УДК 517.537.72

M. M. Sheremeta

ESTIMATES OF THE MAXIMAL TERM OF ENTIRE DIRICHLET SERIES IN TERMS OF TWO-MEMBER ASYMPTOTICS

M. M. Sheremeta. *Estimates of the maximal term of entire Dirichlet series in terms of two-member asymptotics*, Matematychni Studii, **14** (2000) 159–164.

Conditions on coefficients and exponents of an entire Dirichlet series are found in order that the logarithm of the maximal term $\mu(\sigma, F)$ admits an upper estimate of the form $\ln\,\mu(\sigma, F) \le \Phi_1(\sigma) + (1 + o(1))\tau\Phi_2(\sigma)$ as $\sigma \to +\infty$, and the same lower estimate, where τ is a real number, Φ_1, Φ_2 are positive functions satisfying some condtions.

М. Н. Шеремета. *Оценки максимального члена целого ряда Дирихле в терминах двучленной асимптотики* // Математичні Студії. – 2000. – Т.14, №2. – С.159–164.

Найдены условия на коэффициенты и показатели целого ряда Дирихле для того, чтобы логарифм максимального члена $\mu(\sigma, F)$ допускал оценку $\ln\,\mu(\sigma, F) \le \Phi_1(\sigma) + (1+o(1))\tau\Phi_2(\sigma)$ $(\sigma \to +\infty)$, и такую же оценку снизу, где τ — действительное число, а Φ_1, Φ_2 — положительные функции, удовлетворяющие некоторым условиям.

1°. Introduction. Let $0 \le \lambda_n \uparrow +\infty$ $(0 \le n \to \infty)$ and the Dirichlet series

$$F(s) = \sum_{n=0}^{\infty} a_n \exp(s\lambda_n), \quad s = \sigma + it, \tag{1}$$

has the abscissa of absolute convergence $\sigma_a = +\infty$. For $\sigma \in (-\infty, +\infty)$ let $\mu(\sigma, F) = \max\{|a_n|\exp(\sigma\lambda_n) : n \ge 1\}$ be the maximal term of series (1).

We will study the conditions on a_n and λ_n under which $\ln\mu(\sigma, F) \ge \Phi_1(\sigma) + (1 + o(1))\tau_1\Phi_2(\sigma)$ $(\sigma \to +\infty)$, and $\ln\mu(\sigma, F) \le \Phi_1(\sigma) + (1 + o(1))\tau_2\Phi_2(\sigma)$ $(\sigma \to +\infty)$, where $\tau_1, \tau_2 \in \mathbb{R}$ and Φ_1, Φ_2 are positive functions satisfying some conditions.

Put

$$P(t) = \begin{cases} \ln\,|a_n|, & t = \lambda_n\,(n \in \mathbb{Z}_+), \\ -\infty, & t \in (0, +\infty) \setminus \{\lambda_n\}, \end{cases} \qquad Q(\sigma) = \sup\{P(t) + \sigma t : t > 0\}. \tag{2}$$

Then $\ln\,\mu(s, F) = Q(\sigma)$ and the problem is reduced to the study of validity of the relation $Q(\sigma) \ge \Phi_1(\sigma) + (1 + o(1))\tau_1\Phi_2(\sigma)$ $(\sigma \to +\infty)$, and $Q(\sigma) \le \Phi_1(\sigma) + (1 + o(1))\tau_2\Phi_2(\sigma)$ $(\sigma \to +\infty)$.

2000 *Mathematics Subject Classification:* 30B50.

Further we assume that P is an arbitrary function different from $+\infty$ (it can achieve the value $-\infty$ but $P \not\equiv -\infty$). The functions P and Q are said to be Young conjugated functions.

Let Ω be the class of positive unbounded on $(-\infty, +\infty)$ functions Φ such that the derivative Φ' is positive continuously differentiable and increasing to $+\infty$ on $(-\infty, +\infty)$. Clearly, the function $\Phi \in \Omega$ is convex on $(-\infty, +\infty)$, $\Phi(x) \to c \geq 0$ $(x \to -\infty)$, $\Phi'(x) \to 0$ $(x \to -\infty)$. From now on, we denote by φ the inverse function to Φ', and let $\Psi(x) = x - \Phi(x)/\Phi'(x)$ be the function associated with Φ in the sense of Newton. It is clear that the function φ is continuously differentiable and increasing to $+\infty$ on $(0, +\infty)$. The function Ψ is continuously differentiable and increasing to $+\infty$ on $(-\infty, \infty)$.

