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THE FOURIER PROBLEM FOR ONE NONLINEAR PSEUDOPARABOLIC EQUATION IN UNBOUNDED DOMAIN

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In this paper the well-posedness of a problem without initial data for one nonlinear pseudoparabolic equation in an unbounded (on space variables) domain is investigated. We prove existence and uniqueness of the solution of this problem. Since the penalty operator is presented in the equation, the results of the paper can be applied in investigation of the well-posedness classes of the problem without initial conditions for pseudoparabolic variational inequalities.

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Рассматривается задача без начальных условий для одного нелинейного псевдопараболического уравнения в неограниченной (по пространственным переменным) области. Доказано существование и единственность решения этой задачи. В силу наличия в уравнении оператора штрафа, результаты этой статьи могут быть использованы при исследовании классов корректности задачи без начальных условий для псевдопараболических вариационных неравенств.

A problem without initial data for heat conduction equation first was considered in [1]. A. Tikhonov established that it was sufficient to indicate the solution behaviour as $t \to -\infty$ for unique solvability of the problem without initial data for parabolic equation. This behaviour depends on the coefficients of equation and form of the boundary conditions. The problems without initial data for general linear parabolic equations and systems were investigated later in [2–5]. Analogous results for the linear pseudoparabolic systems were obtained in [6–10].

It was first ascertained in the papers [11, 12] that problem without initial data for some nonlinear parabolic equations was well posed and did not depend on the solution behaviour as $t \to -\infty$ and as $|x| \to \infty$.

In this paper we assert the existence of nonlinear pseudoparabolic equations for which well-posedness of the problem without initial data is determined only by the properties of this equations.

Let Ω be an unbounded domain in $\mathbb{R}^n$ and Γ be its boundary; suppose that there exists a sequence $\{\Omega_\tau\}$ of bounded subdomains of the domain Ω which depend on the parameter $\tau \in \Pi$ (here Π is a countable subset of the set of positive real numbers) and have the properties:

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1) $\Omega = \bigcup_{\tau\in\Pi}\Omega_\tau$; 2) $\tau \le \tau' \to \Omega_\tau \subset \Omega'_\tau$; 3) $\partial\Omega_\tau = \Gamma^1_\tau \cup \Gamma^2_\tau$ where $\Gamma^1_\tau, \Gamma^2_\tau$ is a partially smooth hypersurface; $\mathrm{mes}\{\Gamma^1_\tau \cap \Gamma^2_\tau\} = 0$, $\Gamma^1_\tau \neq \oslash$, $\Gamma^1_\tau \cap \Gamma \neq \oslash$, $\forall\tau \in \Pi$; 4) $\Gamma = \bigcup_{\tau\in\Pi}\Gamma^1_\tau$.

By $L^r_{\mathrm{loc}}(\overline{\Omega})$ we denote the space of functions belonging to $L^r(\Omega_\tau)$ for every $\tau \in \Pi$ and by $L^r_{\mathrm{loc}}\big((-\infty,T];X\big)$ the space of functions belonging to the space $L^r\big((\tau,T);X\big)$ for every τ, $-\infty < \tau < T$; here $1 \le r \le \infty$, X is some Banach space.

We assume that $h(x)$, $a(x)$, $\alpha(x)$ are some positive functions from the space $L^\infty_{\mathrm{loc}}(\overline{\Omega})$.

By $\overset{\circ}{H}{}^1_{h,a}(\Omega)$, $\overset{\circ}{W}{}^{1,r}_\alpha(\Omega)$, $1 < r < \infty$ we denote the closure of $C^\infty_0(\Omega)$ (the space of infinitely differentiable functions with compact support in Ω) respectively in the norms

$$\|u\|_H = \left(\int_\Omega \left[h(x)u^2 + a(x)\sum_{i=1}^n u^2_{x_i}\right]dx\right)^{\frac{1}{2}},\ \|u\|_W = \left(\int_\Omega \alpha(x)\left[|u|^r + \sum_{i=1}^n |u_{x_i}|^r\right]dx\right)^{\frac{1}{r}}.$$

Let K be a convex closed subset of the space $V = \overset{\circ}{H}{}^1_{h,a}(\Omega) \cap \overset{\circ}{W}{}^{1,p}_\alpha(\Omega)$, $p > 2$, which contains the zero element. By the usual way we determine the penalty operator on V: $B(u) = J(u - P_k u)$; here J is a dual operator between the spaces V and V^*, P_k is a projection operator on K [13].

Let $Q_T = \Omega \times (-\infty,T), T < \infty$; $Q_{t_1,t_2} = \Omega \times (t_1,t_2)$; $S_T = \Gamma \times (-\infty,T)$. We consider the equation

$$h(x)u_t - \sum_{i,j=1}^n (a_{ij}(x)u_{x_it})_{x_j} - \sum_{i,j=1}^n (b_{ij}(x,t)u_{x_i})_{x_j} - \sum_{i=1}^n c_i(x,t)u_{x_i} + c_0(x,t)u -$$
$$- \sum_{i=1}^n (a_i(x,u,\nabla u))_{x_i} + a_0(x,u) + \gamma B(u) = f_0(x,t) - \sum_{i=1}^n (f_i(x,t))_{x_i}, \tag{1}$$
$$u|_{S_T} = 0 \tag{2}$$

in the domain Q_T. It is known [13], that $B(u)$ is bounded and monotonic. Here γ is a positive number.

