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NONLOCAL PROBLEM FOR THE NONLINEAR HYPERBOLIC SYSTEM OF THE FIRST ORDER WITHOUT INITIAL CONDITIONS

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In the paper we consider the nonlocal problem for a system of hyperbolic equations of the first order with two independent variables in the case when nonlinear functions satisfy the Caratheodory conditions and without any initial data. Some conditions of uniqueness and existence of a solution are obtained.

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Рассматривается нелокальная задача без начальных условий для нелинейной гиперболической системы первого порядка с двумя независимыми переменными в случае, когда нелинейные функции удовлетворяют условиям Каратеодори. Получены некоторые условия существования и единственности решения.

Nonlocal problems for hyperbolic equations of the first order describe the dynamic of population [1–3]. In 70s-80s, different authors [4–8] considered the existence and uniqueness of the solution of nonlocal problems for the system of hyperbolic equations. The authors assumed that the nonlinear functions satisfy the Lipschitz condition with respect to certain unknown functions. In this paper we consider the nonlocal problem for the system of hyperbolic equations of the first order with two independet variables in the case when nonlinear functions satisfy the Caratheodory conditions and without any initial data.

In the domain $\Omega_T = \{(x, y) : 0 < x < c, -\infty < t < T\}$, where $T < +\infty$, $0 < c < +\infty$ we shall consider a system of hyperbolic equations of the type

$$u_t + A(x,t)u_x + C(x,t)u + G(x,u) + \int_a^b Q(\xi,t)u d\xi = F(x,t) \tag{1}$$

with boundary condition

$$u(0,t) = \Lambda(t)u(c,t) \tag{2}$$

where A, C, Q, Λ are square matrices of order n, $n \in \mathbb{N}$ and

$$u = \mathrm{colon}(u_1, ..., u_n),\ \ G = \mathrm{colon}(g_1, ..., g_n),\ \ F = \mathrm{colon}(f_1, ..., f_n),\ \ 0 \le a < b \le c.$$

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Let $\Omega_{t_1,t_2} = (0,c)\times(t_1,t_2)$ for arbitrary t_1, t_2, $-\infty < t_1 < t_2 \le T$. Denote by $L^r_{l,loc}(\Omega_T)$ $(1 < r < \infty)$ the set of functions $u = \text{colon}(u_1,\dots,u_l)$ where $u_j \in L^r(\Omega_{\tau,T})$ $(j = 1,\dots,l)$, $l \in \mathbb{N}$.

For equation (1) we give the following system of conditions:

(**A**) $A \in C^1_{n^2}([0,c])$; $A(x,t) = A^t(x,t)$, for all $(x,t) \in \Omega_T$, A^t is the transposed matrix to A; $(A_x(x)\xi,\xi) \le A_1|\xi|^2$ for every $\xi \in \mathbb{R}^n$ and $(x,t) \in \Omega_T$, where $A_1 = \text{const}$; $A(c,t) = \Lambda^t(t)A(0,t)\Lambda(t)$ by $(\cdot,\cdot)$ and $|\cdot|$ we denote the scalar product and the norm in $\mathbb{R}^n$, respectively).

(**C**) $C \in L^\infty_{n^2,loc}(\Omega_T)$; $(C(x,t)\xi,\xi) \ge c_0()|\xi|^2$ for every $\xi \in \mathbb{R}^n$) and almost all $(x,t) \in \Omega_T$ where $c_0 \in C((-\infty,T])$.

(**G**) The function G is continuous with respect to ξ for almost all $x \in (0,c)$ and is measurable with respect to x for every $\xi \in \mathbb{R}^n$, and for some $p > 2$ the following inequalities hold: $(G(x,\xi) - G(x,\mu), \xi - \mu) \ge G_0|\xi - \mu|^p$, $G_0 = \text{const} > 0$, $|g_i(x,\xi)| \le G_1 \sum\limits_{j=1}^n |\xi_j|^{p-1}$, $G_1 = \text{const} > 0$ for $i = 1,...,n$ and for every $\mu,\xi \in \mathbb{R}^n$ and almost all $x \in (0,c)$.

(**Q**) $Q \in L^\infty_{n^2}(\Omega_T)$.

