УДК 517.53

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# ASYMPTOTIC VALUES OF ENTIRE DIRICHLET SERIES WITH RESPECT TO ITS MAXIMAL TERM

S. I. Fedynyak, M. M. Sheremeta. *Asymptotic values of entire Dirichlet series with respect to its maximal term* , Matematychni Studii, **19** (2003) 31–36.

For an entire Dirichlet series $F(s)=1+\sum_{n=1}^{\infty}a_n\exp\{s\lambda_n\}$, $s=\sigma+it$, with the maximal term $\mu(\sigma,F)$ and the central index $\nu(\sigma,F)$ we investigate behaviour of $F(\gamma(\tau))/\mu(\gamma(\tau))$ as $\tau\to+\infty$, where $\mu(s)=\mu(\sigma,F)\exp\{it\lambda_{\nu(\sigma,F)}\}$ and $\gamma(\tau)$ is a continuous curve such that $\operatorname{Re}\gamma(\tau)\to+\infty$ as $\tau\to+\infty$.

С. И. Федыняк, М. Н. Шеремета. *Асимптотические значения целого ряда Дирихле относительно его максимального члена* // Математичні Студії. – 2003. – Т.19, №1. – С.31–36.

Для целого ряда Дирихле $F(s)=1+\sum_{n=1}^{\infty}a_n\exp\{s\lambda_n\}$, $s=\sigma+it$, с максимальным членом $\mu(\sigma,F)$ и центральным индексом $\nu(\sigma,F)$ изучается поведение $F(\gamma(\tau))/\mu(\gamma(\tau))$ при $\tau\to+\infty$, где $\mu(s)=\mu(\sigma,F)\exp\{it\lambda_{\nu(\sigma,F)}\}$ и $\gamma(\tau)$ — непрерывная кривая такая, что $\operatorname{Re}\gamma(\tau)\to+\infty$ при $\tau\to+\infty$.

**1°. Introduction.** For an entire function $f(z)=1+\sum_{n=1}^{\infty}a_nz^n$, $z=re^{i\theta}$, with the maximal term $\mu_f(r)$ and the central index $\nu_f(r)$ A. Gray and S. M. Shah [1] introduced $\mu(z)=\mu_f(r)\exp\{i\theta\nu_f(r)\}$ and $m(z)=\mu_f(r)\exp\{i\theta\nu_f(r)+i\arg a_{\nu_f(r)}\}$ and investigated behaviour of $f(\gamma(t))/\mu(\gamma(t))$ and $f(\gamma(t))/m(\gamma(t))$ as $t\to+\infty$, where $\gamma(t)$ is a continuous curve such that $|\gamma(t)|\to+\infty$ as $t\to+\infty$.

Here we obtain an analogue of Gray-Shah's theorem for entire (absolutely convergent in $\mathbb{C}$) Dirichlet series

$$F(s)=1+\sum_{n=1}^{+\infty}a_n\exp\{s\lambda_n\},\quad s=\sigma+it, \tag{1}$$

where $0<\lambda_n\uparrow+\infty$ $(n\to+\infty)$. We put $M(\sigma,F)=\max\{|F(\sigma+it)|:t\in\mathbb{R}\}$ and let $\mu(\sigma,F)=\max\{|a_n|\exp\{\sigma\lambda_n\}:n\geq 0\}$ be the maximal term and $\nu(\sigma,F)=\max\{n:\exp\{\sigma\lambda_n\}=\mu(\sigma,F)\}$ be the central index of series (1). We assume that $a_n=|a_n|e^{i\alpha_n}$, $0\leq\alpha_n<2\pi$, and put

$$\mu(s)=\mu(s,F)=\mu(\sigma+it,F)=\mu(\sigma,F)\exp\{it\lambda_{\nu(\sigma,F)}\}$$

and

$$m(s)=m(s,F)=m(\sigma+it,F)=\mu(\sigma,F)\exp\{it\lambda_{\nu(\sigma,F)}+i\alpha_{\nu(\sigma,F)}\}.$$

2000 *Mathematics Subject Classification:* 30B50.

We remark that the functions $\mu(s)$ and $m(s)$ are continuous in each vertical strip in which the function $\nu(\operatorname{Re} s, F)$ is continuous.