Definition. A function $u(x,t)$ is a solution of problem (1),(2) if:

1) $u \in C\big((-\infty,T];\overset{\circ}{H}{}^1_{h,a}(\Omega)\big) \cap L^p_{\mathrm{loc}}\big((-\infty,T];\overset{\circ}{W}{}^{1,p}_\alpha(\Omega)\big)$;

2) the function u satisfies the integral equality

$$\int_{Q_{t_1,t_2}} \left[-h(x)uv_t - \sum_{i,j=1}^n a_{ij}(x)u_{x_i}v_{x_jt} + \sum_{i,j=1}^n b_{ij}(x,t)u_{x_i}v_{x_j} - \sum_{i=1}^n c_i(x,t)u_{x_i}v + \right.$$
$$\left. + c_0(x,t)uv + \sum_{i=1}^n a_i(x,u,\nabla u)v_{x_i} + a_0(x,u)v\right]dxdt + \gamma\int_{t_1}^{t_2}\langle B(u),v\rangle dt +$$
$$+ \int_{\Omega_{t_2}}\left[h(x)uv + \sum_{i,j=1}^n a_{ij}(x)u_{x_i}v_{x_j}\right]dx - \int_{\Omega_{t_1}}\left[h(x)uv + \sum_{i,j=1}^n a_{ij}(x)u_{x_i}v_{x_j}\right]dx =$$
$$= \int_{Q_{t_1,t_2}}\left[f_0(x,t)v + \sum_{i=1}^n f_i(x,t)v_{x_i}\right]dxdt \qquad \forall (t_1,t_2) \subset (-\infty,T],$$
$$\forall v \in H^1_{\mathrm{loc}}\big((-\infty,T];\overset{\circ}{H}{}^1_{h,a}(\Omega)\big) \cap L^p_{\mathrm{loc}}\big((-\infty,T];\overset{\circ}{W}{}^{1,q}_\alpha(\Omega)\big), \quad \frac{1}{p} + \frac{1}{q} = 1. \tag{3}$$

We say that coefficients of equation (1) satisfy conditions $(A_0), (B_0), (C_0), (A_1)$, if:

$$(A_0): a(x)|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \le Aa(x)|\xi|^2, \ \ \forall x \in \Omega, \ \forall \xi \in \mathbb{R}^n, \ A = \text{const};$$

$$a_{ij}(x) = a_{ji}(x), \ \forall x \in \Omega, \ \forall i,j \in \{1,...,n\}; \ \ a_{ij} \in L^\infty_{\text{loc}}(\overline{\Omega}), \ \forall i,j \in \{1,...,n\};$$

$$(B_0): b_0|\xi|^2 \le \sum_{i,j=1}^{n} b_{ij}(x,t)\xi_i\xi_j, \ \ \forall (x,t) \in Q_T, \ \forall \xi \in \mathbb{R}^n; \ b_0 > 0;$$

$$b_{ij} \in L^\infty_{\text{loc}}\big((-\infty,T]; L^\infty(\Omega)\big), \ \forall i,j \in \{1,...,n\};$$

$$(C_0): 0 < c_0 \le c_0(x,t), \ \ \forall (x,t) \in Q_T; \ c_o \in L^\infty_{\text{loc}}\big((-\infty,T]; L^\infty(\Omega)\big);$$

(A_1): $a_i(\cdot,\xi)$ are measurable on Ω $\forall \xi \in \mathbb{R}^{n+1}$,

$a_i(x,\cdot)$ are continuous in $\mathbb{R}^{n+1}$ for a.e. $x \in \Omega$,

$a_0(\cdot,\eta)$ is measurable on Ω $\forall \eta \in \mathbb{R}$, $a_0(x,\cdot)$ is continuous in $\mathbb{R}$ for a.e. $x \in \Omega$,

$$\sum_{i=1}^{n}\big(a_i(x,u,\nabla u) - a_i(x,v,\nabla v)\big)\big(u_{x_i} - v_{x_i}\big) + \big(a_0(x,u) - a_0(x,v)\big)\times$$

$$\times\,(u-v) \ge \alpha(x)\left(\sum_{i=1}^{n}\big|u_{x_i} - v_{x_i}\big|^p + \big|u-v\big|^p\right),$$

$$|a_i(x,u,\nabla u)| \le \Lambda\alpha(x)\left(\sum_{j=1}^{n}|u_{x_j}|^{p-1} + |u|^{p-1}\right), \ \forall i \in \{1,...,n\},$$

$$|a_0(x,u)| \le \Lambda_0\alpha(x)|u|^{p-1}, \quad \forall x \in \Omega, \ \forall u,v \in V.$$

Theorem 1. *Let the coefficients of equation (1) satisfy conditions* (A_0), (B_0), (C_0), (A_1) *and besides that there exists a positive function* $\rho(x)$ *on* Ω *such that* $\rho^{-1} \in C(\overline{\Omega}) \cap L^{\frac{2}{p-2}}(\Omega)$, $\rho(x)\big(h(x)\big)^{\frac{p}{2}} \le \alpha(x)$, $\rho(x)\big(Aa(x)\big)^{\frac{p}{2}} \le \alpha(x)$; *and the following inequality holds:* $4b_0c_0 - \hat{c} \ge 0$, *where* $\hat{c} = \sup_{Q_T}\sum_{i=1}^{n}\big(c_i(x,t)\big)^2$. *Then problem (1), (2) has no more than one solution.*

Proof. Let u_1, u_2 be the solutions of problem (1), (2). For each of them we write the integral equality (3), deduct these equalities and put $u = v = u_1 - u_2$. After using conditions of the theorem and estimates

$$I_1 = \int_{Q_{t_1,t_2}}\left[-h(x)uu_t - \sum_{i,j=1}^{n} a_{ij}(x)u_{x_i}u_{x_jt}\right]dxdt + \int_{\Omega_{t_2}}\left[h(x)u^2 + \sum_{i,j=1}^{n} a_{ij}(x)u_{x_i}u_{x_j}\right]dx-$$

$$-\int_{\Omega_{t_1}}\left[h(x)u^2 + \sum_{i,j=1}^{n} a_{ij}(x)u_{x_i}u_{x_j}\right]dx = \frac{1}{2}\int_{\Omega_{t_2}}\left[h(x)u^2 + \sum_{i,j=1}^{n} a_{ij}(x)u_{x_i}u_{x_j}\right]dx-$$

$$-\frac{1}{2}\int_{\Omega_{t_1}}\left[h(x)u^2 + \sum_{i,j=1}^{n} a_{ij}(x)u_{x_i}u_{x_j}\right]dx;$$

$$I_2 = \gamma\int_{t_1}^{t_2}\langle B(u_1) - B(u_2), u_1 - u_2\rangle dt \ge 0;$$

