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A NOTE ON INDUCED MAPPINGS

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Hosokawa gave an example of an open mapping f between continua such that the induced mapping $C(f)$ between hyperspaces of all subcontinua is not open. In this note we show a much simpler example than one of Hosokawa.

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Хосакава привел пример открытого отображения f между континуумами такого, что индуцированное отображение $C(f)$ между гиперпространствами подконтинуумов не является открытым. В этой заметке мы приводим значительно более простой пример чем пример Хосокавы.

Hosokawa in [1] proved that if a mapping $f\colon X\to Y$ between continua is monotone (quasi-interior) then the induced mapping $C(f)\colon C(X)\to C(Y)$ between hyperspaces of all nonempty subcontinua is monotone (quasi-interior), and $C(f)$ is confluent if f is confluent and Y is locally connected. He also gave an example ([1], Example 3.2, p.4) of an open mapping f from a continuum X onto a continuum Y such that the induced mapping $C(f)$ is not open. This continuum X is rather complicated (it is not locally connected and not acyclic). Later in [2] Hosokawa gave an example of an open mapping from two-dimensional locally connected continuum into itself such that the induced mapping is not open. Both continua of examples of Hosokawa have infinite dimensional hyperspaces of subcontinua.

In this note we provide an example of an open mapping f between the simplest continua, i.e. from the unit interval onto itself such that $C(f)$ need not be open.

Example. *There is an open mapping f from the closed unit interval $[0,1]$ onto itself such that $C(f)\colon C([0,1])\to C([0,1])$ is not open.*

Proof. It is known that the mapping $h([a,b])=(a,b)$, for $[a,b]\in C([0,1])$ is a homeomorphism between $C([0,1])$ and the triangle T in Euclidean plane of vertices $(0,0)$, $(1,1)$ and $(0,1)$ (see [3], Example (0.54), p.30).

Define $f\colon[0,1]\to[0,1]$ as follows

$$f(x)=\begin{cases} 2x & \text{for } x\in\left[0,\frac12\right]\\ 2-2x & \text{for } x\in\left(\frac12,1\right].\end{cases}$$

It is clear that f is open. Let T_1 be a triangle of vertices $(0,0)$, $(\frac12,\frac12)$, $(0,\frac12)$, T_2 a triangle of vertices $(0,\frac12)$, $(\frac12,\frac12)$, $(0,1)$, T_3 a triangle of vertices $(0,1)$, $(\frac12,\frac12)$, $(1,1)$ and note that $T=$

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$T_1 \cup T_2 \cup T_3$. It is easy to verify that f induces the following mapping $C(f) = g_1 \circ g_2 \circ g_3$ from T onto T, where $g_3 : T \to T_1 \cup T_2$ is defined as follows:

$$g_3 \mid T_1 \cup T_2 = \mathrm{id}_{T_1 \cup T_2}$$

and $g_3 \mid T_3$ is the symmetry with respect to the line $y = 1 - x$, $g_2 : T_1 \cup T_2 \to T_1$ is defined as follows:

$$g_2 \mid T_1 = \mathrm{id}_{T_1}$$

and $g_2 \mid T_2$ is the orthogonal projection into the line $y = \frac{1}{2}$, $g_1 : T_1 \to T$ is the homothety with degree two.

It is easy to see that the mapping $g_1 \circ g_2 \circ g_3$ transforms triangle T_2 with the nonempty interior onto a straight line segment with the empty interior, and thus it is not open.

REFERENCES

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