Let $\gamma(\tau)$, $\tau \geq \tau_0$, be a continuous curve such that $\operatorname{Re}\gamma(\tau) \to +\infty$ as $\tau \to +\infty$.

If

$$\lim_{\tau\to+\infty} \frac{F(\gamma(\tau))}{\mu(\gamma(\tau))} = \omega \quad \left(\lim_{\tau\to+\infty} \frac{F(\gamma(\tau))}{m(\gamma(\tau))} = \omega\right),$$

where $0 \leq |\omega| \leq +\infty$, then we say that $\omega$ is a *$\mu$-asymptotic ($m$-asymptotic) value of $F$*, and the curve $\gamma(\tau)$ is a $\mu$-asymptotic ($m$-asymptotic) path.

For all $l \in (0, +\infty)$ let $R_{lt} = \{\tau : \operatorname{Re}\gamma(t) \leq \operatorname{Re}\gamma(\tau) < \operatorname{Re}\gamma(t) + l\}$. An asymptotic ($\mu$- or $m$-) path is said to be *uniformly oscillating* if $\max_t\{\max_{\tau\in R_{lt}}\{|\operatorname{Im}\gamma(\tau) - \operatorname{Im}\gamma(t)|\}\} < Q(l) < +\infty$. The corresponding asymptotic ($\mu$- or $m$-) value is said to be uniformly oscillating.

Let $(\sigma_n)$ be the sequence of jump points of $\nu(\sigma, F)$, counting multiplicity, and let $(n_k)$ be the range of $\nu(\sigma, F)$, that is $\nu(\sigma, F) = n_k$ for $\sigma_{n_k} \leq \sigma < \sigma_{n_{k+1}}$ and $\sigma_{n_k+1} = \sigma_{n_k+2} = \cdots = \sigma_{n_{k+1}}$. Clearly, we assume that $n_0 = 0$ and $-\infty = \sigma_0 < \sigma_{n_1}$.

**2°. Auxiliary lemmas.** We need two lemmas.

**Lemma 1.** *For all $x \in [0, \sigma_{n_{k+1}} - \sigma_{n_k}]$ we have*

$$M(\sigma_{n_k} + x, F) \geq \frac{\pi}{4}\mu(\sigma_{n_k} + x, F)(1 + \exp\{-(\lambda_{n_k} - \lambda_{n_{k-1}})x\}). \tag{2}$$

*Proof.* Using the equality [2, p. 124] we have

$$a_n e^{(\sigma+iy)\lambda_n} = \lim_{T\to+\infty} \frac{1}{T}\int_{t_0}^{T} F(\sigma+it)\exp\{i(y-t)\lambda_n\}dt, \quad \{\sigma, y\} \in \mathbb{R}.$$

Then for $\sigma = \sigma_{n_k} + x$

$$|a_{n_k}\exp\{(\sigma_{n_k} + x + iy)\lambda_{n_k}\} + a_{n_{k-1}}\exp\{(\sigma_{n_k} + x + iy)\lambda_{n_{k-1}}\}| \leq$$

$$\leq M(\sigma_{n_k} + x, F)\varlimsup_{T\to+\infty}\frac{1}{T}\int_{t_0}^{T} |\exp\{i(y-t)\lambda_{n_k}\}) + \exp\{i(y-t)\lambda_{n_{k-1}}\}|)dt =$$

$$= M(\sigma_{n_k} + x, F)\varlimsup_{T\to+\infty}\frac{1}{T}\int_{t_0}^{T} |1 + \exp\{i(t-y)(\lambda_{n_k} - \lambda_{n_{k-1}})\}|)dt. \tag{3}$$

It is not difficult to verify, that for all $y \in \mathbb{R}$ and $p > 0$

$$\varlimsup_{T\to+\infty}\frac{1}{T}\int_{t_0}^{T} |1 + \exp\{i(t-y)p\}|dt = \frac{4}{\pi}. \tag{4}$$

We choose $y = -(\alpha_{n_k} - \alpha_{n_{k-1}})/(\lambda_{n_k} - \lambda_{n_{k-1}})$. Then

$$|a_{n_k}\exp\{(\sigma_{n_k} + x + iy)\lambda_{n_k}\} + a_{n_{k-1}}\exp\{(\sigma_{n_k} + x + iy)\lambda_{n_{k-1}}\}| =$$
$$= |a_{n_k}|\exp\{(\sigma_{n_k} + x)\lambda_{n_k}\} + |a_{n_{k-1}}|\exp\{(\sigma_{n_k} + x\lambda_{n_{k-1}}\}.$$