$$I_3=\int_{Q_{t_1,t_2}}\Big[\sum_{i=1}^{n}\big(a_i(x,u_1,\nabla u_1)-a_i(x,u_2,\nabla u_2)\big)\big(u_{1x_i}-u_{2x_i}\big)+$$
$$+\big(a_0(x,u_1)-a_0(x,u_2)\big)(u_1-u_2)\Big]dxdt\geq\int_{Q_{t_1,t_2}}\alpha(x)\Big[\sum_{i=1}^{n}|u_{x_i}|^p+|u|^p\Big]dxdt;$$
$$I_4=\int_{Q_{t_1,t_2}}\Big[\sum_{i,j=1}^{n}b_{ij}(x,t)u_{x_i}u_{x_j}-\sum_{i=1}^{n}c_i(x,t)u_{x_i}u+c_0(x,t)u^2\Big]dxdt\geq$$
$$\geq\int_{Q_{t_1,t_2}}\Big(c_0-\frac{\hat{c}}{4b_0}\Big)\sum_{i=1}^{n}|u_{x_i}||u|dxdt,$$

we obtain

$$\int_{Q_{t_1,t_2}}\alpha(x)\Big[\sum_{i=1}^{n}|u_{x_i}|^p+|u|^p\Big]dxdt\leq\frac{1}{2}\int_{\Omega_{t_1}}\Big[h(x)u^2+\sum_{i,j=1}^{n}a_{ij}(x)u_{x_i}u_{x_j}\Big]dx. \quad (4)$$

It is easy to see that

$$\int_{\Omega}\Big[h(x)u^2+\sum_{i,j=1}^{n}a_{ij}(x)u_{x_i}u_{x_j}\Big]dx\leq\rho_0C_p^{\frac{2}{p}}\Big(\int_{\Omega}\rho(x)\Big[\Big(h(x)\Big)^{\frac{p}{2}}|u|^p+$$
$$+\Big(Aa(x)\Big)^{\frac{p}{2}}\sum_{i=1}^{n}|u_{x_i}|^p\Big]dx\Big)^{2/p}\leq\rho_0C_p^{\frac{2}{p}}\Big(\int_{\Omega}\alpha(x)\Big[|u|^p+\sum_{i=1}^{n}|u_{x_i}|^p\Big]dx\Big)^{2/p}, \quad (5)$$

where $\rho_0=\Big(\int_{\Omega}\big(\rho(x)\big)^{\frac{2}{2-p}}dx\Big)^{\frac{p-2}{2}}$. From estimates (4) and (5) it follows

$$\int_{Q_{t_1,t_2}}\alpha(x)\Big[\sum_{i=1}^{n}|u_{x_i}|^p+|u|^p\Big]dxdt\leq\frac{\rho_0C_p^{\frac{2}{p}}}{2}\Big(\int_{\Omega}\alpha(x)\Big[|u|^p+\sum_{i=1}^{n}|u_{x_i}|^p\Big]dx\Big)^{2/p}. \quad (6)$$

If we make the notation $t_1=\tau$, $t_2=T$, $y(\tau)=\int_{Q_{\tau,T}}\alpha(x)\Big[|u|^p+\sum_{i=1}^{n}|u_{x_i}|^p\Big]dxdt$, in (6) then we obtain

$$\Big(\frac{2}{\rho_0C_p^{\frac{2}{p}}}\Big)^{p/2}\big(y(\tau)\big)^{p/2}+\frac{dy(\tau)}{d\tau}\leq 0.$$

Thus, $y(\tau)\equiv 0$ [11]. $\square$

Theorem 2. *Let the coefficients of equation (1) fulfill all the conditions of Theorem 1;* $4b_0c_0-\hat{c}>0$ *and besides that*

$$b_{ij},c_i,c_o\in C\big((-\infty,T];L^{\infty}(\Omega)\big),\quad \rho^{-\frac{1}{p}}h^{-\frac{1}{2}}f_0\in C\big((-\infty,T];L^{q}(\Omega)\big),$$
$$\rho^{-\frac{1}{p}}a^{-\frac{1}{2}}f_i\in C\big((-\infty,T];L^{q}(\Omega)\big),\ i=\overline{1,n}.$$

Then a solution of problem (1), (2) exists.

Proof. Without losing generality we accept $\Omega \cap \{x \in \mathbb{R}^n : |x| < R\} = \Omega^R \subset \{\Omega_\tau\}$. Consider the problem

$$h(x)u_t + A(t)u + \gamma B(u) = F^{t_0,R}, \tag{7}$$

$$u|_{\partial\Omega^R \times (t_0,T)} = 0, \tag{8}$$

$$u|_{t=t_0} = 0 \tag{9}$$

in the domain $Q^R_{t_0,T} = \Omega^R \times (t_0, T)$. Here

$$\langle A(t)u, v\rangle = \int_{\Omega^R} \Bigg[\sum_{i,j=1}^{n} a_{ij}(x)u_{x_i t}v_{x_j} + \sum_{i,j=1}^{n} b_{ij}(x,t)u_{x_i}v_{x_j} -$$

$$- \sum_{i=1}^{n} c_i(x,t)u_{x_i}v + c_0(x,t)uv + \sum_{i=1}^{n} a_i(x,u,\nabla u)v_{x_i} + a_0(x,u)v\Bigg]dx,$$

$$\langle F^{t_0,R}, v\rangle = \int_{\Omega^R}\Big[f_0^{t_0,R}(x,t)v + f_i^{t_0,R}(x,t)v_{x_i}\Big]dx,$$

and

$$f_i^{t_0,R} = \begin{cases} f_i(x,t), & (x,t) \in Q^R_{t_0,T}; \\ 0, & (x,t) \in Q_T \setminus Q^R_{t_0,T}; \end{cases} \qquad i = \overline{0,n}.$$

The solution of problem (7)–(9) will be found by the Galerkin method. Let $\{\hat{\varphi}^{R,k}(x)\}$ be a fundamental system of functions in $\overset{\circ}{W}{}^{1,p}_{\alpha}(\Omega^R)$. After orthogonalization of this system in the norm $\|\cdot\|_H$ we obtain the system $\{\varphi^{R,k}(x)\}$.

