УДК 512.544

ON GROUPS RELATED TO THE ČARIN GROUPS

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O.D. Artemovych. *On groups related to the Čarin groups*, Matematychni Studii, **11**(1999) 141–148.

We establish some properties of groups G with the hypercentral commutator subgroup G' which is a p'-group and the divisible p-Černikov quotient group G/G', and construct several examples of those groups.

О.Д. Артемович. *Группы, близкие к группам Чарина*// Математичні Студії. – 1999. – Т.11, № 2. – С.141–148.

Описаны некоторые свойства групп G с гиперцентральным коммутантом G', являющимся p'-группой, и фактор-группой G/G', являющейся черниковской делимой p-группой. Приведены некоторые примеры таких групп.

0. A group G is called an HM^*-group if its commutator subgroup G' is hypercentral (i. e. G' possesses a central ascending series) and the quotient group G/G' is a divisible Černikov p-group. The class of HM^*-groups contains the Čarin groups [1–2], the groups of Heineken-Mohamed type (i.e. the non-nilpotent groups with all proper subgroups nilpotent and subnormal) [3–7], the minimal non-"hypercentral-by-finite" groups [1–2, 8–14]. The concept of HM^*-p-groups G with nilpotent commutator subgroup G' were investigated by A. Azar [14].

In this paper we establish some properties of HM^*-groups related to the Čarin groups and construct several examples of these groups.

Throughout this paper p and q will always denote distinct primes, $G' = [G, G]$ will indicate the commutator subgroup of G, and $\mathbb{C}_{p^\infty}$ stands for the quasicyclic p-group.

Let us also agree that $J(R)$ will stand for the Jacobson radical of a ring R, and $U(R)$ for the unit group of R. We will also apply some other standard notations and terminology used, for example, in [15–17].

1. Clearly that the groups of Heineken-Mohamed type are HM^*-groups with the normalizer condition. In this part we construct several examples of HM^*-groups without the normalizer condition.

Recall that a semidirect product $G = F \rtimes H$ is called a Frobenius group with a kernel F and a complement H if $H \cap H^g = 1$ for all $g \in G \setminus H$ and $F \setminus \{1\} = G \setminus \bigcup_{g \in G} H^g$.

1991 *Mathematics Subject Classification.* 20F19.

Let R be an associative ring with the identity 1, T a nontrivial subgroup of the unit group $\mathcal{U}(R)$ of R, k an integer, $G = G(T, R, k)$ the set of pairs $\{(x, y) |\ x \in T,\ y \in R\}$ with the algebraic operation given by

$$(x, y)(u, v) = (xu, x^k v + y). \tag{1}$$

Firstly, the groups of type $G(T, R, k)$ were considered by V.S. Čarin. Namely in [1] he considered the group $G(X, Y, 1)$, where Y is a locally finite field of characteristic q and X is the quasicyclic p-subgroup of the multiplicative group Y^* of Y. Later, V.V. Belyaev [2] considered the groups $G(X, Y, p^m)$, where m is a nonnegative integer, and named them the Čarin groups.

Proposition 1.1. *Let R be an associative domain with the identity 1. Then $G(T, R, k)$ is a group with respect to the operation given by rule (1) if and only if $(xy)^k = x^k y^k$ for all x, y in T.*

Proof. Straightforward.

We will show that the HM^*-groups G, where the commutator subgroup G' is a p'-group and the quotient G/G' is a Černikov divisible p-group, are related in some sence to the Čarin groups.

Proposition 1.2. *Let R be an associative domain with the identity 1, and $G = G(T, R, k)$ a group. Then*

1. *$G = M \rtimes N$ with $M = \{(1, y) |\ y \in R\}$, $N = \{(x, 0) |\ x \in T\}$, $M \cong R^+$, $N \cong T$;*
2. *the centre $Z(G(T, R, 1))$ is trivial;*
3. *if $\exp T > k \geq 2$ then $Z(G(T, R, k)) = \{(x, 0) |\ x^k = 1 \text{ and } x \in Z(T)\}$;*
4. *if $\exp T \leq k$ and $k \geq 2$ then $Z(G(T, R, k)) = \{(x, y) |\ x \in Z(T) \text{ and } y \in R\}$;*
5. *if $k = 1$ then $N \cap N^{(x,y)} = \langle (1, 0) \rangle$ for all $(x, y) \in G \setminus N$;*
6. *if R is a skew field then $G(T, R, 1)$ is a Frobenius group with the kernel M and the complement N;*
7. *if R is a skew field then the complement N acts irreducibly on the kernel M of Frobenius group $G(R^*, R, 1)$ and $[M, N] = M$ (here R^* is the multiplicative group of the skew field R).*

Proof. Properties (1)–(4) and (7) are clear.

