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L. Ye. Bazylevych, M. M. Zarichnyi

ON VERSIONS OF THE GIBBARD-SATTERTHWEIT THEOREM

Let A be a nonempty finite set. The elements of A are called *alternatives*. We denote by $\mathcal{O}(A)$ the set of all order relations on A. If F is a (covariant) endofunctor acting on the category of sets then a map $f\colon \mathcal{O}(A)^n \to F(A)$ is called an *F-valued social choice function* (see [1]).

Suppose $\mathbb{F} = (F, \eta, \mu)$ is a monad on the category of sets [2] and there exists an $\mathbb{F}$-algebra structure on the segment $[0, 1]$ (and therefore, on every segment).

Given an order relation $\preceq \in \mathcal{O}(A)$, consider its Borda utility function $B_\preceq\colon A \to [0, |A|-1]$, $B_\preceq(a) = |\{b \in A \mid b \prec a\}|$. Define the function $u_\preceq\colon F(A) \to [0, |A| - 1]$ by $u_\preceq = \alpha \circ F(B_\preceq)$ (here $\alpha\colon F([0, |A| - 1]) \to [0, |A| - 1]$ is a fixed $\mathbb{F}$-algebra structural map). The function $u_\preceq$ is a utility function of a pre-order relation $\tilde{\preceq}$ on $F(A)$.

We say that f is *non-manipulable (strategy-proof)* if there is no $\preceq_i' \in \mathcal{O}(A)$ and $(\preceq_1, \dots, \preceq_n) \in \mathcal{O}(A)^n$ such that

$$f(\preceq_1, \dots, \preceq_n) \tilde{\prec}_i f(\preceq_1, \dots, \preceq_{i-1}, \preceq_i', \preceq_{i+1}, \dots, \preceq_n)$$

(compare with [4], [3]). A general problem is: *describe all non-manipulable F-valued social choice functions*. The classical Gibbard-Satterthwaite theorem gives such a description for the identity functor; some results are known also in the case of power-set functor and probability measure functor).

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Department of Applied Mathematics, National University "Lviv Polytechnica"
Department of Mechanics and Mathematics, Lviv National University
mzar@litech.lviv.ua

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