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ON VERSIONS OF THE GIBBARD-SATTERTHWEIT THEOREM

Let A be a nonempty finite set. The elements of A are called *alternatives*. We denote by $\mathcal{O}(A)$ the set of all order relations on A . If F is a (covariant) endofunctor acting on the category of sets then a map $f: \mathcal{O}(A)^n \rightarrow F(A)$ is called an *F -valued social choice function* (see [1]).

Suppose $\mathbb{F} = (F, \eta, \mu)$ is a monad on the category of sets [2] and there exists an \mathbb{F} -algebra structure on the segment $[0, 1]$ (and therefore, on every segment).

Given an order relation $\preceq \in \mathcal{O}(A)$, consider its Borda utility function $B_{\preceq}: A \rightarrow [0, |A|-1]$, $B_{\preceq}(a) = |\{b \in A \mid b \prec a\}|$. Define the function $u_{\preceq}: F(A) \rightarrow [0, |A|-1]$ by $u_{\preceq} = \alpha \circ F(B_{\preceq})$ (here $\alpha: F([0, |A|-1]) \rightarrow [0, |A|-1]$ is a fixed \mathbb{F} -algebra structural map). The function u_{\preceq} is a utility function of a pre-order relation $\tilde{\preceq}$ on $F(A)$.

We say that f is *non-manipulable (strategy-proof)* if there is no $\preceq'_i \in \mathcal{O}(A)$ and $(\preceq_1, \dots, \preceq_n) \in \mathcal{O}(A)^n$ such that

$$f(\preceq_1, \dots, \preceq_n) \tilde{\prec}_i f(\preceq_1, \dots, \preceq_{i-1}, \preceq'_i, \preceq_{i+1}, \dots, \preceq_n)$$

(compare with [4], [3]). A general problem is: *describe all non-manipulable F -valued social choice functions*. The classical Gibbard-Satterthwaite theorem gives such a description for the identity functor; some results are known also in the case of power-set functor and probability measure functor).

REFERENCES

1. Bazylevych L. and Zarichnyi M., *Monads in the social choice theory*, in Ukrainian (to appear).
2. Barr M., Wells Ch. *Toposes, triples and theories*, Berlin: Springer-Verlag, 1985.
3. Satterthwaite M. A. *Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions*, J. Econom. Theory **10** (1975), no. 2, 187–217.
4. Gibbard A. *Manipulation of schemes that mix voting with chance*, Econometrica **45** (1977), no. 3, 665–681.

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