(5). If $(a, 0) \in N^{(x,y)}$ then $(a, 0) = (x, y)^{-1}(b, 0)(x, y)$ for some elements y, b of T and x of R with $xy \neq 0$. Then $a = x^{-1}bx$, $x^{-1}(b - 1)y = 0$ and consequently $(a, 0) = (1, 0)$.

(6). Put $A = G \setminus \bigcup \{N^{(a,b)} |\ (a, b) \in G\}$. Since

$$(x, y) = \left(1, (x - 1)^{-1}y\right)^{-1}(x, 0)(1, (x - 1)^{-1}y) \in N^{(1,(x-1)^{-1}y)}$$

for all nontrivial x in T and all y in R, we have $A = M \setminus \{(1, 0)\}$. Now (6) yields that $G(T, R, 1)$ is a Frobenius group with the kernel M and the complement N, as desired.

Proposition 1.3. *There exist a countable metabelian Frobenius group $G = A \rtimes B$ with a torsion-free divisible abelian kernel A and a p-quasicyclic complement B. Also there exist uncountable groups with the same properties.*

Proof. Let R_n be a subfield of complex field $\mathbb{C}$ generated by the roots of polynomial $x^{p^n} - 1$ over the rational field $\mathbb{Q}$ ($n \in \mathbb{N}$). Since $R_n \le R_{n+1}$, we see that $R = \bigcup_{n=1}^{\infty} R_n$ is a field. It is clear that the multiplicative group R^* of R has a quasicyclic p-subgroup F. Then $G = G(T, R, 1) = A \rtimes B$ is a countable Frobenius group with the kernel $A \cong R^+$ and the complement $B \cong F$.

The group $G(X, \mathbb{C}, 1)$, where X is a quasicyclic p-subgroup of the multiplicative group $\mathbb{C}^*$, is the example of an uncountable Frobenius group with the kernel $A \cong \mathbb{C}^+$ and a complement $B \cong X$. This proves the proposition.

Proposition 1.4. *If $G = A \rtimes B$ is a Frobenius group with a kernel A and an abelian complement B then $A = [G, A]$.*

Proof. Let a be a nontrivial element of A and b be a nontrivial element of B. Then $ab \in B^x$ for some $x \in A$ and $ab = x^{-1}dx$ for some $d \in B$. Since $abd^{-1} \in A$, we have $b = d$ and $a = x^{-1}dxd^{-1} \in [A, B]$, as desired.

Recall that a group G is called decomposable if it is generated by two proper subgroups.

Lemma 1.4, Corollary 1.3 and Lemma 2.3 of [11] yield the following corollaries.

Corollary 1.5. *For each prime p there is a countable decomposable HM^*-p-group G with the abelian commutator subgroup G' and the quasicyclic quotient group G/G' (and consequently G does not satisfy the normalizer condition).*

Corollary 1.6. *For each set π of primes and for each prime p there is a HM^*-group G such that the commutator subgroup G' is an abelian periodic π-group and the quotient group G/G' is a quasicyclic p-group.*

Let A be an associative ring (perhaps, without an identity element), M be a right A-module, T a subgroup of the adjoint group A° of A, I a submodule of M. Define on the set of pairs $H(I, T) = \{(x, y)|\ x \in I,\ y \in T\}$ the algebraic operation by the rule

$$(x, y)(u, v) = (uy + u + x, y \circ v). \tag{2}$$

If A is an associative ring with an identity element and T a subgroup of the unit group $U(A)$ of A we denote by $H^*(I, T)$ the set of pairs $\{(x, y)|\ x \in I,\ y \in T\}$ with the algebraic operation given by the rule $(x, y)(u, v) = (uy + x, yv)$.