From (3) and (4) it follows that

$$|a_{n_k}\exp\{(\sigma_{n_k}+x+iy)\lambda_{n_k}\}+a_{n_{k-1}}\exp\{(\sigma_{n_k}+x+iy)\lambda_{n_{k-1}}\}|\le\frac{4}{\pi}M(\sigma_{n_k}+x,F). \qquad (5)$$

Since $\sigma_{n_k}\le\sigma_{n_k}+x\le\sigma_{n_{k+1}}$, we have $|a_{n_k}|\exp\{(\sigma_{n_k}+x)\lambda_{n_k}\}=\mu(\sigma_{n_k}+x)$. On the other hand, $|a_{n_{k-1}}|\exp\{\sigma_{n_k}\lambda_{n_{k-1}}\}=|a_{n_k}|\exp\{\sigma_{n_k}\lambda_{n_k}\}$. Therefore, from (5) we obtain (2). Lemma 1 is proved. □

**Lemma 2.** *For all* $n\ge 1$

$$|a_n|\le\exp\left\{-\sum_{k=1}^{n}\sigma_k(\lambda_k-\lambda_{k-1})\right\}. \qquad (6)$$

*The proof* is elementary.

**3°. Main result.** Now, we prove the following result.

**Theorem.** *If* $\varliminf\limits_{n\to\infty}(\lambda_{n+1}-\lambda_n)>0$, $\varlimsup\limits_{k\to\infty}(\sigma_{n_{k+1}}-\sigma_{n_k})=L>0$ *and*

$$\varlimsup_{k\to\infty}(\lambda_{n_{k+1}}-\lambda_{n_k})=S<+\infty,$$

*then* $F$ *has no uniformly oscillating* $\mu$*-asymptotic and* $m$*-asymptotic values.*

*Proof.* From the condition $\varlimsup\limits_{k\to\infty}(\sigma_{n_{k+1}}-\sigma_{n_k})=L>0$ it follows that for each $L_1\in(0,L)$ there exists a sequence $(\sigma_{n_{k_p}})$ such that $\sigma_{n_{k_p+1}}-\sigma_{n_{k_p}}>L_1$.

Suppose that $F$ has a uniformly oscillating $\mu$-asymptotic value $\omega$. Then there exists a continuous curve $\gamma(\tau)$, $\tau\ge\tau_0$, such that $\operatorname{Re}\gamma(\tau)\to+\infty$ as $\tau\to+\infty$,

$$\max_t\{\max_{\tau\in R_{L_1t}}\{|\operatorname{Im}\gamma(\tau)-\operatorname{Im}\gamma(t)|\}\}<Q<+\infty \text{ and } \lim_{\tau\to+\infty}\frac{F(\gamma(\tau))}{\mu(\gamma(\tau))}=\omega.$$

There exists an unbounded set $I$ with the following property: for each $\tau\in I$ there exists a unique integer $p$ and such that $\sigma_{n_{k_p}}\le\operatorname{Re}\gamma(\tau)<\sigma_{n_{k_p}+1}=\sigma_{n_{k_p+1}}$. For each $\sigma_{n_{k_p}}$ let $\tau_p=\max\{\tau\in I:\ \operatorname{Re}\gamma(\tau)=\sigma_{n_{k_p}}\}$.

For $w\in\Omega_1=\{w:\,0<\operatorname{Re}w<L_1\}$ we define

$$\Phi_p(w)=\frac{F(\sigma_{n_{k_p}}+i\operatorname{Im}\gamma(\tau_p)+w)}{\mu(\sigma_{n_{k_p}}+i\operatorname{Im}\gamma(\tau_p)+w)}.$$

From the condition $\varliminf\limits_{n\to+\infty}(\lambda_{n+1}-\lambda_n)>0$ it follows that $\lambda_{n+1}-\lambda_n\ge h>0$, i.e. $|\lambda_{n_{k_p}}-\lambda_n|\ge h|n_{k_p}-n|$.