We put $u^{R,N} = \sum_{s=1}^{N} c_k^N(t)\varphi^{R,k}(x)$ where $c_k^N(t)$, $k = 1,\dots,N$ may be found from the system of equations

$$\int_{\Omega^R}\Bigg[h(x)u_t^{R,N}\varphi^{R,k}(x) + \sum_{i,j=1}^{n} a_{ij}(x)u^{R,N}_{x_i t}\varphi^{R,k}_{x_j}(x) + \sum_{i,j=1}^{n} b_{ij}(x,t)u^{R,N}_{x_i}\varphi^{R,k}_{x_j}(x) -$$

$$- \sum_{i=1}^{n} c_i(x,t)u^{R,N}_{x_i}\varphi^{R,k}(x) + c_0(x,t)u^{R,N}\varphi^{R,k}(x) + a_0(x,u^{R,N})\varphi^{R,k}(x) +$$

$$+ \sum_{i=1}^{n} a_i(x,u^{R,N},\nabla u^{R,N})\varphi^{R,k}_{x_i}(x)\Bigg]dx + \gamma\langle B(u^{R,N}), \varphi^{R,k}(x)\rangle =$$

$$= \int_{\Omega^R}\Bigg[f_0^{t_0,R}(x,t)\varphi^{R,k}(x) + \sum_{i=1}^{n} f_i^{t_0,R}(x,t)\varphi^{R,k}_{x_i}(x)\Bigg]dx, \qquad k = \overline{1,N} \tag{10}$$

or

$$\sum_{s=1}^{N}\Big(c_s^N(t)\Big)'\left\{\int_{\Omega^R}\Bigg[h(x)\varphi^{R,s}(x)\varphi^{R,k}(x) + \sum_{i,j=1}^{n} a_{ij}(x)\varphi^{R,s}_{x_i}(x)\varphi^{R,k}_{x_j}(x)\Bigg]dx\right\} =$$

$$= \Phi\Big(c_1^N(t), \dots c_N^N(t)\Big), \qquad k = \overline{1,N}, \tag{11}$$

and conditions

$$c_k^N(t_0) = 0. \tag{12}$$

We extend all the functions $u^{R,N}(x,t), N = 1,2,...,$ by zero on Q_T and consider (7)–(9) in Q_T. After multiplication of each equation of system (10) on $c_k^N(t)q_\tau(t)$ respectively (where $0 \le q_\tau(t) \le 1$; $q_\tau(t) = 0$ if $t \le t_1$; $q_\tau(t) = 1$ if $t \ge t_2$; $\frac{q'_\tau(t)}{q_\tau^\beta(t)} \le q_p$ and besides that the function $q_\tau(t)$ is sufficiently smooth [11]), summation on k and integration over the interval $[t_1, t_2] \subset (-\infty, T]$ we obtain

$$\begin{aligned}
&\int_{Q_{t_1,t_2}} \Big[h(x)u_t^{R,N}u^{R,N} + \sum_{i,j=1}^n a_{ij}(x)u_{x_it}^{R,N}u_{x_j}^{R,N} + \sum_{i,j=1}^n b_{ij}(x,t)u_{x_i}^{R,N}u_{x_j}^{R,N} - \\
&\quad - \sum_{i=1}^n c_i(x,t)u_{x_i}^{R,N}u^{R,N} + c_0(x,t)u^{R,N}u^{R,N} + a_0(x,u^{R,N})u^{R,N} + \\
&\quad + \sum_{i=1}^n a_i(x,u^{R,N},\nabla u^{R,N})u_{x_i}^{R,N} \Big] dxdt + \gamma \int_{t_1}^{t_2} \langle B(u^{R,N}), u^{R,N}\rangle dt = \\
&\quad = \int_{Q_{t_1,t_2}} \Big[f_0^{t_0,R}(x,t)u^{R,N} + \sum_{i=1}^n f_i^{t_0,R}(x,t)u_{x_i}^{R,N} \Big] dxdt.
\end{aligned} \tag{13}$$

We estimate each summand of the equation (13):

$$\begin{aligned}
I_{11} &= \int_{Q_{t_1,t_2}} \Big[h(x)u_t^{R,N}u^{R,N} + \sum_{i,j=1}^n a_{ij}(x)u_{x_it}^{R,N}u_{x_j}^{R,N} \Big] q_\tau(t)dxdt \ge \\
&\ge \frac{1}{2}\int_{\Omega_{t_2}} \Big[h(x)\big(u^{R,N}\big)^2 + \sum_{i,j=1}^n a_{ij}(x)u_{x_i}^{R,N}u_{x_j}^{R,N} \Big] dx - \frac{\mu_{11}}{\delta^{p/(p-2)}} - \\
&- \frac{\delta^{p/2}C_p^{p/2}}{p} \int_{Q_{t_1,t_2}} \rho(x)\Big[\big(h(x)\big)^{p/2}|u^{R,N}|^p + \big(Aa(x)\big)^{p/2}\sum_{i=1}^n |u_{x_i}^{R,N}|^p \Big] q_\tau(t)dxdt; \\
I_{12} &= \gamma\int_{t_1}^{t_2} \langle B(u^{R,N}), u^{R,N}\rangle q_\tau(t)dt \ge 0; \\
I_{13} &= \int_{Q_{t_1,t_2}} \Big[\sum_{i=1}^n a_i(x,u^{R,N},\nabla u^{R,N})u_{x_i}^N + a_o(x,u^{R,N})u^{R,N} \Big] q_\tau(t)dxdt \ge \\
&\ge \int_{Q_{t_1,t_2}} \alpha(x)\Big[\sum_{i=1}^n |u_{x_i}^{R,N}|^p + |u^{R,N}|^p \Big] q_\tau(t)dxdt; \\
I_{14} &= \int_{Q_{t_1,t_2}} \Big[\sum_{i,j=1}^n b_{ij}(x,t)u_{x_i}^{R,N}u_{x_j}^{R,N} - \sum_{i=1}^n c_i(x,t)u_{x_i}^{R,N}u^{R,N} + c_0(x,t)\big(u^{R,N}\big)^2 \Big] \times \\
&\times q_\tau(t)dxdt \ge \int_{Q_{t_1,t_2}} \Big(c_0 - \frac{\hat{c}}{4b_0} \Big) \sum_{i=1}^n |u_{x_i}^{R,N}||u^{R,N}|dxdt \ge 0;
\end{aligned}$$

$$I_{15}=\int_{Q_{t_1,t_2}}\left[f_0^{t_0,R}(x,t)u^{R,N}+\sum_{i=1}^{n}f_i^{t_0,R}(x,t)u_{x_i}^{R,N}\right]q_\tau(t)dxdt\le\frac{\mu_{15}}{\delta^{p/(p-1)}}+$$
$$+\frac{\delta^p}{p}\int_{Q_{t_1,t_2}}\rho(x)\left[\big(h(x)\big)^{p/2}\left|u^{R,N}\right|^p+\big(Aa(x)\big)^{p/2}\sum_{i=1}^{n}\left|u_{x_i}^{R,N}\right|^p\right]q_\tau(t)\,dxdt,$$