Remark 1.7. Firstly, the group $H(R, R^\circ)$, where R is a radical ring, was constructed by Ya.P. Sysak [19] (see also [20, chapter 6]).

For the construction of Examples 1.9–1.11 we need the following

Proposition 1.8 ([18, Theorem 2.3]). *Let M be a right A-module, I a nontrivial submodule of M, A an associative ring with nontrivial adjoint group A° and T a nontrivial subgroup of A°. Then $G = H(I, T)$ is a Frobenius group with a kernel B and a complement C, where B is isomorphic to the additive subgroup I^+ of I and C is isomorphic to T, if and only if the following hold:*

(*i*) $\mathrm{ann}_T(i) = \{t \in T|\ it = 0\} = 0$ *for every nontrivial element i of I.*

(*ii*) $I = Ia$ *for every nontrivial element a of T.*

Example 1.9. Let W be a nontrivial linear space over a locally finite field X of characteristic q, where the multiplicative group X^* contains a quasicyclic p-group H (p and q are distinct primes and $q \geq 3$).

If $R = X \oplus X$ is a ring direct sum of two copies of X then the operation given by the rule $v(r_1, r_2) = v(r_1 + r_2)$, where $v \in W$ and $(r_1, r_2) \in R$, defines the right R-module W.

Let $T = \{(t_1, t_2) |\ t_1, t_2 \in H\}$. Suppose that $h = (u, v)$ is a nontrivial element of T such that $W(h - (1, 1)) \neq W$. Then $u + v = 2$. Since $u = a^{p^k}$, $v = a^{p^m}$ for some element a of H and some positive integers k and m, where $k < m$ and $p^m < |a| = p^s$, we have

$$a^{p^k}(1 + a^{p^m - p^k}) = 2,$$

and therefore

$$a^{p^m - p^k}(2a^{p^s - p^m} - 1) = 1.$$

Thus

$$2a^{p^s - p^m} - 1 = \left(a^{p^m - p^k}\right)^{-1} = a^{p^s - p^m}$$

and consequently

$$a^{p^s - p^m} = 1,$$

a contradiction. Hence $W(h - (1, 1)) = W$ for every nontrivial element h of T.

Let i be a nontrivial element of T and $i(a-(1, 1)) = 0$ for some element $a = (x, y)$ of T. Then $x + y - 2 = 0$. As above we can prove that $x = y = 1$.

Thus $G = H^*(I, T)$ is a Frobenius group by Remark 1.9 and consequently G is a HM^*-group with the abelian commutator subgroup G' of exponent q and the quotient group G/G' isomorphic to $\mathbb{C}_{p^\infty} \times \mathbb{C}_{p^\infty}$.

Example 1.10. Let p and q be distinct primes, $\mathbb{Z}_q$ the field with q elements, $\mathbb{Z}_q(\alpha)$ will indicate the algebraic closure of $\mathbb{Z}_q$ generated by α. If ϵ_i is a primitive p^i-th root of 1 $(i = 0, 1, 2, \dots)$, put $F_i = \mathbb{Z}_q(\epsilon_i)$ and $F = \bigcup_{i=0}^{\infty} F_i$. By Theorem 2.5 of [21] the field F has a nontrivial automorphism σ.

Let $F[x, \sigma]$ be the skew polynomial ring with $ax = xa^\sigma$ for every element a of F, and $R = F[x, \sigma]/(x^n)$ $(n \geq 2)$. Then R is a local ring with the nilpotent Jacobson radical xR.

Suppose that $n = 2$. Then

$$[1 + xf, u] = (1 - xf)u(1 + xf)u^{-1} = 1 + x(u^\sigma - u)fu^{-1} \qquad (3)$$

for all elements f and u of F. Since $F = (u^\sigma - u)u^{-1}F$ for some u of F, we conclude that $[1 + J(R), F^*] = 1 + J(R)$. Let now $n \geq 2$. In the same manner as above we can prove that $[1 + J(R)^k, F^*] = 1 + J(R)^k$, where $1 \leq k \leq n$. From [22] and Lemma 2.4 of [23] follows that $1 + J(R)$ is a nilpotent q-group and, moreover, by Theorem 2 of [24] F^* is a locally cyclic abelian q'-group. Lemma 2.2 of [25] yields that $U(R) = (1 + J(R)) \rtimes F^*$. Thus $(1 + J(R)) \rtimes H$ is a HM^*-group with the nilpotent commutator subgroup $1 + J(R)$ of class n $(n \geq 2)$.