By L^0 we denote the class of positive continuous on $[x_0, +\infty)$ functions l such that $l\big((1 + o(1))\, x\big) = (1 + o(1))l(x)$ as $x \to +\infty$.

Finally, we will say that a positive twice continuously differentiable increasing to $+\infty$ on $(-\infty, +\infty)$ function Φ_2 is subordinated to $\Phi_1 \in \Omega$, if $\Phi_2'(\varphi_1) \in L^0$, $\Phi_2'(\sigma) = o(\sigma\Phi_1''(\sigma))$ and $\Phi_2''(\sigma) = o(\Phi_1''(\sigma))$ as $\sigma \to +\infty$.

2°. Upper estimates. We need the following

Lemma 1 [1]. *Let $\Phi \in \Omega$. In order that $Q(\sigma) \leq \Phi(\sigma)$ for all $\sigma \geq \sigma_0$, it is necessary and sufficient that $P(t) \leq -t\Psi(\varphi(t))$ for all $t \geq t_0$.*

We prove here the following

Theorem 1. *Let $\Phi_1 \in \Omega$, $\varphi_1' \in L^0$, and a function Φ_2 be subordinated to Φ_1. In order that*

$$Q(\sigma) \leq \Phi_1(\sigma) + (1 + o(1))\tau\Phi_2(\sigma), \quad \sigma \to +\infty, \tag{3}$$

it is necessary and sufficient that

$$P(t) \leq -t\Psi_1(\varphi_1(t)) + (1 + o(1))\tau\Phi_2(\varphi_1(t)), \quad t \to \infty. \tag{4}$$

Proof. Consider the function $\Phi(\sigma) = \Phi_1(\sigma) + \tau\Phi_2(\sigma)$. Since $\Phi_1 \in \Omega$ and $\Phi_2''(\sigma) = o(\Phi_1''(\sigma))$ as $\sigma \to A$, we have $\Phi''(\sigma) = (1 + o(1))\Phi_1''(\sigma)$, $\Phi_2'(\sigma) = o(\Phi_1'(\sigma))$ and $\Phi_2(\sigma) = o(\Phi_1(\sigma))$ as $\sigma \to A$, whence in particular it follows that Φ' is positive and increasing to $+\infty$ on $[\sigma_0, A)$. For simplicity we assume that $\sigma_0 = -\infty$, i. e. $\Phi \in \Omega(A)$.

Clearly, the inverse function φ to Φ' satisfies the equation

$$\Phi_1'(\sigma) + \tau\Phi_2'(\sigma) = x. \tag{5}$$

Since $\Phi_2'(\sigma) = o(\Phi_1'(\sigma))$ $(\sigma \to +\infty)$, we look for a solution of (5) in the form

$$\sigma = \varphi_1(x - y), \quad y = y(x) = o(x), \quad x \to +\infty. \tag{6}$$

Substituting (6) in (5) and taking into account the condition $\Phi_2'(\varphi_1) \in L^0$, we obtain

$$y = \tau\Phi_2'(\varphi_1((1 + o(1))\, x) = (1 + o(1))\tau\Phi_2'(\varphi_1(x)), \quad x \to +\infty.$$

Therefore, from (6) in view of the condition $\varphi_1' \in L^0$ we have

$$\begin{aligned}\varphi(x) &= \varphi_1(x - (1 + o(1))\tau\Phi_2'(\varphi_1(x))) = \\ &= \varphi_1(x) - \{\varphi_1(x) - \varphi_1(x - (1 + o(1))\tau\Phi_2'(\varphi_1(x)))\} = \\ &= \varphi_1(x) - \varphi_1'(x + O(\Phi_2'(\varphi_1(x))))(1 + o(1))\tau\Phi_2'(\varphi_1(x))),\end{aligned}$$

whence in virtue of the relation $\Phi_2'(\sigma) = o(\Phi_1'(\sigma))\,(\sigma \to +\infty)$ and the condition $\varphi_1' \in L^0$, it follows that

$$\varphi(x) = \varphi_1(x) - (1+o(1))\tau\Phi_2'(\varphi_1(x)))\varphi_1'(x), \quad x \to +\infty. \tag{7}$$

(We remark that the equality $\Phi_1''(\varphi_1(x))\varphi_1'(x) \equiv 1$ and the condition $\Phi_2'(\sigma) = o(\sigma\Phi_1''(\sigma))$ $(\sigma \to +\infty)$, imply $\Phi_2'(\varphi_1(x)))\varphi_1'(x) = o(\varphi_1(x))$ $(x \to +\infty)$.)