(**F**) $F \in L^2_{n,loc}(\Omega_T)$

Let $\frac{1}{p} + \frac{1}{q} = 1$.

Definition 1. The function

$$u \in L^p_{n,loc}(\Omega_T), \quad u_t \in L^2_{n,loc}(\Omega_T), \quad u_x \in L^2_{n,loc}(\Omega_T) + L^q_{n,loc}(\Omega_T)$$

is called a *solution of problem (1)–(2)*, if u satisfies (1),(2) for almost all $(x,t) \in \Omega_T$.

We denote $Q_0 := \sup\limits_{a<x<b,-\infty<t<T} \|Q(x,t)\|$ where $\|\cdot\|$ is the Euclidean norm of the matrix Q.

Theorem 1. *If conditions* (A), (C), (G), (Q), (F) *hold, and*

$$2 \lim_{\tau\to-\infty} \inf_{t\le\tau} c_0(t) - A_1 - Q_0(c(b-a)+1) > 0 \tag{3}$$

then problem (1)–(2) has at most one solution.

Proof. To obtain a contradiction, suppose that there exist two solutions u^1, u^2 of problem (1)-(2) such that $u^1 \ne u^2$. Let $u = u^1 - u^2$. It is easy to show that for every $t_1, t_2 \in (-\infty, T]$ $(t_1 < t_2)$ the following equality holds:

$$\begin{aligned}\int\limits_{\Omega_{t_1,t_2}} \Big[&(u_t,u) + (A(x)u_x,u) + (C(x,t)u,u) + \\ &+((G(x,u^1) - G(x,u^2)),u) + \Big(\int\limits_a^b Q(\xi,t)u\,d\xi, u\Big)\Big]dxdt = 0.\end{aligned} \tag{4}$$

Considering every term of (4) separately we get

$$I_1 = \int\limits_{\Omega_{t_1,t_2}} (u_t,u)\,dx\,dt = \frac{1}{2}\int\limits_{t_1}^{t_2}\int\limits_0^c \frac{d}{dt}|u(x,t)|^2\,dx\,dt,$$

$$I_2 = \int\limits_{\Omega_{t_1,t_2}} (Au_x,u)dxdt = \frac{1}{2}\int\limits_{\Omega_{t_1,t_2}} (Au,u)_x dxdt - \frac{1}{2}\int\limits_{\Omega_{t_1,t_2}} (A_x u,u)dxdt.$$

From (A) we have

$$I_2 \geq -\frac{1}{2} A_1 \int\limits_{\Omega_{t_1,t_2}} |u(x,t)|^2 dxdt.$$

By (C)

$$I_3 = \int\limits_{\Omega_{t_1,t_2}} (C(x,t)u, u) dxdt \geq \int\limits_{\Omega_{t_1,t_2}} c_0(t) |u(x,t)|^2 dxdt.$$

From (G) we get

$$I_4 = \int\limits_{\Omega_{t_1,t_2}} (G(x,u^1) - G(x,u^2), u) dxdt \geq G_0 \int\limits_{\Omega_{t_1,t_2}} |u(x,t)|^p dxdt.$$

Besides,

$$I_5 = \int\limits_{\Omega_{t_1,t_2}} \left(\int\limits_a^b (Q(\xi,t) u d\xi, u \right) dxdt \leq \frac{1}{2} Q_0 (c(b-a)+1) \int\limits_{\Omega_{t_1,t_2}} |u(x,t)|^2 dxdt.$$

Due to condition (3) there exists a number $\tau_0 \in (-\infty, T]$ such that $2c_0(t) - A_1 - Q_0(c(b-a)+1) > 0$ for every $t \in (-\infty, \tau_0]$. From estimates of the integrals $I_1, \dots, I_5$ and from (4) we obtain

$$\int\limits_{t_1}^{t_2} \frac{d}{dt} \int\limits_0^c |u(x,t)|^2 \, dxdt + 2G_0 \int\limits_{t_1}^{t_2} \int\limits_0^c |u(x,t)|^p dxdt \leq 0 \tag{5}$$

for $t_1, t_2 \in (-\infty, \tau_0]$.