Then by Lemma 2 for $n<n_{k_p}$, $z=\sigma_{n_{k_p}}+i\operatorname{Im}\gamma(\tau_p)+w$

$$\frac{|a_ne^{z\lambda_n}|}{|\mu(z)|}=\frac{|a_ne^{z\lambda_n}|}{\mu(\sigma_{n_{k_p}}+\operatorname{Re}w,F)}=$$

$$=\exp\left\{\sum_{k=n+1}^{n_{k_p}}\sigma_k(\lambda_k-\lambda_{k-1})+(\sigma_{n_{k_p}}+\operatorname{Re}w)(\lambda_n-\lambda_{n_{k_p}})\right\}\le$$

$$\le\exp\left\{\sum_{k=n+1}^{n_{k_p}}\sigma_{n_{k_p}}(\lambda_k-\lambda_{k-1})+(\sigma_{n_{k_p}}+\operatorname{Re}w)(\lambda_n-\lambda_{n_{k_p}})\right\}\le$$

$$\le\exp\left\{-(\lambda_{n_{k_p}}-\lambda_n)\operatorname{Re}w\right\}\le\exp\left\{-h(n_{k_p}-n)\operatorname{Re}w\right\}.$$

Similarly for $n > n_{k_p}$

$$\frac{|a_n e^{z\lambda_n}|}{|\mu(z)|} = \exp\left\{-\sum_{k=n_{k_p}+1}^{n} \sigma_k(\lambda_k - \lambda_{k-1}) + (\sigma_{n_{k_p}} + \operatorname{Re} w)(\lambda_n - \lambda_{n_{k_p}})\right\} \leq$$
$$\leq \exp\left\{-(\lambda_n - \lambda_{n_{k_p}})(\sigma_{n_{k_p}+1} - \sigma_{n_{k_p}} - \operatorname{Re} w)\right\} \leq \exp\left\{-h(n - n_{k_p})(L_1 - \operatorname{Re} w)\right\}.$$

and

$$\frac{1}{|\mu(z)|} = \exp\left\{\sum_{k=1}^{n_{k_p}} \sigma_k(\lambda_k - \lambda_{k-1}) - (\sigma_{n_{k_p}} + \operatorname{Re} w)\lambda_{n_{k_p}}\right\} \leq$$
$$\leq \exp\left\{-\lambda_{n_{k_p}} \operatorname{Re} w)\right\} \leq \exp\left\{-h n_{k_p} \operatorname{Re} w)\right\}.$$

Therefore,

$$|\Phi_p(w)| \leq 1 + \sum_{n=0}^{n_{k_p}-1} \exp\{-h(n_{k_p} - n)\operatorname{Re} w)\} + \sum_{n=n_{k_p}+1}^{\infty} \exp\{-h(L_1 - \operatorname{Re} w)(n - n_{k_p})\} \leq$$
$$\leq 1 + \sum_{n=1}^{\infty} \exp\{-hn \operatorname{Re} w)\} + \sum_{n=1}^{\infty} \exp\{-hn(L_1 - \operatorname{Re} w)\}. \tag{7}$$

From (7) it follows that the family $\{\Phi_p(w)\}$ is uniformly bounded in each close strip $\Omega_2 = \{w : \alpha \leq \operatorname{Re} w \leq \beta\} \subset \Omega_1$ and, thus, is a compact and normal family. All functions $\Phi_p(w)$ are analytic in $\Omega_2$. Therefore, there exists a subsequence of $\{\Phi_p(w)\}$ (we denote it again by $\{\Phi_p(w)\}$) convergent in $\Omega_2$ to an analytic function $\Phi(w)$.

We shall prove that $\Phi(w)$ is not constant. Indeed, we assume on the contrary that $\Phi(w) \equiv C$. From the proof of (7) we see that all $\Phi_p(w)$ have the form $1 + \sum_k b_{k,p} \exp\{\lambda_k^* w\}$, where $|\lambda_k^*| \geq h > 0$, and $\sum_k |b_{k,p}| \exp\{\lambda_k^* \sigma\} \leq K(\sigma)$, $0 < \sigma = \operatorname{Re} w < L_1$, for all $p$. Therefore,