But

$$x\Psi(\varphi(x)) - x_0\Psi(\varphi(x_0)) = \int_{x_0}^{x} (x\varphi(x) - \Phi(\varphi(x)))'dx = \int_{x_0}^{x} \varphi(x)dx. \tag{8}$$

Therefore, from (7) we have

$$\begin{aligned} t_n\Psi(\varphi(t_n)) &= \int_{x_0}^{t_n} \varphi_1(x)dx - \int_{x_0}^{t_n} (1+o(1))\tau\Phi_2'(\varphi_1(x)))\varphi_1'(x)dx + O(1) = \\ &= t_n\Psi_1(\varphi_1(t_n)) - (1+o(1))\tau\Phi_2(\varphi_1(t_n))) + O(1) = \\ &= t_n\Psi_1(\varphi_1(t_n)) - (1+o(1))\tau\Phi_2(\varphi_1(t_n))), \quad n \to \infty, \end{aligned}$$

Finally, Lemma 1 completes the proof. □

Choosing $P(t)$ as in (2), we obtain from Theorem 1 the following

Corollary 1. *Let $\Phi_1 \in \Omega$, $\varphi_1' \in L^0$, and the function Φ_2 is subordinate to Φ_1. In order that*

$$\ln \mu(\sigma, F) \le \Phi_1(\sigma) + (1+o(1))\tau\Phi_2(\sigma), \quad \sigma \to +\infty,$$

it is necessary and sufficient that

$$\ln |a_n| \le -\lambda_n\Psi_1(\varphi_1(\lambda_n)) + (1+o(1))\tau\Phi_2(\varphi_1(\lambda_n)), \quad n \to \infty.$$

3°. Lower estimates. For $\Phi \in \Omega$, $0 < a < b < +\infty$ and $q > 0$ we put

$$G_1(a,b,\Phi) = \frac{ab}{b-a}\int_a^b \frac{\Phi(\varphi(t))}{t^2}dt, \quad G_2(a,b,\Phi) = \Phi\left(\frac{1}{b-a}\int_a^b \varphi(t)dt\right),$$

where φ is the inverse function to Φ'.

Lemma 2 [2]. *The inequality $G_1(a,b,\Phi) < G_2(a,b,\Phi)$ holds.*

Lemma 3 [1]. *Let $A \in (-\infty, +\infty)$, $\Phi \in \Omega$ and*

$$P(t_n) \ge -t_n\Psi(\varphi(t_n)) \tag{9}$$

for a some increasing to $+\infty$ sequence (t_n) of positive numbers.

Then for all $n \ge n_0$ and $\sigma \in [\varphi(t_n),\, \varphi(t_{n+1})]$

$$Q(\sigma) \ge \Phi(\sigma) + G_1(t_n, t_{n+1}, \Phi) - G_2(t_n, t_{n+1}, \Phi). \tag{10}$$

Theorem 2. *Let $\Phi_1 \in \Omega$, $\varphi_1' \in L^0$, a function Φ_2 be subordinated to Φ_1, and*

$$\Phi_j'\left(\sigma + O\left(\frac{\Phi_2'(\sigma)}{\Phi_1''(\sigma)}\right)\right) \sim \Phi_j'(\sigma) \quad (\sigma \to +\infty), \quad j = 1,\, 2. \tag{11}$$

If

$$P(t_n) \geq -t_n\Psi_1(\varphi_1(t_n)) + (1+o(1))\tau\Phi_2(\varphi_1(t_n)), \quad n\to\infty, \tag{12}$$

for a some increasing to $+\infty$ *sequence* (t_n) *such that* $t_{n+1}\sim t_n\,(n\to\infty)$ *and*

$$\frac{G_2(t_n,t_{n+1},\Phi_1) - G_1(t_n,t_{n+1},\Phi_1)}{\Phi_2(\varphi_1(t_n))} \to 0, \quad n\to\infty, \tag{13}$$

then

$$Q(\sigma) \geq \Phi_1(\sigma) + (1+o(1))\tau\Phi_2(\sigma), \quad \sigma\to+\infty, \tag{14}$$