Example 1.11. Let $p_1, \dots, p_s, p$ be distinct primes, X_i the splitting field of the polynomials $x^{p^n} - 1$ $(n \in \mathbb{N} \cup \{0\})$ over the field $\mathbb{Z}_{p_i}$, $A = X_1 \oplus \cdots \oplus X_s$ a ring direct sum and σ_i a nontrivial automorphism of X_i $(i = 1, \dots, s)$. Then $R = A[x; \sigma_1, \dots, \sigma_s]/(x^n)$ $(n \geq 2)$, where $(a_1, \dots, a_s)x = x({a_1}^{\sigma_1}, \dots, {a_s}^{\sigma_s})$ for all elements $(a_1, \dots, a_s) \in A$, is a semiperfect ring with the unit group

$$U(R) = (1 + J(R)) \rtimes ({X_1}^* \times \cdots \times {X_s}^*).$$

Obviously, $U(X_i)$ contains the quasicyclic p-subgroup H_i of finite index $(i = 1, \dots, s)$. Therefore

$$G = (1 + J(R)) \rtimes (H_1 \times \cdots \times H_s)$$

is a HM^*-group with the nilpotent π-subgroup $1 + J(R)$ of class n $(n \geq 2)$, where $\pi = \{p_1, \dots, p_s\}$.

The next proposition produces more examples of this sort.

Proposition 1.12. *Let R be a left and right Noetherian ring which satisfies the following condition:*

(α) *every left ideal of R is principal;*

(β) *the set of all left ideals of R is linearly ordered;*

(γ) *the Jacobson radical $J(R)$ is hypercentral (i.e. every quotient ring of $J(R)$ has a nontrivial left annihilator);*

(ζ) *the quotient $R/J(R)$ is the splitting field of the polynomials $x^{p^n} - 1$ $(n \in \mathbb{N} \cup \{0\})$ over the field $\mathbb{Z}_q$ (p and q are distinct primes).*

Then either $U(R)$ is a hypercentral group or there is a positive integer m such that

$$G = (1 + J(R)^m) \rtimes H$$

is a HM^*-group with the hypercentral commutator subgroup $(1 + J(R)^m)$, where H is isomorphic to the quasicyclic p-subgroup of multiplicative group $(R/J(R))^*$.

Proof. By Theorem 6.7 of [26] $\bigcap_{n=1}^{\infty} J(R)^n = (0)$ and by Lemma of [19, p.27] $1 + J(R)$ is a hypercentral subgroup. Put $B_m = R/J(R)^m$ $(m \geq 2)$. Obvious that B_m is a local ring. Since

$$[u, 1 + j] = u^{-1}(1 - j)u(1 + j) = 1 + u^{-1}(uj - ju)$$

for all elements $u \in U(B_m)$, $j \in J(B_m)^{m-1}$, we obtain

$$[1 + J(B_m)^{m-1}, U(B_m)] \leq 1 + J(B_m)^{m-1}.$$

If $[1 + J(B_m)^{m-1}, U(B_m)] = 1$ for all $m \in \mathbb{N}$ then U(R) is a hypercentral group. Therefore we suppose that

$$[1 + J(B_m)^{m-1}, U(B_m)] = 1 + J(B_m)^{m-1}$$

for some positive integer m. Then $G = (1 + J(R)^{m-1}) \rtimes X$, where X is isomorphic to the quasicyclic p-subgroup of $(R/J(R))^*$, is an HM^*-group with the hypercentral commutator subgroup $(1 + J(R)^{m-1})$, as desired.