$$\left|\frac{1}{2T}\int_{-T}^{T} \Phi_p(\sigma + it)dt - 1\right| = \left|\sum_k b_{k,p} e^{\lambda_k^* \sigma} \frac{1}{2T}\int_{-T}^{T} e^{it\lambda_k^*} dt\right| =$$
$$= \left|\sum_k \frac{b_{k,p}}{\lambda_k^*} e^{\lambda_k^* \sigma} \frac{\sin T\lambda_k^*}{T}\right| \leq \frac{K(\sigma)}{Th} \to 0, \quad T \to +\infty,$$

and, thus,

$$C = \lim_{T\to+\infty} \frac{1}{2T}\int_{-T}^{T} \Phi_p(\operatorname{Re} w + it)dt = \lim_{p\to\infty} \lim_{T\to+\infty} \frac{1}{2T}\int_{-T}^{T} \Phi_p(\operatorname{Re} w + it)dt = 1.$$

So, $\Phi(w) \equiv 1$. From boundedness of all $\Phi_p$ on $\{w : \operatorname{Re} w = \sigma\}$ boundedness of $M(\sigma, \Phi)$ follows. It is easy to show that $M(\sigma, \Phi_p) \to M(\sigma, \Phi)$ as $p \to \infty$. Therefore, by Lemma 1 for $\alpha \leq \sigma \leq \beta$ we have

$$1 = M(\sigma, \Phi) = \lim_{p\to\infty} M(\sigma, \Phi_p) \geq \frac{\pi}{4}(1 + \exp\{-(\lambda_{n_{k_p}} - \lambda_{n_{k_{p+1}}})\sigma\}) \geq \frac{\pi}{4}(1 + \exp\{-S\sigma\})$$

and if $\alpha < \dfrac{1}{S}\ln\dfrac{\pi}{4-\pi}$ then $M(\alpha,\Phi) > 1$, which is impossible. Thus, the function $\Phi(w)$ is not constant.

We denote $\gamma_p(\tau) = \gamma(\tau) - \sigma_{n_{k_p}} - i\,\mathrm{Im}\,\gamma(\tau_p)$. Then $0 \le \mathrm{Re}\,\gamma_p(\tau) \le L + o(1)$ and $-Q \le \mathrm{Im}\,\gamma_p(\tau) \le Q$, so $\gamma_p(\tau)$ is bounded.

We now consider the set $T$ of limit points of $\gamma_p(t)$ as $t \to +\infty$, $t \in I$, which lie in $\Omega_3 = \{w : \alpha \le \mathrm{Re}\,w \le \beta,\ -Q \le \mathrm{Im}\,w \le Q\} \subset \Omega_2$, and prove that they are an uncountable set on which $\Phi(w)$ is constant.

In fact, let $\Sigma$ be the intersection of $\Omega_3$ with the real axis, and define $\varphi\colon \Sigma \to \Omega_3$ as follows. For each $x \in \Sigma$ there exists $t_p \in I$ such that $\mathrm{Re}\,\gamma(t_p) = \sigma_{n_{k_p}} + x$. Then $\mathrm{Re}\,\gamma_p(\tau) = x$. Choose a limit point $v$ of $\gamma_p(\tau_p)$ and define $\varphi(x) = v$. Then $\varphi$ is one-to-one, because $\mathrm{Re}\,\varphi(x) = x$. Thus, $T$ is uncountable, since so is $\Sigma$. Furthermore, $\Phi(w)$ is constant on $T$, because if $\gamma_p(t_k) \to b \in T$ for a sequence $(t_k)$ with $t_k \in I$ then in virtue of uniform convergence $\Phi_p(\gamma_p(t_k)) \to \Phi(b)$. But we are assuming that $\omega$ is a uniformly oscillating $\mu$-asymptotic value and so $\Phi(b) = \omega$. Hence $\Phi$ is constant on $T$. This is a contradiction. Therefore, $\Phi$ has no uniformly oscillating $\mu$-asymptotic values.