Since

$$\int\limits_0^c |u(x,t)|^p dx = \int\limits_0^c (|u(x,t)|^2)^{p/2} dx \geq \mu_1 \left(\int\limits_0^c |u(x,t)|^2 dx \right)^{p/2}$$

where the constant μ_1 depends on c, p, we may write (5) in the form

$$\int\limits_{t_1}^{t_2} y'(t) dt + 2G_0 \mu_1 \int\limits_{t_1}^{t_2} (y(t))^{p/2} dt \leq 0 \tag{6}$$

where $y(t) = \int_0^c |u(x,t)|^2 dx$. Taking into account that t_1, t_2 are arbitrary we obtain from (6) the inequality

$$y'(t) + 2G_0 \mu_1 y^{p/2}(t) \leq 0 \tag{7}$$

for every $t \in (-\infty, \tau_0]$. According to a lemma [9, p. 10], from inequality (7) we obtain $y(t) = 0$ for $t \in (-\infty, \tau_0)$ which means that $u(x,t) = 0$ for almost all $(x,t) \in \Omega_{\tau_0}$. Now it is easy to show that $u(x,t) = 0$ for almost all $(x,t) \in \Omega_T$. This finishes the proof of Theorem 1. □

Denote by J the Jacobian matrix for the function $G(x, \mu)$

$$J(x,\mu) = \left[\frac{\partial g_i(x,\mu)}{\partial \mu_j} \right]_{i,j=1}^n$$

for almost all $x \in (0,c)$ and all $\mu \in \mathbb{R}^n$.

Theorem 2. *If conditions* $(A), (C), (G), (Q), (F)$ *hold, and* $C_t, Q_t \in L^{\infty}_{n^2}(\Omega_T)$, $F_t \in L^2_{n,loc}(\Omega_T)$, Λ *is a constant matrix,* $A(x,t) \equiv A(x)$, $\det A(x) \neq 0$ *for all* $x \in [0,c]$,

$$(J(x,\mu)\xi,\xi) \geq 0 \tag{8}$$

for all $\mu, \xi \in \mathbb{R}^n$ *and almost all* $x \in (0,c)$*, and there exists a function* $\phi(t)$, $\phi \in C^1((-\infty,T])$, $\phi(t) > 0$, $t \in (-\infty,T]$, $\phi'(t) > 0$, $t \in (-\infty,T]$ *such that*

$$\lim_{\tau\to-\infty} \inf_{t\leq\tau} \left[2c_0(t) - \frac{\phi'(t)}{\phi(t)}\right] > A_1 + Q_0(c(b-a)+1) \tag{9}$$

and

$$F_0 = \int\limits_{\Omega_T} (|F(x,t)|^2 + |F_t(x,t)|^2)\phi(t)dxdt < \infty,$$

then there exists a solution u *of problem (1)-(2) which satisfies the inequality*

$$\int\limits_{\Omega_T} u^2(x,t)\phi(t)dxdt \leq \mu_2 F_0 \tag{10}$$

with some constant μ_2.

Proof. We consider the following problem for the eigenfunction:

$$y'' = \lambda y, \tag{11}$$

$$y(0) = \Lambda y(c), \quad y'(c) = \Lambda^t y(0) \tag{12}$$

where $y = \mathrm{colon}(y_1, ..., y_n)$. Then there exists an orthogonal system of eigenfunctions for problem (11), (12) [10] which forms a basis of the space $L^2_n(0,c)$. Let $\{w^k(x)\}$ where $w^k(x) = \mathrm{colon}(w^k_1(x), ..., w^k_n(x))$ will be the such a system of eigenfunctions. We consider the sequence $\{u^N(x,t)\}$ of functions of the form $u^N(x,t) = \sum_{k=1}^N C^N_k(t)w^k(x)$ for $N = 1, 2, ...$ where the functions $C^N_1(t), ..., C^N_N(t)$ are solutions of the following Cauchy problem:

$$\begin{gathered}\int\limits_0^c \Bigg[(u^N_t, w^k) + (A(x)u^N_x, w^k) + (C(x,t)u^N, w^k) + +(G(x,u^N), w^k) + \\ +\Bigg(\int\limits_a^b Q(\xi,t)u^N(\xi,t)d\xi, w^k(x)\Bigg) - (F_N(x,t), w^k)\Bigg]dx = 0 \quad \text{for } k = 1, ..., N\end{gathered} \tag{13}$$

with

$$C^N_k(T-N) = 0 \quad \text{for } k = 1, ..., N, \tag{14}$$

where $F_N(x,t) = F(x,t)\eta_N(t)$,

$$\eta_N(t) = \begin{cases} 1 & \text{for } \ t \in [T-N+1, T], \\ t-T+N & \text{for } \ t \in [T-N, T-N+1), \\ 0 & \text{for } \ t \in (-\infty, T-N). \end{cases}$$

