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TOPOLOGIES ON $\mathbb{Z}$ DETERMINED BY SEQUENCES: SEVEN OPEN PROBLEMS

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A sequence $\langle a_n\rangle_{n\in\omega}$ of integers is called a T-sequence if there exists a Hausdorff group topology τ on $\mathbb{Z}$ such that $\langle a_n\rangle_{n\in\omega}$ converges to zero. Given any T-sequence $\langle a_n\rangle_{n\in\omega}$, denote by $(\mathbb{Z}\mid\langle a_n\rangle_{n\in\omega})$ the group $\mathbb{Z}$ endowed with strongest group topology in which $\langle a_n\rangle_{n\in\omega}$ converges to zero. We say that a group topology τ on $\mathbb{Z}$ is determined by T-sequence if $(\mathbb{Z},\tau)=(\mathbb{Z}\mid\langle a_n\rangle_{n\in\omega})$ for some T-sequence $\langle a_n\rangle_{n\in\omega}$. For more detailed information concerning T-sequences see [1, 2].

1. Is $\langle 2^n+3^n\rangle_{n\in\omega}$ a T-sequence?

2. Let $\langle a_n\rangle_{n\in\omega}$ be a sequence of integers such that $a_{n+2}=pa_{n+1}+qa_n$ for some $p,q\in\mathbb{Z}$ and for all $n\in\omega$. Find necessary and sufficient conditions under which $\langle a_n\rangle_{n\in\omega}$ is a T-sequence.

3. Does there exist an increasing T-sequence $\langle a_n\rangle_{n\in\omega}$ such that $a_n<f(n)$ for some polynomial $f(x)$ and every $n\in\omega$?

4. Let $\langle a_n\rangle_{n\in\omega}$ be an increasing sequence of integers such that $\lim_{n\to\infty}a_{n+1}/a_n=\alpha$ and α is transcendental. Does there exist a Hausdorff ring topology on $\mathbb{Z}$ in which $\langle a_n\rangle_{n\in\omega}$ converges to zero?

5. Let τ_1,τ_2 be topologies on $\mathbb{Z}$ determined by T-sequences. Is $\tau_1\vee\tau_2$ determined by T-sequence?

6. Does there exist an increasing (decreasing) well ordered chain of length 2^ω of topologies on $\mathbb{Z}$ determined by T-sequence? This is so under CH.

7. A sequence $\langle F_n\rangle_{n\in\omega}$ of pairwise disjoint finite subsets of topological space is called an expansive sequence if, for every open subset U, there exists $m\in\omega$ such that $F_n\cap U\neq\varnothing$ for every $n>m$. Let τ be a nondiscrete group topology on $\mathbb{Z}$. Does there exist an expansive sequence in $(\mathbb{Z},\tau)$? This is true if τ is either totally bounded or determined by T-sequence.

REFERENCES

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