where

$$\mu_{11}=\frac{q_p^{p/(p-2)}(p-2)}{2p}\int_{Q_{t_1,t_2}}\big(\rho(x)\big)^{\frac{2}{2-p}}dxdt,$$
$$\mu_{15}=\frac{1}{q}\int_{Q_{t_1,t_2}}\big(\rho(x)\big)^{-\frac{q}{p}}\left[\big(h(x)\big)^{-\frac{q}{2}}\left|f_0^{t_0,R}(x,t)\right|^q+\big(Aa(x)\big)^{-\frac{q}{2}}\sum_{i=1}^{n}\left|f_i^{t_0,R}(x,t)\right|^q\right]q_\tau(t)dxdt.$$

From the above-stated estimates we infer

$$\frac{1}{2}\int_{\Omega_{t_2}}\left[h(x)\big(u^{R,N}\big)^2+\sum_{i,j=1}^{n}a_{ij}(x)u_{x_i}^{R,N}u_{x_j}^{R,N}\right]dxdt+\gamma\int_{t_1}^{t_2}\langle B(u^{R,N}),u^{R,N}\rangle q_\tau(t)dt+$$
$$+\int_{Q_{t_1,t_2}}q_\tau(t)\left[\sum_{i=1}^{n}|u_{x_i}^{R,N}|^p\left(\alpha(x)-\left(\frac{\delta^{p/2}}{p}C_p^{p/2}+\frac{\delta^p}{p}\right)\big(Aa(x)\big)^{p/2}\rho(x)\right)+\right.$$
$$\left.+\left|u^{R,N}\right|^p\left(\alpha(x)-\left(\frac{\delta^{p/2}}{p}C_p^{p/2}+\frac{\delta^p}{p}\right)\big(h(x)\big)^{p/2}\rho(x)\right)\right]dxdt\le\frac{\mu_{11}}{\delta^{p/(p-2)}}+\frac{\mu_{15}}{\delta^{p/(p-1)}}=\frac{\mu}{2}.$$

As far as the products $\rho(x)\big(h(x)\big)^{\frac{p}{2}}$ and $\rho(x)\big(a(x)\big)^{\frac{p}{2}}$ are bounded we may choose δ in such a way that:

$$\frac{\alpha(x)}{2}-\rho(x)\big(h(x)\big)^{p/2}\left(\frac{\delta^{p/2}}{p}C_p^{p/2}+\frac{\delta^p}{p}\right)\ge 0,\ \frac{\alpha(x)}{2}-\rho(x)\big(Aa(x)\big)^{p/2}\left(\frac{\delta^{p/2}}{p}C_p^{p/2}+\frac{\delta^p}{p}\right)\ge 0,$$

and then

$$\int_{\Omega_{t_2}}\left[h(x)\left(u^{R,N}\right)^2+\sum_{i,j=1}^{n}a_{ij}(x)u_{x_i}^{R,N}u_{x_j}^{R,N}\right]dxdt+2\gamma\int_{t_1+1}^{t_2}\langle B(u^{R,N}),u^{R,N}\rangle dt+$$
$$+\int_{Q_{t_1+1,t_2}}\alpha(x)\left[\sum_{i=1}^{n}\left|u_{x_i}^{R,N}\right|^p+\left|u^{R,N}\right|^p\right]dxdt\le\mu, \tag{14}$$

namely

$$\left\|u^{R,N}\right\|_{L^\infty\left((t_1+1,t_2);\overset{\circ}{H}{}^1_{h,a}(\Omega)\right)}\le M;\quad \left\|u^{R,N}\right\|_{L^p\left((t_1+1,t_2);\overset{\circ}{W}{}^{1,p}_\alpha(\Omega)\right)}\le M;$$
$$\int_{t_1+1}^{t_2}\langle B(u^{R,N}),u^{R,N}\rangle dt\le M. \tag{15}$$

The sequence $\{u^{R,N}\}$ is uniformly bounded. We'll show that there exists subsequence $\{u^{R,N_k}\}\subset\{u^{R,N}\}$ of functions that are determined for all $t\in(-\infty,T]$ and have values on V and this subsequence is uniformly continuous on every segment $[T_1, T_2] \subset (-\infty, T]$. So long as the second estimate of (15) is true for the functions $u^{R,N}$ then by Fatou lemma we have

$$\int_{t_1+1}^{T} \liminf \int_{\Omega_t} \alpha(x) \left[\sum_{i=1}^{n} \left|u_{x_i}^{R,N}\right|^p + \left|u^{R,N}\right|^p\right] dxdt \leq M,$$

and the latter inequality implies

$$\liminf \int_{\Omega_t} \alpha(x) \left[\sum_{i=1}^{n} \left|u_{x_i}^{R,N}\right|^p + \left|u^{R,N}\right|^p\right] dx < \infty$$

for almost all $t \in [t_1+1, T]$. There exist $\hat{T} \in [t_1+1, T]$ and a sequence $\{u^{R,N_k}\}$ for which

$$\liminf \int_{\Omega_t} \alpha(x) \left[\sum_{i=1}^{n} \left|u_{x_i}^{R,N_k}\right|^p + \left|u^{R,N_k}\right|^p\right] dx = \lim_{k\to\infty} \inf \int_{\Omega_{\hat{T}}} \alpha(x) \left[\sum_{i=1}^{n} \left|u_{x_i}^{R,N_k}\right|^p + \left|u^{R,N_k}\right|^p\right] dx.$$

Thereto estimates (15) hold for $\{u^{R,N_k}\}$. We put $\hat{T} = t_1 + 1$ and obtain

$$\int_{\Omega_{t_1+1}} \left[\sum_{i,j=1}^{n} a_{ij}(x) u_{x_i}^{R,N_k} u_{x_j}^{R,N_k} + h(x) \left(u^{R,N_k}\right)^2\right] dx \leq M.$$