Observe that the assumptions of Theorem 2 guarantee the existence of a solution of problem (13), (14) in the interval $[T-N, T-N+\eta]$, $\eta > 0$ which is differentiable in this interval. The following estimation shows that $\eta = N$. We can extend every function u^k by zero onto $(-\infty, T-N)$. Then we obtain a functional sequence $\{u^N(x,t)\}$ defined on Ω_T. Let t_0, τ_1 be numbers belonging to $(-\infty, T)$, $t_0 < \tau_1$. Multiply (13) by the functions $C_k^N(t)\phi(t)$ respectively, then summing by k from 1 to N and integrating with respect to t from t_0 to τ, $t_0 < \tau < \tau_1$ we obtain

$$\int\limits_{\Omega_{t_0,\tau}} \Big[(u_t^N, u^N) + (A(x)u_x^N, u^N) + (C(x,t)u^N, u^N) + \\ +(G(x,u^N),u^N) + \Big(\int\limits_a^b Q(\xi,t)u^N d\xi, u^N\Big) - (F_N(x,t), u^N)\Big]\phi(t)dxdt = 0. \tag{15}$$

Analogously to the proof of Theorem 1, considering the integrals in the last equality we obtain

$$I_6 = \int\limits_{\Omega_{t_0,\tau}} \Big[(u_t^N, u^N) + (A(x)u_x^N, u^N) + (C(x,t)u^N, u^N) + \Big(\int\limits_a^b Q(\xi,t)u^N d\xi, u^N\Big)\Big]\phi(t)dxdt \geq \\ \geq \frac{1}{2}\int\limits_0^c |u^N(x,\tau)|^2\phi(\tau)dx + \frac{1}{2}\int\limits_{\Omega_{t_0,\tau}} \Big[2c_0(t) - A_1 - Q_0(c(b-a)+1) - \frac{\phi'(t)}{\phi(t)}\Big]|u^N|^2\phi(t)dxdt - \\ -\frac{1}{2}\int\limits_0^c |u^N(x,t_0)|^2\phi(t_0)dx.$$

Moreover from (G) we have

$$I_7 = \int\limits_{\Omega_{t_0,\tau}} (G(x,u^N), u^N)\phi(t)dxdt \geq G_0 \int\limits_{\Omega_{t_0,\tau}} |u^N|^p\phi(t)dxdt$$

and, besides,

$$I_8 = \int\limits_{\Omega_{t_0,\tau}} (F_N(x,t), u^N)\phi(t)dxdt \leq \frac{1}{2\delta_0}\int\limits_{\Omega_{t_0,\tau}} |F(x,t)|^2\phi(t)dxdt + \frac{\delta_0}{2}\int\limits_{\Omega_{t_0,\tau}} |u^N|^2\phi(t)dxdt$$

where δ_0 is arbitrary positive number. From (15) and estimates of the integrals I_6, I_7, I_8 we obtain the following inequality

$$2G_0\int\limits_{\Omega_{t_0,\tau}} |u^N(x,t)|^p\phi(t)dxdt + \int\limits_0^c |u^N(x,\tau)|^2\phi(\tau)dx + \\ + \int\limits_{\Omega_{t_0,\tau}} \Big[2c_0(t) - A_1 - Q_0(c(b-a)+1) - \frac{\phi'(t)}{\phi(t)} - \delta_0\Big]|u^N(x,t)|^2\phi(t)dxdt \leq \\ \leq \int\limits_0^c |u^N(x,t_0)|^2\phi(t_0)dx + \frac{1}{\delta_0}\int\limits_{\Omega_{t_0,\tau}} |F(x,t)|^2\phi(t)dxdt. \tag{16}$$