For uniformly oscillating $m$-asymptotic values we define

$$\Psi_p(w) = \frac{F(\sigma_{n_{k_p}} + i\,\mathrm{Im}\,\gamma(\tau_p) + w)}{m(\sigma_{n_{k_p}} + i\,\mathrm{Im}\,\gamma(\tau_p) + w)}$$

and still have (7) holding with $\Phi_p$ replaced by $\Psi_p$. Thus, $\{\Psi_p(w)\}$ is a normal family and the rest of the proof goes through in exactly the same manner as for uniformly oscillating $\mu$-asymptotic values. $\square$

**4°. Remarks.** Directly from Theorem, for example, the corollary follows.

**Corollary 1.** *Suppose that in (1) all* $a_n > 0$ *and* $\varkappa_n = \frac{\ln a_n - \ln a_{n+1}}{\lambda_n - \lambda_{n+1}} \nearrow +\infty\,(n \to \infty)$. *If* $0 < h \le \lambda_{n+1} - \lambda_n \le H < +\infty$ *and* $\varlimsup\limits_{n\to\infty}(\varkappa_{n+1} - \varkappa_n) = L > 0$ *then* $\mu(\sigma,F)/F(\sigma)$ *has no limit as* $\sigma \to +\infty$.

One cannot remove the condition $\varlimsup\limits_{n\to\infty}(\varkappa_{n+1} - \varkappa_n) = L > 0$ in general. Indeed, for the entire Dirichlet series

$$F(s) = \exp\{e^s\} = 1 + \sum_{n=1}^{\infty} \exp\{sn\}/n!$$

we have $\lambda_{n+1} - \lambda_n = 1$, $\varkappa_n = \ln(n+1)$, $\varkappa_{n+1} - \varkappa_n \to 0\,(n \to \infty)$ and [6] $\mu(\sigma,F)/F(\sigma) \to 0\,(\sigma \to +\infty)$.

In general, one cannot remove also the condition $\lambda_{n+1} - \lambda_n \ge h > 0$. Indeed, let

$$F(s) = 1 + \sum_{n=2}^{\infty} \exp\{-n + s\ln n\}. \tag{8}$$

It is easy to show that $\lambda_{n+1} - \lambda_n \to 0$, $\varkappa_{n+1} - \varkappa_n = 1 + o(1)$ $(n \to \infty)$, $\varkappa_n = 1/(\ln(n+1) - \ln n)$, and

$$\mu(\sigma,F) \le \exp\{\max\{-x + \sigma\ln x : x \ge 1\}\} = (\sigma/e)^{\sigma}.$$

On the other hand, for Dirichlet series (10) we have

$$F(\sigma) = \int_1^\infty e^{-t+\sigma \ln t} dt + O(\mu(\sigma, F)) = \int_0^\infty \exp\{-e^t + (\sigma+1)t\} dt + O((\sigma/e)^\sigma), \quad \sigma \to +\infty.$$

Using the Laplace method [4, p. 20–22] we can show that

$$I(\sigma) = \int_0^\infty \exp\{-e^t + (\sigma+1)t\} dt = (1+o(1)) \exp\left\{(\sigma+1)\ln \frac{\sigma+1}{e}\right\} \sqrt{\frac{2\pi}{\sigma+1}}, \quad \sigma \to +\infty,$$

and, since $\left(\frac{\sigma}{e}\right)^\sigma / I(\sigma) = \dfrac{1+o(1)}{\sqrt{2\pi\sigma}}$ $(\sigma \to +\infty)$, we have $\mu(\sigma, F)/F(\sigma) \to 0$, $(\sigma \to +\infty)$.

Therefore, for the constructed function $F$ $+\infty$ is a $\mu$- (and $m$-) uniformly oscillating asymptotic value.

The condition $\lambda_{n+1} - \lambda_n \le H < +\infty$ arose in virtue of the applied method. It seems that it is unnecessary, because, for example, if $\sum_{n=1}^\infty \frac{1}{\lambda_{n+1}-\lambda_n} < +\infty$ then [5] $F(\sigma) \sim m(\sigma, F)$ as $\sigma \to +\infty$ outside a set of finite measure and at the same time $\underline{\lim}_{\sigma\to+\infty} \mu(\sigma, F)/F(\sigma) \le 2/\pi$, when $a_n > 0$ $(n \ge 1)$.

Finally, the question arises whether it is possible to replace in Theorem the uniformly oscillating $\mu$-asymptotic ($m$-asymptotic) value by the $\mu$-asymptotic ($m$-asymptotic) value. The following assertion seems to be true.

**Conjecture.** *There exists an entire Dirichlet series (1) such that the conditions of Theorem hold, but $F$ has no $\mu$-asymptotic ($m$-asymptotic) value.*

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*Received 1.10.2003*