So $\|u^{R,N_k}(\cdot, t_1+1)\| \leq \mu_{16}$. Suppose that $\lim_{k\to\infty} \|u^{R,N_k}(\cdot, t_1+1)\|_V = U > 0$. Then

$$\gamma\langle B\big(u^{R,N_k}(\cdot, t_1+1)\big), u^{R,N_k}(\cdot, t_1+1)\rangle = \gamma \left\|B\big(u^{R,N_k}(\cdot, t_1+1)\big)\right\|_{V^*} \left\|u^{R,N_k}(\cdot, t_1+1)\right\|_V$$

and $\left\|u^{R,N_k}(\cdot, t_1+1)\right\|_V \geq U/2$ beginning from some number k_0. The Fatou lemma implies $\gamma \left\|B\big(u^{R,N_k}(\cdot, t_1+1)\big)\right\|_{V^*} \left\|u^{R,N_k}(\cdot, t_1+1)\right\|_V \leq \mu_{17}$. Therefore

$$\gamma \left\|B\big(u^{R,N_k}(\cdot, t_1+1)\big)\right\|_{V^*} \leq 2\mu_{17}/U, \ k \geq k_0.$$

The analogous estimate is true under $U = 0$ too.

Next we put $t_1 + 1 = \tau$ and consider system (10) for $\{u^{R,N_k}\}$ in the point $\tau + t$. After multiplication this system by $c_k^N(\tau + t)$ and summation on k we obtain

$$\int_{\Omega} \Bigg[h(x) u_t^{R,N_k}(x,\tau+t) u^{R,N_k}(x,\tau+t) + \sum_{i,j=1}^{n} a_{ij}(x) u_{x_i t}^{R,N_k}(x,\tau+t) u_{x_j}^{R,N_k}(x,\tau+t) +$$
$$+ \sum_{i,j=1}^{n} b_{ij}(x,\tau+t) u_{x_i}^{R,N_k}(x,\tau+t) u_{x_j}^{R,N_k}(x,\tau+t) + c_0(x,\tau+t)\left(u^{R,N_k}(x,\tau+t)\right)^2 -$$
$$- \sum_{i=1}^{n} c_i(x,\tau+t)) u_{x_i}^{R,N_k}(x,\tau+t) u^{R,N_k}(x,\tau+t) + a_0(x, u^{R,N_k}(x,\tau+t)) \times \tag{16}$$
$$\times u^{R,N_k}(x,\tau+t) + \sum_{i=1}^{n} a_i(x, u^{R,N_k}(x,\tau+t), \nabla u^{R,N_k}(x,\tau+t)) u_{x_i}^{R,N_k}(x,\tau+t)\Bigg] dx +$$
$$+ \gamma\langle B\big(u^{R,N_k}(x,\tau+t)\big), u^{R,N_k}(x,\tau+t)\rangle =$$
$$= \int_{\Omega} \left[f_0^{t_0,R}(x,\tau+t) u^{R,N_k}(x,\tau+t) + \sum_{i=1}^{n} f_i^{t_0,R}(x,\tau+t) u_{x_i}^{R,N_k}(x,\tau+t)\right] dx.$$

Analogously to the above-said we consider system (10) for $\{u^{R,N_k}\}$ in the point $\tau+t$. We again multiply it by $c_k^N(\tau)$ and summarize. After that we deduct found equality from (16) and integrate between 0 and δ:

$$\int_0^\delta \int_\Omega \Big[h(x)u_t^{R,N_k}(x,\tau+t)\hat{u}^{R,N_k}(x,\tau) + \sum_{i,j=1}^n a_{ij}(x)u_{x_it}^{R,N_k}(x,\tau+t)\hat{u}_{x_j}^{R,N_k}(x,\tau)+$$
$$+\sum_{i,j=1}^n b_{ij}(x,\tau+t)u_{x_i}^{R,N_k}(x,\tau+t)\hat{u}_{x_j}^{R,N_k}(x,\tau) + c_0(x,\tau+t)u^{R,N_k}(x,\tau+t)\hat{u}^{R,N_k}(x,\tau)-$$
$$-\sum_{i=1}^n c_i(x,\tau+t))u_{x_i}^{R,N_k}(x,\tau+t)\hat{u}^{R,N_k}(x,\tau) + a_0(x,u^{R,N_k}(x,\tau+t))\hat{u}^{R,N_k}(x,\tau)+$$
$$+\sum_{i=1}^n a_i(x,u^{R,N_k}(x,\tau+t),\nabla u^{R,N_k}(x,\tau+t))\hat{u}_{x_i}^{R,N_k}(x,\tau)\Big]dxdt+$$
$$+\gamma\int_0^\delta \langle B\big(u^{R,N_k}(x,\tau+t)\big),\hat{u}^{R,N_k}(x,\tau)\rangle dt =$$
$$= \int_0^\delta \int_\Omega \Big[f_0^{t_0,R}(x,\tau+t)\hat{u}^{R,N_k}(x,\tau) + \sum_{i=1}^n f_i^{t_0,R}(x,\tau+t)\hat{u}_{x_i}^{R,N_k}(x,\tau)\Big]dxdt \tag{17}$$

where $\hat{u}^{R,N_k}(x,\tau) = u^{R,N_k}(x,\tau+t) - u^{R,N_k}(x,\tau)$.

From the estimates

$$I_{21} = \int_0^\delta \int_\Omega \Big[h(x)u_t^{R,N_k}(x,\tau+t)\hat{u}^{R,N_k}(x,\tau) + \sum_{i,j=1}^n a_{ij}(x)u_{x_it}^{R,N_k}(x,\tau+t)\times$$
$$\times\hat{u}_{x_j}^{R,N_k}(x,\tau)\Big]dxdt = \frac{1}{2}\int_\Omega \Big[h(x)\Big(u^{R,N_k}(x,\tau+\delta) - u^{R,N_k}(x,\tau)\Big)^2+$$
$$+\sum_{i,j=1}^n a_{ij}(x)\Big(u_{x_i}^{R,N_k}(x,\tau+\delta) - u_{x_i}^{R,N_k}(x,\tau)\Big)\Big(u_{x_j}^{R,N_k}(x,\tau+\delta) - u_{x_j}^{R,N_k}(x,\tau)\Big)\Big]dx;$$