Due to (9) we can choose numbers τ_1, δ_0 such that

$$2c_0(t) - A_1 - Q_0(1 + c(b-a)) - \frac{\phi'(t)}{\phi(t)} - \delta_0 > 0 \tag{17}$$

for all $t \in (-\infty, \tau_1]$. The equality $\lim_{t_0 \to -\infty} \int\limits_0^c |u^N(x,t_0)|\phi(t_0)dx = 0$ and (16), (17) imply the following inequalities:

$$\int\limits_{\Omega_{\tau_1}} |u^N(x,t)|^2 \phi(t) dxdt \le \mu_2, \tag{18}$$

$$\int\limits_{\Omega_{\tau_1}} |u^N(x,t)|^p \phi(t) dxdt \le \mu_2, \tag{19}$$

where μ_2 does not depend on N.

Differentiating (13) with respect to t, next multiplying respectively by functions $C_{kt}^N(t)\phi(t)$, the summing by k from 1 to N and finnaly integrating by t from t_1 to τ, $t_1 < \tau < \tau_1$ we obtain

$$\begin{gathered}\int\limits_{\Omega_{t_1,\tau}} \Big[(u_{tt}^N, u_t^N) + (A(x)u_{xt}^N, u_t^N) + (C(x,t)u_t^N, u_t^N) + \\ + \int\limits_a^b (Q(\xi,t)u_t^N, u_t^N)d\xi - (F_{Nt}(x,t), u^N) + (C_t(x,t)u^N, u_t^N) + \\ +(J(x,u^N)u_t^N, u_t^N) + \Big(\int\limits_a^b Q_t(\xi,t)u^N d\xi, u_t^N\Big)\Big]\phi(t)dxdt = 0.\end{gathered} \tag{20}$$

Analogously to the proof of Theorem 1 we obtain the following estimation

$$\begin{aligned} I_9 = &\int\limits_{\Omega_{t_1,\tau}} \Big[(u_{tt}^N, u_t^N) + (A(x)u_{xt}^N, u_t^N) + (C(x,t)u_t^N, u_t^N) + \\ &+ \Big(\int\limits_a^b Q(\xi,t)u_t^N d\xi, u_t^N\Big) - (F_{Nt}(x,t), u^N)\Big]\phi(t)dxdt \ge \\ &\ge \frac{1}{2}\int\limits_0^c |u_t^N(x,\tau)|^2\phi(\tau)dx - \frac{1}{2}\int\limits_0^c |u_t^N(x,t_1)|^2\phi(t_1)dx - \\ &- \frac{1}{2\delta_2}\int\limits_{\Omega_{t_1,\tau}} |F_{Nt}(x,t)|^2\phi(t)dxdt + \\ &+ \frac{1}{2}\int\limits_{\Omega_{t_1,\tau}} \Big[2c_0(t) - A_1 - Q_0(c(b-a)+1) - \frac{\phi'(t)}{\phi(t)} - \delta_2\Big]|u_t^N(x,t)|^2\phi(t)dxdt, \end{aligned}$$

where $\delta_2 > 0$. Next from the conditions of Theorem 2 we have

$$\begin{aligned} I_{10} &= \int\limits_{\Omega_{t_1,\tau}} (C_t(x,t)u^N, u_t^N)\phi(t)dxdt \le \frac{\delta_2}{2}\int\limits_{\Omega_{t_1,\tau}} |u_t^N(x,t)|^2\phi(t)dxdt + \\ &+\frac{1}{2\delta_2}\sup_{\Omega_T}\|C_t(x,t)\|^2 \int\limits_{\Omega_{t_1,\tau}} |u^N(x,t)|^2\phi(t)dxdt, \\ I_{11} &= \int\limits_{\Omega_{t_1,\tau}} (J(x,u^N)u_t^N, u_t^N)\phi(t)dxdt \ge 0. \\ I_{12} &= \int\limits_{\Omega_{t_1,\tau}} \Big(\int\limits_a^b Q_t(\xi,t)u^N d\xi, u_t^N\Big)\phi(t)dxdt \le \frac{\delta_2}{2}\int\limits_{\Omega_{t_1,\tau}} |u_t^N(x,t)|^2\phi(t)dxdt + \\ &+\frac{1}{2\delta_2}\sup_{a<x<b,-\infty<t<T}\|Q_t(x,t)\|^2(c(b-a)+1)\int\limits_{\Omega_{t_1,\tau}} |u^N(x,t)|^2\phi(t)dxdt. \end{aligned}$$

