subseteq X\} \subseteq \{f \in C(I^n, M)\colon f(I^n) \cap F = \varnothing\},$$

it follows that the set $\{f \in C(I^n, M)\colon f(I^n) \cap F = \varnothing\}$ is dense in $C(I^n, M)$. Consequently, F is a Z_σ-subset of M. This proves property (a).

Next observe that since M is homeomorphic to N_n^{2n+1}, it can be identified with the pseudo-interior ν^n of the universal n-dimensional Menger compactum (see [6, Theorem 5.5.5]). Let Y be a G_δ-subspace of M containing X. By (a) and [6, Proposition 5.7.7], the inclusion $Y \hookrightarrow M$ is a near-homeomorphism. In particular, Y is homeomorphic to N_n^{2n+1}. This proves (b). Properties (c) and (d) are proved similarly. □

A $\mathcal{K}(n)$-absorbing set is called *representable* in $\mathbb{R}^k$ [11] if $M \subset \mathbb{R}^k$ and the complement $\mathbb{R}^k - M$ is locally n-homotopy negligible in $\mathbb{R}^k$.

Proposition 2.2. *Every $\mathcal{K}(n)$-absorbing set is representable in $\mathbb{R}^k$ for $k \geq 2n+1$.*

Proof. As it is shown in the proof of Proposition 2.1, there exists an embedding of a $\mathcal{K}(n)$-absorbing set M into N_n^k with locally n-homotopy negligible image in N_n^k, $k \geq 2n+1$. Now observe that the complement $\mathbb{R}^k - N_n^k$ as a σZ_n-set [6] in $\mathbb{R}^k$ is locally n-homotopy negligible in $\mathbb{R}^k$. This obviously implies that the complement of the image of M in $\mathbb{R}^k$ is also n-negligible in $\mathbb{R}^k$ as required. □

Proposition 2.2 solves affirmatively Problem 555 from [11].

Lemma 2.3. *Let X be an at most n-dimensional separable metrizable LC^{n-1}-space. If $X = \cup\{X_i\colon i \in \omega\}$, where each X_i is a strong Z-set in X, then each compact subset of X is a strong Z-set in X.*

Proof. Let $\widetilde{X}$ be an n-dimensional separable completely metrizable space containing X as a subspace in such a way that X is LC^{n-1} rel. $\widetilde{X}$. As in the proof of Proposition 2.1, we conclude that

(∗) the set $\{f \in C(I^n, \widetilde{X})\colon f(I^n) \subseteq X\}$ is dense in $C(I^n, \widetilde{X})$.

First we need the following observation.

Claim. *A compact subset K of X is a Z-set in X if and only if K is a Z-set in $\widetilde{X}$.*

Proof of Claim. First let K be a Z-set in X. Consider a map $f\colon I^n \to \widetilde{X}$ and open covers $\mathcal{U}, \mathcal{V} \in \mathrm{cov}(\widetilde{X})$ such that $\mathrm{St}(\mathcal{V})$ refines $\mathcal{U}$. By (∗), there exists a $\mathcal{V}$-close to f map $g \in C(I^n, \widetilde{X})$ such that $g(I^n) \subseteq X$. Since K is a Z-set in X, there exists a $\mathcal{V}$-close to g map $h\colon I^n\colon X$ such that $h(I^n) \cap K = \varnothing$. Since h is $\mathcal{U}$-close to f, it follows that K is a Z-set in $\widetilde{X}$.

Conversely, let K be a Z-set in $\widetilde{X}$. Consider a map $f\colon I^n \to X$ and open covers $\mathcal{U}, \mathcal{V} \in \operatorname{cov}(X)$ so that $\operatorname{St}(\mathcal{V})$ refines $\mathcal{U}$. For each $V \in \mathcal{V}$ choose an open subset $\widetilde{V} \subseteq \widetilde{X}$ such that $V = \widetilde{V} \cap X$. It is easy to see that K is a Z-set in $Y = \cup\{\widetilde{V}\colon V \in \mathcal{V}\}$. Consequently there exists a $\widetilde{\mathcal{V}}$-close to f map $g\colon I^n \to Y$ such that $g(I^n) \cap K = \varnothing$, where $\widetilde{\mathcal{V}} = \{\widetilde{V}\colon V \in \mathcal{V}\} \in \operatorname{cov}(Y)$. Let G be an open subset of Y such that $K \cap G = \varnothing$ and $g(I^n) \subseteq G$. By $(*)$, there exists a map $h\colon I^n \to X$ which is $\widetilde{\mathcal{V}} \wedge \{G, Y - g(I^n)\}$-close to g. Obviously, h is $\mathcal{U}$-close to f and $h(I^n) \cap K = \varnothing$. This shows that K is a Z-set in X and completes the proof of Claim. □