$$I_{22} = \gamma\int_0^\delta \langle B\big(u^{R,N_k}(x,\tau+t)\big),\hat{u}^{R,N_k}(x,\tau)\rangle dt \geq \gamma\int_0^\delta \langle B\big(u^{R,N_k}(x,\tau)\big),\hat{u}^{R,N_k}(x,\tau)\rangle dt \geq$$
$$\geq -\gamma\int_0^\delta \big\|B\big(u^{R,N_k}(x,\tau)\big)\big\|_{V^*}\big\|\hat{u}^{R,N_k}(x,\tau)\big)\big\|_V\,dt \geq -\frac{2(p-1)\mu_{17}\delta}{Up\sigma^{(p-1)/p}}-$$
$$-\frac{\gamma\sigma^p}{p}\int_0^\delta\int_\Omega \alpha(x)\Big[\big|\hat{u}^{R,N_k}(x,\tau)\big|^p + \sum_{i=1}^n\big|\hat{u}_{x_i}^{R,N_k}(x,\tau)\big|^p\Big]dxdt;$$

$$I_{23} = \int_0^\delta\int_\Omega\Big[\sum_{i=1}^n a_i(x,u^{R,N_k}(x,\tau+t),\nabla u^{R,N_k}(x,\tau+t))\hat{u}_{x_i}^{R,N_k}(x,\tau)+$$
$$+a_0(x,u^{R,N_k}(x,\tau+t))\hat{u}^{R,N_k}(x,\tau)\Big]dxdt \geq -\frac{M(p-1)\delta}{p\sigma^{p/(p-1)}}\Big(n\Lambda+\Lambda_0\Big)+$$
$$+\int_0^\delta\int_\Omega \alpha(x)\Big[\Big(1-\frac{\sigma^p(n+1)}{p}\Lambda\Big)\sum_{i=1}^n\big|\hat{u}_{x_i}^{R,N_k}(x,\tau)\big|^p + \Big(1-\frac{\sigma^p}{p}\Lambda_0\Big)\big|\hat{u}^{R,N_k}(x,\tau)\big|^p\Big]dxdt;$$

$$I_{24}=\int_0^\delta\int_\Omega\Big[\sum_{i,j=1}^n b_{ij}(x,\tau+t)u_{x_i}^{R,N_k}(x,\tau+t)\hat u_{x_j}^{R,N_k}(x,\tau)+c_0(x,\tau+t)\hat u^{R,N_k}(x,\tau)\times$$

$$\times\, u^{R,N_k}(x,\tau+t)-\sum_{i=1}^n c_i(x,\tau+t)u_{x_i}^{R,N_k}(x,\tau+t)\hat u^{R,N_k}(x,\tau)\Big]dxdt\geq$$

$$-\frac{M\delta}{2}\Big(\hat bn+\hat c+\hat{c_0}\Big)+\int_0^\delta\int_\Omega\bigg[\Big(b_0-\frac{\hat c}{2}-\frac{\hat bn}{2}\Big)\sum_{i=1}^n\big(\hat u_{x_i}^{R,N_k}(x,\tau)\big)^2+$$

$$+\Big(c_0-\frac{\hat{c_0}}{2}-\hat cn\Big)\big(\hat u^{R,N_k}(x,\tau)\big)^2\bigg]dxdt;$$

$$I_{25}=\int_0^\delta\int_\Omega\Big[f_0^{t_0,R}(x,\tau+t)\hat u^{R,N_k}(x,\tau)+\sum_{i=1}^n f_i^{t_0,R}(x,\tau+t)\hat u_{x_i}^{R,N_k}(x,\tau)\Big]dxdt\leq$$

$$\leq\hat F\delta+\frac{\varepsilon^p}{p}\int_0^\delta\int_\Omega\alpha(x)\Big[\big|\hat u^{R,N_k}(x,\tau)\big|^p+\sum_{i=1}^n\big|\hat u_{x_i}^{R,N_k}(x,\tau)\big|^p\Big]dxdt,$$

where

$$\hat b=\sup_{Q_T}\max_{i,j=\overline{1,n}}|b_{ij}(x,t)|;\quad \hat{c_0}=\sup_{Q_T}|c_0(x,t)|;$$

$$\hat F=\frac{1}{q\varepsilon^q}\sup_{t\in[-\tau,-\tau+\delta]}\int_\Omega\Big[(h(x))^{-q/2}\Big|f_0^{t_0,R}(x,t)\Big|^q+$$

$$+(Aa(x))^{-q/2}\sum_{i=1}^n\Big|f_i^{t_0,R}(x,t)\Big|^q\Big](\rho(x))^{-q/p}dx,$$

we obtain

$$\begin{aligned}&\frac12\int_{\Omega_\delta}\Big[h(x)\big(\hat u^{R,N_k}(x,\tau)\big)^2+\sum_{i,j=1}^n a_{ij}(x)\hat u_{x_i}^{R,N_k}(x,\tau)\hat u_{x_j}^{R,N_k}(x,\tau)\Big]dx+\\&+\int_0^\delta\int_\Omega\alpha(x)\bigg[\Big(1-\frac{\gamma\sigma^p}{p}-\frac{\varepsilon^p}{p}-\frac{\Lambda_0\sigma^p}{p}\Big)\big|\hat u^{R,N_k}(x,\tau)\big|^p+\\&+\Big(1-\frac{\gamma\sigma^p}{p}-\frac{\varepsilon^p}{p}-\frac{(n+1)\Lambda\sigma^p}{p}\Big)\sum_{i=1}^n\big|\hat u_{x_i}^{R,N_k}(x,\tau)\big|^p\bigg]dxdt+\\&+\int_0^\delta\int_\Omega\bigg[\Big(c_0-\hat cn-\frac{\hat{c_0}}{2}\Big)\big(\hat u^{R,N_k}(x,\tau)\big)^2+\Big(b_0-\frac{\hat c}{2}-\frac{\hat bn}{2}\Big)\sum_{i=1}^n\big(\hat u_{x_i}^{R,N_k}(x,\tau)\big)^2\bigg]dxdt\leq\\&\leq\frac{\delta}{2}\Big(2\hat F+\frac{2M(p-1)}{p\sigma^{p/(p-1)}}\Big(\Lambda_0+n\Lambda\Big)+\hat bnM+\hat{c_0}M+\hat cM+\frac{4(p-1)\mu_{17}}{Up\sigma^{(p-1)/p}}\Big).\end{aligned}\tag{18}$$