Let $3\delta_2 = \delta_1$. Taking into account (9), (18), estimates of the integrals $I_9, I_{10}, I_{11}, I_{12}$ and the fact that

$$\int\limits_0^c |u_t^N(x,t_1)|\phi(t)dx = 0 \quad for \quad t_1 < N-T$$

we can choose a number $\tau_1 < T$ such that for $N-T<\tau_1$ (20) implies the inequality

$$\int\limits_{\Omega_\tau} |u_t^N(x,t)|^2\phi(t)dxdt \le \mu_3 F_0 \tag{21}$$

where constant μ_3 does not depend on N. Moreover, from (G) and from (19) we obtain

$$\int\limits_{\Omega_\tau} |g_i(x,u^N)|^q\phi(t)dxdt \le \int\limits_{\Omega_{\tau_1}} \Big(G_1\sum_{i=1}^n |u_i^N|^{p-1}\Big)^q\phi(t)dxdt \le \mu_4\int\limits_{\Omega_{\tau_1}} |u^N(x,t)|^p\phi(t)dxdt \le \mu_4\mu_2 \tag{22}$$

for $i=1,2,...,n$. Denote by $L^r_{n,\phi}(\Omega_\tau)$ the space with the norm

$$\|u\|_{r,\phi} = \Big(\int\limits_{\Omega_\tau} |u(x,t)|^r\phi(t)dxdt\Big)^{1/r}$$

where $1<r<\infty$. Due to (18), (21), (22) there exists a subsequence $\{u^m\}$ of the sequence $\{u^N\}$ such that $u^m \to u$ weakly in $L^2_{n,\phi}(\Omega_{\tau_1})$, $u_t^m \to u_t$ weakly in $L^2_{n,\phi}(\Omega_{\tau_1})$, $G(x,u^m)\to\omega$ weakly in $L^q_{n,\phi}(\Omega_{\tau_1})$ for $m\to\infty$. Taking into account (13) and (A) it is easy to prove that for the functions u, ω the following equality is satisfied

$$\begin{aligned} &\int\limits_{\Omega_\tau}\Big[(u_t,v)+(A_xu,v)-(Au,v_x)+(C(x,t)u,v)+ \\ &+(\omega,v)+\Big(\int\limits_a^b Q(\xi,t)u\,d\xi,v\Big)-(F(x,t),v)\Big]\phi(t)dxdt = 0 \end{aligned} \tag{23}$$

for all functions $v \in C^1(\overline{\Omega}_{\tau_1})$, satisfying condition (2) and having bounded support. In view of (A) and (23) we get

$$u_x \in L^2_{n,loc}(\Omega_{\tau_1}) + L^q_{n,loc}(\Omega_{\tau_1}).$$

Analogously to [11], p. 171, it is easy to prove that $\omega = G(x,u)$ in Ω_{τ_1}. Then the function u is a solution of system (1) in the domain Ω_{τ_1}. Moreover u satisfies condition (2) and estimation (10) in Ω_{τ_1}. Let $u(x,\tau_1) = \Phi(x)$. Now we consider problem (1), (2) in the domain $\Omega_{\tau_1,T}$ with the initial condition

$$u(x,\tau_1) = \Phi(x). \tag{24}$$

Then using conditions of Theorem 2 one can prove that there exists a solution u of (1), (2), (24), where

$$u \in L^p(\Omega_{\tau_1,T}) \quad u_t \in L^2(\Omega_{\tau_1,T}), \quad u_x \in L^2(\Omega_{\tau_1,T}) + L^q(\Omega_{\tau_1,T}).$$

Thus the theorem is proved. □

Remark 1. If

$$\lim_{t\to-\infty} \frac{\phi'(t)}{\phi(t)} = 0,$$

then condition (9) is equivalent to (3).

Remark 2. Consider the function

$$G(x,u) = \mathrm{colon}(g_1(x)|u|^{p-2}u_1, ..., g_n(x)|u|^{p-2}u_n)$$

where $g_i \in L^\infty(0,c)$ for $i = 1, ..., n$. It is easy to show that G satisfies conditions (G).

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