Proof. As above, let $\Phi(\sigma)=\Phi_1(\sigma)+\tau\Phi_2(\sigma)$. Put

$$\varkappa_n = \frac{1}{t_{n+1}-t_n}\int_{t_n}^{t_{n+1}} \varphi(t)dt.$$

In view of (7),

$$\begin{aligned}\varkappa_n =& \frac{1}{t_{n+1}-t_n}\int_{t_n}^{t_{n+1}} \varphi_1(t)dt - \frac{(1+o(1))\tau}{t_{n+1}-t_n}\int_{t_n}^{t_{n+1}} \Phi_2'(\varphi_1(t))\varphi_1'(t)dt = \\ =& \varkappa_n^{(1)} - (1+o(1))\tau\xi_n, \quad \xi_n = \frac{\Phi_2(\varphi_1(t_{n+1}))-\Phi_2(\varphi_1(t_n))}{t_{n+1}-t_n}.\end{aligned}$$

Therefore,

$$\begin{aligned}&G_2(t_n,t_{n+1},\Phi) = \Phi(\varkappa_n) = \Phi_1(\varkappa_n^{(1)} - (1+o(1))\tau\xi_n) + \tau\Phi_2(\varkappa_n^{(1)} - (1+o(1))\tau\xi_n) = \\ &= \Phi_1(\varkappa_n^{(1)}) - (1+o(1))\tau\xi_n\Phi_1'(\varkappa_n^{(1)} - (1+o(1))\eta\tau\xi_n) + \tau\Phi_2(\varkappa_n^{(1)} - (1+o(1))\tau\xi_n) = \\ &= G_2(t_n,t_{n+1},\Phi_1) - (1+o(1))\tau\xi_n\Phi_1'(\varkappa_n^{(1)} - (1+o(1))\eta\tau\xi_n) + \\ &+ \tau\Phi_2(\varkappa_n^{(1)} - (1+o(1))\tau\xi_n), \quad n\to\infty, \quad 0<\eta<1.\end{aligned}$$

Further, in view of (8) and (7),

$$\begin{aligned}&G_1(t_n,t_{n+1},\Phi) = \frac{t_{n+1}t_n}{t_{n+1}-t_n}\int_{t_n}^{t_{n+1}}\Phi(\varphi(t))d\left(-\frac{1}{t}\right) = \\ &= \frac{t_{n+1}t_n}{t_{n+1}-t_n}\left(\frac{\Phi(\varphi(t_n))}{t_n} - \frac{\Phi(\varphi(t_{n+1}))}{t_{n+1}} + \int_{t_n}^{t_{n+1}}\varphi'(t)dt\right) = \\ &= \frac{t_{n+1}t_n}{t_{n+1}-t_n}(\Psi(\varphi(t_{n+1})) - \Psi(\varphi(t_n))) = \\ &= t_n\left(\frac{t_{n+1}\Psi(\varphi(t_{n+1})) - t_n\Psi(\varphi(t_n))}{t_{n+1}-t_n} - \Psi(\varphi(t_n))\right) = t_n\varkappa_n - t_n\Psi(\varphi(t_n)) = \\ &= t_n\varkappa_n^{(1)} - (1+o(1))t_n\tau\xi_n - t_n\Psi_1(\varphi_1(t_n)) + (1+o(1))\tau\Phi_2(\varphi_1(t_n)) = \\ &= G_1(t_n,t_{n+1},\Phi_1) - (1+o(1))t_n\tau\xi_n + (1+o(1))\tau\Phi_2(\varphi_1(t_n)), \quad n\to\infty.\end{aligned}$$

Therefore,

$$\begin{aligned}&G_2(t_n,t_{n+1},\Phi) - G_1(t_n,t_{n+1},\Phi) = G_2(t_n,t_{n+1},\Phi_1) - G_1(t_n,t_{n+1},\Phi_1) + \\ &+ (1+o(1))t_n\tau\xi_n - (1+o(1))\tau\xi_n\Phi_1'(\varkappa_n^{(1)} - (1+o(1))\eta\tau\xi_n) + \\ &+ \tau\Phi_2(\varkappa_n^{(1)} - (1+o(1))\tau\xi_n) - (1+o(1))\tau\Phi_2(\varphi_1(t_n)), \quad n\to\infty.\end{aligned} \tag{15}$$

It is easy to show that when the function l is continuously differentiable then $l \in L^0$ if and only if $xl'(x)/l(x) = O(1)$ $(x \to +\infty)$. Hence it follows that $\lambda \in L^0$ if $l \in L^0$ and $\lambda(x) = \int_{x_0}^{x} l(t)dt$.