We continue the proof of Lemma 2.3. Let K be a compact subset of X. Clearly $K \cap X_i$ is a compact Z-set in X for each $i \in \omega$. By the above Claim, $K \cap X_i$ is a Z-set in $\widetilde{X}$. This means that the set

$$\{f \in C(I^n, \widetilde{X})\colon f(I^n) \cap (X_i \cap K) = \varnothing\}$$

is open and dense in the space $C(I^n, \widetilde{X})$. Since $\widetilde{X}$ is completely metrizable, the space $C(I^n, \widetilde{X})$ has the Baire property (see, for instance, [6, Proposition 2.1.7]) and consequently, the set

$$\begin{aligned}\{f \in C(I^n, \widetilde{X})\colon f(I^n) \cap K = \varnothing\} \\ = \{f \in C(I^n, \widetilde{X})\colon f(I^n) \cap (\cup\{X_i \cap K\colon i \in \omega\}) = \varnothing\} \\ = \bigcap \Big\{\{f \in C(I^n, \widetilde{X})\colon f(I^n) \cap (X_i \cap K) = \varnothing\}\colon i \in \omega\Big\}\end{aligned}$$

is also dense in the space $C(I^n, \widetilde{X})$. This simply means that K is a Z-set in $\widetilde{X}$. By the above Claim, we conclude that K is a Z-set in X as well. □

The proof of the following statement uses Lemma 2.3 and follows verbatim the proof of [11, Lemma 1.9].

Proposition 2.4. *Let X be an at most n-dimensional separable metrizable LC^{n-1}-space. If $X = \bigcup\{X_i\colon i \in \omega\}$, where each X_i is a strong Z-set in X, then $X \in n$-SDAP.*

Now we are in position to prove the uniqueness result.

Theorem 2.5. *Let $n \geq 0$ and $\mathcal{K}$ be a class of spaces that is topological, finitely additive and hereditary with respect to closed subspaces. Then any two $\mathcal{K}(n)$-absorbing sets are homeomorphic.*

Proof. Let X and Y be $\mathcal{K}(n)$-absorbing sets. Proposition 2.4 guarantees that $X, Y \in n$-SDAP. Embed X and Y into a copy M of the universal n-dimensional Nöbeling space N_n^{2n+1} in such a way that properties (a)–(d) of Proposition 2.1 are satisfied. Next we follow the argument presented in the prove of [5, Theorem 3.1] (use Z-set Unknotting Theorem for N_n^{2n+1} at the appropriate place). □

2.3. Characterization of σ_n^{2n+1}. In order to obtain a topological characterization of a $K(n)$-absorbing set, Theorem 2.5 must be combined with the corresponding existence result. In other words, we need to know that there exists a $K(n)$-absorbing set. For certain choices of K it is even possible to explicitly construct corresponding absorbing sets.

Let us recall that for each space X and for each ordinal $\alpha < \omega_1$, we can define two classes of subspaces of X – the *additive Borelian class* α, $\mathcal{A}_\alpha(X)$, and the *multiplicative*

Borelian class α, $\mathcal{M}_\alpha(X)$, – as follows: $\mathcal{A}_0(X)$ is the collection of all open subsets of X and $\mathcal{M}_0(X)$ is the collection of all closed subsets of X. Assuming that for each ordinal $\beta < \alpha$, where $\alpha < \omega_1$, the classes $\mathcal{A}_\beta$ and $\mathcal{M}_\beta$ have already been constructed, we proceed as follows: the class $\mathcal{A}_\alpha$ consists of countable unions of elements of $\cup\{\mathcal{M}_\beta \colon \beta < \alpha\}$ and the class $\mathcal{M}_\alpha$ consists of countable intersections of elements of $\cup\{\mathcal{A}_\beta \colon \beta < \alpha\}$.

