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ELEMENTARY REDUCTION OF MATRICES OVER ADEQUATE DOMAIN

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We prove that if R is an adequate domain, then every $k\times(k+2)$ and $(k+2)\times k$ matrices, where $k\geq 2$, admits a diagonal reduction by elementary transformations.

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Доказано, что если R — адекватная область, то произвольные $k\times(k+2)$ и $(k+2)\times k$ матрицы, где $k\geq 2$, допускают диагональною редукцию элементарными преобразованиями.

The aim of this note is to study the question of diagonalizability for matrices over an adequate domain. We prove that if R is an adequate domain, then every $k\times k+2$ and $k+2\times k$ matrices, where $k\geq 2$, admits a diagonal reduction by elementary transformations.

All the rings considered will be commutative and have the identity.

A ring is a *Bezout ring* if every finitely generated ideal is principal.

A ring R said to be an *adequate domain* if R is a Bezout domain and for $a,b\in R$ with $a\neq 0$ there exist $r,s\in R$ such that $a=rs$, $rR+bR=R$, and if a nonunit element s' divides s then $s'R+bR\neq R$ [1].

Two matrices A and B over a ring R are said to be *equivalent* if there exist invertible matrices P, Q such that $B=PAQ$. A matrix A *admits diagonal reduction* if A is equivalent to a diagonal matrix [2].

We denote by R_n the ring of all $n\times n$ matrices over R, and by $GL_n(R)$ its group of unites. We write $GE_n(R)$ for the subgroup of $GL_n(R)$ generated by the elementary matrices. Denote by (a,b) the greatest common divisor of $a,b\in R$.

Theorem. *Over a commutative adequate domain R, every $k\times k+2$ and $k+2\times k$ matrix, where $k\geq 2$, admits a diagonal reduction by elementary transformations.*

Proof. We prove the theorem by induction on the number of rows. Let A be a 2×4 matrix. Without loss of generality we may change notations and assume that the greatest

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common divisor of all elements, of a is 1 [2]. By [3], the matrix A is reduced by elementary transformations on the left to the form

$$A_1 = \begin{pmatrix} a & 0 & 0 & 0 \\ b & c & d & k \end{pmatrix}.$$

If $b = 0$ by [3], the matrix A can be reduced by elementary transformations to a diagonal form.

If $b \neq 0$, write $(a, b, c) = \delta$, $a = a_0\delta$, $b = b_0\delta$, $c = c_0\delta$ and $(a_0, b_0, c_0) = 1$. Write $c_0 = rs$ with the property asserted above. Then

$$(ra + b, c) = (a, b, c) = \delta.$$

Multiplying the first row of a matrix A_1 by r and adding it to the second row, we obtain the matrix

$$A_2 = \begin{pmatrix} a & 0 & 0 & 0 \\ ra + b & c & d & k \end{pmatrix}.$$

Since $(ra + b, c) = (a, b, c)$ and $(a, b, c, d, k) = 1$, we have

$$(ra + b, c, d, k) = 1.$$

By [3], the matrix A_2 is reducible by elementary transformations to the form

$$A_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ * & * & * & * \end{pmatrix}.$$

It is obvious that A_3 is reducible by elementary transformations to a diagonal form. Induction on the number of rows completes the proof. □

Corollary. *Over a commutative adequate domain R, for every $n \times m$ matrix A, where n, $m \geq 2$, there exist invertible matrices $P \in GE_n(R)$, $Q \in GE_m(R)$ such that PAQ is diagonal matrix.*

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