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E. D. Tymchatyn, M. M. Zarichnyi*

A NOTE ON A QUESTION OF NAGATA

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An example is described that answers in the negative a question of Nagata on metrization of rim-compact spaces.

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Приведен пример, дающий отрицательный ответ на один вопрос Нагаты о метризации периферически компактных пространств.

1. Introduction. A metrizable space is *rim-compact* (respectively *rim-finite*) if it has a base of topology consisting of sets with compact (respectively finite) boundaries.

The following question is formulated in [1]; see also [2].

Question. *Let X be a rim-compact metrizable space. Is there a compatible metric on X such that the boundary $\partial B_\varepsilon(x)$ of the ball $B_\varepsilon(x)$ is compact for all $x \in X$ and $\varepsilon > 0$?*

The aim of this note is to answer the question in the negative.

2. Example. We will construct a subset in plane $\mathbb{R}^2$.

Given a binary rational number, $r = m/(2^n)$, where $m, n \in \mathbb{N}$, m is odd, $0 < m < 2^n$, denote by $S(r)$ the (closed) upper semicircle of radius $1/(2^n)$ centered at $(r, 0)$. Let

$$X = \bigcup\{S(r) \mid r \text{ is binary rational, } 0 < r < 1\}.$$

Then it is easy to see that X is a rim-finite space.

Let d be any metric on X inducing its initial topology. There exists ε such that $0 < \varepsilon < d((0,0),(1,0))$ and

$$\varepsilon \notin \{d((0,0),(r,0)) \mid r \text{ is binary rational, } 0 < r < 1\}.$$

Show that $\partial B_\varepsilon(0,0)$ is not compact. For every $n \in \mathbb{N}$ denote by X_n the arc in X that is the union of all semicircles of radius 2^{-n} in X. Then X_n connects the points $(0,0)$ and $(1,0)$ and therefore the set $X_n \cap \partial B_\varepsilon(0,0)$ is nonempty for every $n \in \mathbb{N}$. The sets $X'_n = \{(x,t) \in X_n \mid t > 0\}$, $n \in \mathbb{N}$, form disjoint open cover of $\partial B_\varepsilon(0,0)$ that has no finite subcover.

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Department of Mathematics and Statistics, University of Saskatchewan,
Room 142 McLean Hall, 106 Wiggins Road, Saskatoon, SK. S7N 5E6 Canada
tymchat@math.usask.ca

Department of Mechanics and Mathematics, Lviv National University,
Universytetska 1, 79000 Lviv, Ukraine
topos@franko.lviv.ua, mzar@litech.lviv.ua

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