Further, let $\alpha < \omega_1$ and X be a separable metrizable space. We say that X belongs to the *absolute additive Borelian class* $\mathcal{A}_\alpha$, if for any embedding $i \colon X \to Y$ into any separable metrizable space Y, we have $i(X) \in \mathcal{A}_\alpha(Y)$. Similarly, X belongs to the *absolute multiplicative Borelian class* $\mathcal{M}_\alpha$ if for any embedding $i \colon X \to Y$ into any separable metrizable space Y, we have $i(X) \in \mathcal{M}_\alpha(Y)$. It is well-known that: (a) $X \in \mathcal{A}_\alpha$, $\alpha \geq 2$, if and only if $X \in \mathcal{A}_\alpha(l_2)$ and (b) $X \in \mathcal{M}_\alpha$, $\alpha \geq 1$, if and only if $X \in \mathcal{M}_\alpha(l_2)$.

Obviously, $\mathcal{A}_0 = \varnothing$ and $\mathcal{M}_0$ coincides with the class of all metrizable compacta. Further, $\mathcal{A}_1 = \{\sigma\text{-compact spaces}\}$, $\mathcal{M}_1 = \{\text{Polish spaces}\}$, etc.

Let $\mathcal{A}_\alpha(n) = \{X \in \mathcal{A}_\alpha \colon \dim X \leq n\}$ and $\mathcal{M}_\alpha(n) = \{X \in \mathcal{M}_\alpha \colon \dim X \leq n\}$ for each $n \in \omega$ and $\alpha < \omega_1$. The existence problem for these classes of spaces is solved in the following statement (see [6, Theorem 5.7.21] and [17, Theorem 2.5]).

Theorem 2.6. *Let* $n \in \omega$ *and* $1 < \alpha < \omega_1$. *Then there exist an* $A_\alpha(n)$*-absorbing set* $\Lambda_\alpha(n)$ *and* $M_\alpha(n)$*-absorbing set* $\Omega_\alpha(n)$.

Theorems 2.5 and 2.6 imply the following characterization result.

Theorem 2.7. *Let* X *be an* n*-dimensional,* $n \geq 0$*, separable metrizable* $AE(n)$*-space and* $1 < \alpha < \omega_1$. *Then* X *is homeomorphic to* $\Omega_\alpha(n)$ *(respectively,* $\Lambda_\alpha(n)$*) if and only if the following two conditions are satisfied:*

(i) $X = \cup\{X_i : i \in \omega\}$, *where each* $X_i \in \mathcal{M}_\alpha$ *(respectively,* $X_i \in \mathcal{A}_\alpha$*) and* X_i *is a strong* Z*-set in* X,

(ii) X *is strongly* $M_\alpha(n)$*-universal (respectively,* $A_\alpha(n)$*-universal).*

In particular ($\alpha = 1$), we obtain a topological characterization of σ_n^{2n+1}.

Corollary 2.8. *Let* X *be an* n*-dimensional,* $n \geq 0$, σ*-compact metrizable* $AE(n)$*-space. Then* X *is homeomorphic to* σ_n^{2n+1} *if and only if the following conditions are satisfied:*

(i) X *has the discrete* n*-cells property,*

(ii) X *is strongly* $\mathcal{A}_1(n)$*-universal.*

3. Open problems

The theory of absorbing systems is a natural generalization of the theory of absorbing sets. Its backgrounds are elaborated in [13] and we leave to the reader the reformulations of the basic definitions in n-dimensional setting.

Problem 3.1. *Construct absorbing systems in the Nöbeling space that are* n*-dimensional counterparts of the systems*

$$(\underbrace{Y_\alpha \times \cdots \times Y_\alpha}_{n} \times \ell^2 \times \ell^2 \times \dots)_{n \in \mathbb{N}}$$

(here Y_α *is either* Ω_α *or* Λ_α*) in a topological copy* $(\ell^2)^{\mathbb{N}}$ *of* ℓ^2.

Problem 3.2. *Prove uniqueness theorem for the above mensioned* n*-dimensional absorbing systems.*

4. Remarks

The text of this paper (with minor changes) circulates as a prepint since November, 1999 (see http://xxx.lanl.gov/abs/math.GN/9911128). The main results are essentially based on results of S. Ageev, in particular, his characterization theorem for Nöbeling manifolds. Ageev's results have not yet appeared in print (note also that newer versions of his preprint contained the statement of Theorem 2.7 for the class $\mathcal{A}_1(n)$), nevertheless we decided to publish our paper. In any case, its results can be considered as conventional ones.

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