Therefore, since $\Phi_2'(\varphi_1) \in L^0$ and $\varphi_1' \in L^0$, we have $\Phi_2(\varphi_1) \in L^0$ and

$$\frac{x\Phi_2'(\varphi_1(x))\varphi_1'(x)}{\Phi_2(\varphi_1(x))} = O(1), \quad x \to +\infty. \tag{16}$$

From the condition $\varphi_1' \in L^0$ it follows also that $\varphi_1 \in L^0$ and $\Phi_1''(\varphi_1) \in L^0$.

Since $\xi_n = \Phi_2'(\varphi_1(\eta_n))\varphi_1'(\eta_n)$ $(t_n < \eta_n < t_{n+1})$, $\Phi_2'(\varphi_1)\varphi_1' \in L^0$ and $t_{n+1} \sim t_n$ $(n \to \infty)$, we have

$$\xi_n = (1 + o(1))\Phi_2'(\varphi_1(t_n))\varphi_1'(t_n), \quad n \to \infty,$$

and in view of (16)

$$\frac{t_n\xi_n}{\Phi_2(\varphi_1(t_n))} = O(1), \quad n \to \infty. \tag{17}$$

From the condition $t_{n+1} \sim t_n$ $(n \to \infty)$ it follows that $\varkappa_n^{(1)} = \varphi_1((1 + o(1))t_n)$ $(n \to \infty)$, and thus

$$\frac{\Phi_2'(\varkappa_n^{(1)})}{\Phi_1''(\varkappa_n^{(1)})} = \frac{\Phi_2'(\varphi_1((1 + o(1))t_n))}{\Phi_1''(\varphi_1((1 + o(1))t_n))} = (1 + o(1))\frac{\Phi_2'(\varphi_1(t_n))}{\Phi_1''(\varphi_1(t_n))} =$$
$$= (1 + o(1))\Phi_2'(\varphi_1(t_n))\varphi_1'(t_n) = (1 + o(1))\xi_n, \quad n \to \infty.$$

Also $\Phi_2(\varkappa_n^{(1)}) = (1 + o(1))\Phi_2(\varphi_1(t_n))$, $n \to \infty$. Therefore, in view of (11) and (17),

$$\xi_n\frac{t_n - \Phi_1'(\varkappa_n^{(1)} - (1 + o(1))\eta\tau\xi_n)}{\Phi_2(\varphi_1(t_n))} = \xi_n\frac{\Phi_1'(\varphi_1(t_n)) - \Phi_1'(\varkappa_n^{(1)} - (1 + o(1))\eta\tau\xi_n)}{\Phi_2(\varphi_1(t_n))} =$$
$$= \xi_n\frac{\Phi_1'(\varkappa_n^{(1)}) - \Phi_1'(\varkappa_n^{(1)} - (1 + o(1))\eta\tau\xi_n)}{\Phi_2(\varphi_1(t_n))} + o\left(\xi_n\frac{\Phi_1'(\varphi_1(t_n))}{\Phi_2(\varphi_1(t_n))}\right) =$$
$$= \frac{\xi_n}{\Phi_2(\varkappa_n^{(1)})}\left(\Phi_1'(\varkappa_n^{(1)}) - \Phi_1'\left(\varkappa_n^{(1)} - (1 + o(1))\eta\tau\frac{\Phi_2'(\varkappa_n^{(1)})}{\Phi_1''(\varkappa_n^{(1)})}\right)\right) + o\left(\frac{t_n\xi_n}{\Phi_2(\varphi_1(t_n))}\right) =$$
$$= o\left(\frac{\xi_n\Phi_1'(\varkappa_n^{(1)})}{\Phi_2(\varkappa_n^{(1)})}\right) + o\left(\frac{t_n\xi_n}{\Phi_2(\varphi_1(t_n))}\right) = o\left(\frac{t_n\xi_n}{\Phi_2(\varphi_1(t_n))}\right) = o(1), \quad n \to \infty. \tag{18}$$