Choose ε and σ in such way that second summand of inequality (18) is nonnegative and

$$\rho_1=\max\{2,\hat bn+\hat c-2b_0,2\hat cn+\hat{c_0}-2c_0\},$$

$$y(\delta)=\int_{\Omega_\delta}\Big[h(x)\Big(\hat u^{R,N_k}(x,\tau)\Big)^2+\sum_{i,j=1}^n a_{ij}(x)\hat u_{x_i}^{R,N_k}(x,\tau)\hat u_{x_j}^{R,N_k}(x,\tau)\Big]dx.$$

Then

$$y(\delta) \le \delta\mu_{20} + \rho_1 \int_0^\delta y(t)dt. \tag{19}$$

The Gronnuoll-Bellmann lemma implies

$$y(\delta) \le \mu_{21}\delta. \tag{20}$$

By similar reasonings we can derive from (10) the following estimate:

$$\begin{aligned} -\frac{1}{2}\int_{\Omega_\tau}\Big[h(x)\big(\tilde{u}^{R,N_k}\big)^2 + \sum_{i,j=1}^n a_{ij}(x)\tilde{u}^{R,N_k}_{x_i}\tilde{u}^{R,N_k}_{x_j}\Big]dx + \\ + \int_\tau^{t_2}\int_\Omega \alpha(x)\Big[\Big(1-\frac{\delta^p}{p}\Big)\big|\tilde{u}^{R,N_k}\big|^p + \Big(1-\frac{\delta^p}{p}\Big)\sum_{i=1}^n\big|\tilde{u}^{R,N_k}_{x_i}\big|^p\Big]dxdt \le \varepsilon\mu_{30}, \end{aligned} \tag{21}$$

where $\tilde{u}^{R,N_k} = u^{R,N_k}(x,t+\delta) - u^{R,N_k}(x,t)$, ε is a small number that satisfies fulfill the condition

$$4b_0c_0 - \hat{c} + \varepsilon^2 n(n+1) - 2\varepsilon\big(nc_0 + (n+1)b_0\big) \ge 0,$$

δ is small too and such that $\frac{1}{2} - \frac{\delta^p}{p} \ge 0$. Then from (20) and (21) we obtain

$$\int_\tau^{t_2}\int_\Omega \alpha(x)\Big[\big|\tilde{u}^{R,N_k}\big|^p + \sum_{i=1}^n\big|\tilde{u}^{R,N_k}_{x_i}\big|^p\Big]dxdt \le 2\big(\varepsilon\mu_{30} + \mu_{21}\delta\big). \tag{22}$$

The latter inequality implies that the functions $\{u^{R,N_k}(x,t)\}$ are uniformly continuous by t on the segment $[T_1,T_2]$. Since the terms of this sequence satisfy estimates (15), we can choose a subsequence $\{u^{R,N_{k_m}}(x,t)\}$ such that under $m\to\infty$

$u^{R,N_{k_m}}(x,t) \to u(x,t)$ $\quad$ $*$-weakly in $L^\infty_{\text{loc}}\big((-\infty,T];\overset{\circ}{H}{}^1_{h,a}(\Omega)\big)$;

$u^{R,N_{k_m}}(x,t) \to u(x,t)$ $\quad$ weakly in $L^p_{\text{loc}}\big((-\infty,T];\overset{\circ}{W}{}^{1,p}_\alpha(\Omega)\big)$;

$u^{R,N_{k_m}}(x,t) \to u(x,t)$ $\quad$ uniformly in $C\big([T_1,T_2];\overset{\circ}{H}{}^1_{h,a,w}(\Omega)\big)$, $\forall [T_1,T_2]\subset(-\infty,T]$.

Here the index "w" means that convergence in specified space is weak.

Without losing generality we admit $\Pi = \mathbb{N}$ and consider problem (7)–(9) in the domains $Q^k_{T-k,T}$, $k\in\mathbb{N}$. By Ascoly-Arzela theorem we can construct the diagonal sequence $\{u^{m,m}(x,t)\}$ for which

$u^{m,m}(x,t) \to u(x,t)$ $\quad$ $*$-weakly in $L^\infty_{\text{loc}}\big((-\infty,T];\overset{\circ}{H}{}^1_{h,a}(\Omega)\big)$;

$u^{m,m}(x,t) \to u(x,t)$ $\quad$ weakly in $L^p_{\text{loc}}\big((-\infty,T];\overset{\circ}{W}{}^{1,p}_\alpha(\Omega)\big)$;

$u^{m,m}(x,t) \to u(x,t)$ $\quad$ uniformly on $C\big([T_1,T_2];\overset{\circ}{H}{}^1_{h,a,w}(\Omega)\big)$, $\forall [T_1,T_2]\subset(-\infty,T]$ under $m\to\infty$. The functions $u^{m,m}$ obviously satisfy (3). Taking into account properties of penalty operator and Caratheodory functions we proceed to limit in

$$\int_{Q_{t_1,t_2}}\Big[-h(x)u^{m,m}v_t-\sum_{i,j=1}^{n}a_{ij}(x)u^{m,m}_{x_i}v_{x_jt}+\sum_{i,j=1}^{n}b_{ij}(x,t)u^{m,m}_{x_i}v_{x_j}+a_o(x,u^{m,m})v-$$

$$-\sum_{i=1}^{n}c_i(x,t)u^{m,m}_{x_i}v+c_0(x,t)u^{m,m}v+\sum_{i=1}^{n}a_i(x,u^{m,m},\nabla u^{m,m})v_{x_i}\Big]dxdt+$$

$$+\gamma\int_{t_1}^{t_2}\langle B(u^{m,m}),v\rangle dt+\int_{\Omega_{t_2}}\Big[h(x)u^{m,m}v+\sum_{i,j=1}^{n}a_{ij}(x)u^{m,m}_{x_i}v_{x_j}\Big]dx-$$

$$-\int_{\Omega_{t_1}}\Big[h(x)u^{m,m}v+\sum_{i,j=1}^{n}a_{ij}(x)u^{m,m}_{x_i}v_{x_j}\Big]dx=\int_{Q_{t_1,t_2}}\Big[f_0(x,t)v+\sum_{i=1}^{n}f_i(x,t)v_{x_i}\Big]dxdt$$

under $m\to\infty$ and obtain that $u(x,t)$ is a solution of (1), (2). $\square$

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