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## ON A GROSS CONJECTURE CONCERNING ENTIRE FUNCTIONS OF BOUNDED INDEX

M. M. Sheremeta. *On a Gross conjecture concerning entire functions of bounded index*, *Matematychni Studii*, **18** (2002) 211–212.

The following Gross conjecture is true: if an entire function  $f$  is of unbounded index and  $p$  is a polynomial, then the entire function  $F(z) = f(p(z))$  is of unbounded index.

М. Н. Шеремета. *Об одной гипотезе Гросса относительно целых функций ограниченного индекса* // *Математичні Студії*. – 2002. – Т.18, №2. – С.211–212.

Верна следующая гипотеза Гросса: если  $f$  — целая функция неограниченного индекса, а  $p$  — многочлен, то  $F(z) = f(p(z))$  — целая функция неограниченного индекса.

Let  $l$  be a positive continuous function on  $[0, +\infty)$ . An entire function  $f$  is said to be of bounded  $l$ -index [1, p. 67] if there exists  $N \in \mathbb{Z}_+$  such that for all  $n \in \mathbb{Z}_+$  and  $z \in \mathbb{C}$

$$\frac{|f^{(n)}(z)|}{n!l^n(|z|)} \leq \max \left\{ \frac{|f^{(k)}(z)|}{k!l^k(|z|)} : 0 \leq k \leq N \right\}.$$

For  $l(r) \equiv 1$ , we obtain a definition of entire function of bounded index [2]. F. Gross [3] formulated the following

**Conjecture.** *If an entire function  $f$  is of unbounded index and  $p$  is a polynomial then the entire function  $F(z) = f(p(z))$  is of unbounded index.*

G. Fricke [4] proved that  $f(az + b)$  is of bounded index if and only if  $f(z)$  is of bounded index. In general case we have [5; 1, p. 97] the following result.

**Theorem.** *Let  $p(z) = a_n z^n + \dots + a_0$ ,  $a_n \neq 0$ ,  $n \geq 2$ , and  $f$  be an entire function. Let  $l$  be a positive continuous on  $[0, +\infty)$  function such that  $l(r) \asymp r^{n-1}$ ,  $1 \leq r \rightarrow +\infty$ . Then  $F(z) = f(p(z))$  is of bounded  $l$ -index if and only if  $f$  is of bounded index.*

Theorem implies that if  $f$  is of unbounded index then  $F(z) = f(p(z))$  is of unbounded  $l$ -index with  $l(r) \asymp r^{n-1}$ ,  $1 \leq r \rightarrow +\infty$ , whence it follows that  $F(z) = f(p(z))$  is of unbounded index, because if  $F$  is of bounded  $l_1$ -index and  $l_2(r) \leq l_1(r)$  then  $F$  is of bounded  $l_2$ -index [1, p. 23].

Thus, the Gross conjecture is true.

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