From (17) and (18) we obtain

$$\frac{(1 + o(1))t_n\tau\xi_n - (1 + o(1))\tau\xi_n\Phi_1'(\varkappa_n^{(1)} - (1 + o(1))\eta\tau\xi_n)}{\Phi_2(\varphi_1(t_n))} = o(1), \quad n \to \infty. \tag{19}$$

From (11) we obtain also

$$\frac{\Phi_2(\varkappa_n^{(1)} - (1 + o(1))\tau\xi_n))}{\Phi_2(\varphi_1(t_n))} = \frac{(1 + o(1))}{\Phi_2(\varkappa_n^{(1)})}\Phi_2\left(\varkappa_n^{(1)} - (1 + o(1))\tau\frac{\Phi_2'(\varkappa_n^{(1)})}{\Phi_1''(\varkappa_n^{(1)})}\right) \to 1, \quad n \to \infty,$$

and thus

$$\frac{\tau\Phi_2(\varkappa_n^{(1)} - (1 + o(1))\tau\xi_n) - (1 + o(1))\tau\Phi_2(\varphi_1(t_n))}{\Phi_2(\varphi_1(t_n))} = o(1), \quad n \to \infty. \tag{20}$$

Finally, from (15), (19), (20) and (13) we have

$$G_2(t_n, t_{n+1}, \Phi) - G_1(t_n, t_{n+1}, 1, \Phi) =$$
$$= G_2(t_n, t_{n+1}, \Phi_1) - G_1(t_n, t_{n+1}, 1, \Phi_1) + o(\Phi_2(\varphi_1(t_n))) =$$
$$= o(\Phi_2(\varphi_1(t_n))) = o(\Phi_2(\varphi(t_n))), \quad n \to \infty,$$

because, in view of (7) and (11),

$$\frac{\Phi_2(\varphi(t)}{\Phi_2(\varphi_1(t))} = \frac{1}{\Phi_2(\varphi_1(t))}\Phi_2\left(\varphi_1(t) - (1+o(1))\frac{\Phi_2'(\varphi_1(t))}{\Phi_1''(\varphi_1(t))}\right) \to 1, \quad t \to +\infty.$$

Therefore, by Lemma 2 for all $\sigma \in [\varphi(t_n),\, \varphi(t_{n+1})]$ we have

$$Q(\sigma) \geq \Phi(\sigma) + o(\Phi_2(\varphi(t_n))) = \Phi(\sigma) + o(\Phi_2(\sigma)) = \Phi_1(\sigma) + (1+o(1))\tau\Phi_2(\sigma), \quad \sigma \to A,$$

and Theorem 2 is proved. □

Corollary 2. *Let functions Φ_1 and Φ_2 satisfy the conditions of Theorem 2. If*

$$\ln|a_{n_k}| \geq -\lambda_{n_k}\Psi_1(\varphi_1(\lambda_{n_k})) + (1+o(1))\tau\Phi_2(\varphi_1(\lambda_{n_k})), \quad k \to \infty,$$

for a some increasing subsequence (λ_{n_k}) such that $\lambda_{n_{k+1}} \sim \lambda_{n_k}$ $(k \to \infty)$ and

$$\frac{G_2(\lambda_{n_k}, \lambda_{n_{k+1}}, \Phi_1) - G_1(\lambda_{n_k}, \lambda_{n_{k+1}}, \Phi_1)}{\Phi_2(\varphi_1(\lambda_{n_k}))} \to 0, \quad k \to \infty,$$

then

$$\ln\mu(\sigma, F) \geq \Phi_1(\sigma) + (1+o(1))\tau\Phi_2(\sigma), \quad \sigma \to A.$$

REFERENCES

1. Sheremeta M. M., Sumyk O. M. *A connection between the growth of Young conjugated functions,* Matematychni studii **11** (1999), no.1, 41–47 (in Ukrainian).
2. Zabolotskyi M. V., Sheremeta M. M. *A generalization of Lindelöf theorem*, Ukr. matem. journ. **50** (1998), no.9, 1177–1192 (in Ukrainian).

Faculty of Mechanics and Mathematics, Lviv National University

Received 19.06.2000