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ON A GROSS CONJECTURE CONCERNING ENTIRE FUNCTIONS OF BOUNDED INDEX

M. M. Sheremeta. *On a Gross conjecture concerning entire functions of bounded index*, *Matematychni Studii*, **18** (2002) 211–212.

The following Gross conjecture is true: if an entire function f is of unbounded index and p is a polynomial, then the entire function $F(z) = f(p(z))$ is of unbounded index.

М. Н. Шеремета. *Об одной гипотезе Гросса относительно целых функций ограниченного индекса* // *Математичні Студії*. – 2002. – Т.18, №2. – С.211–212.

Верна следующая гипотеза Гросса: если f — целая функция неограниченного индекса, а p — многочлен, то $F(z) = f(p(z))$ — целая функция неограниченного индекса.

Let l be a positive continuous function on $[0, +\infty)$. An entire function f is said to be of bounded l -index [1, p. 67] if there exists $N \in \mathbb{Z}_+$ such that for all $n \in \mathbb{Z}_+$ and $z \in \mathbb{C}$

$$\frac{|f^{(n)}(z)|}{n!l^n(|z|)} \leq \max \left\{ \frac{|f^{(k)}(z)|}{k!l^k(|z|)} : 0 \leq k \leq N \right\}.$$

For $l(r) \equiv 1$, we obtain a definition of entire function of bounded index [2]. F. Gross [3] formulated the following

Conjecture. *If an entire function f is of unbounded index and p is a polynomial then the entire function $F(z) = f(p(z))$ is of unbounded index.*

G. Fricke [4] proved that $f(az + b)$ is of bounded index if and only if $f(z)$ is of bounded index. In general case we have [5; 1, p. 97] the following result.

Theorem. *Let $p(z) = a_n z^n + \dots + a_0$, $a_n \neq 0$, $n \geq 2$, and f be an entire function. Let l be a positive continuous on $[0, +\infty)$ function such that $l(r) \asymp r^{n-1}$, $1 \leq r \rightarrow +\infty$. Then $F(z) = f(p(z))$ is of bounded l -index if and only if f is of bounded index.*

Theorem implies that if f is of unbounded index then $F(z) = f(p(z))$ is of unbounded l -index with $l(r) \asymp r^{n-1}$, $1 \leq r \rightarrow +\infty$, whence it follows that $F(z) = f(p(z))$ is of unbounded index, because if F is of bounded l_1 -index and $l_2(r) \leq l_1(r)$ then F is of bounded l_2 -index [1, p. 23].

Thus, the Gross conjecture is true.

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