2. In this part we establish some properties of HM^*-groups.

Theorem 2.1. *Let $G = A \rtimes B$ be a group with a nonperfect locally finite normal q-subgroup A of finite exponent, where the commutator subgroup A' cannot be supplemented nontrivially in A, and an abelian q'-subgroup B. Then*

(*i*) *every G-admissible subgroup of A is contained in a maximal G-admissible subgroup of A;*

(*ii*) *if M is a maximal G-admissible subgroup of A then either $[A, G] \leq M$ or $Z(G/M)$ is a q'-subgroup and $(G/M)/Z(G/M)$ is a Frobenius group.*

Proof. Without restricting generality, we can assume that A is an abelian group of exponent q.

(*i*). Let $R = \mathbb{Z}_q B$ be the group ring of the group B over the field $\mathbb{Z}_q$. By Theorem 1.5 of [27] R is a (von Neumann) regular ring. Moreover, A is a right R-module, with B acting by conjugation. By Theorem 11.24 of [17] and by the Kaplansky Theorem (see [16, Exercise 7.32B]) R is a V-ring. Since every right V-ring is a B-ring (see [17, §18.3]), we conclude that every proper submodule N of A is contained in some maximal submodule of A by Proposition 18.3 of [17].

(*ii*). If M is a maximal submodule of right R-module A then $I = A/M$ is a simple R-module. From $IR = (0)$ it follows that $A/M \leq Z(G/M)$. Therefore we assume that IR is nontrivial. Then $I = iR$ for every nontrivial element i of I. Let $T = \langle 1 - h \mid h \in B \rangle$ is subgroup of R°. If $(u, v) \in Z(H(I, T))$ and (x, y) in a nontrivial element of $H(I, T)$ (see (2)) then $xv = uy$. In particular, $uv(1 - y) = 0$ for nontrivial element u and $x = uy$ of I. Therefore $Iv = (0)$. If now y is an element of T such that $Iy \neq (0)$ we obtain a contradiction. Hence $u = 0$. This means that $Z(H(I, T))$ is a q'-subgroup. Moreover, it is clear that $H(I, T) \cong G/M$.

Let now

$$I_1 = ((A/M)Z(G/M))/Z(G/M), \quad H = (BM/M)/Z(G/M).$$

Then I_1 is a right $\mathbb{Z}_q H$-module. Since I_1 is simple, we obtain $I_1 t = I_1$ for every nontrivial element t of T_1, where $T_1 = \langle 1 - b \mid b \in H \rangle$ is a subgroup of $(\mathbb{Z}_q H)^{\circ}$. It is easy to see that $\operatorname{ann}_{T_1}(i) = (\bar{0})$ for every nontrivial element i of I_1.

Finally, $H(I_1, T_1)$ is a Frobenius group by Proposition 1.8 and

$$H(I_1, T_1) \cong (G/M)/Z(G/M).$$

The theorem is proved.

Corollary 2.2. *Let $G = A \rtimes B$ be a group with a nonperfect locally finite normal q-subgroup A of finite exponent, where the commutator subgroup A' cannot be supplemented nontrivially in A, and a q'-subgroup B. If $\mathbb{Z}_q B$ is a right V-ring (respectively B-ring) then*

(*i*) *every G-admissible subgroup of A is contained in a maximal G-admissible subgroup of A;*

(*ii*) *if M is a maximal G-admissible subgroup of A then either $[G, A] \leq M$ or $Z(G/M)$ is q'-subgroup and $(G/M)/Z(G/M)$ is a Frobenius group.*

Since $A'A^p \neq A$, we obtain from Theorem 2.1 and Lemma 1 of [2] the following

Corollary 2.3. *Let $G = A \rtimes B$ be a nonabelian HM^*-group with a divisible Černikov p-subgroup B and a hypercentral normal p'-subgroup A. If the nontrivial Sylow q-subgroup S of A has a quotient of finite exponent then there exists a quasicyclic p-subgroup D of B such that $S \rtimes D$ has a homomorphic image which is a Čarin group.*

Proposition 2.4. *For any HM^*-group G the following hold:*
(i) $G' = [G', G]$;
(ii) G' has no proper normal supplements in G;
(iii) if N is a proper normal subgroup of G then the quotient group G/N has an infinite exponent; in particular, $G = G^m$ for all positive integers m.

Proof is clear.

Proposition 2.5. *Let G be a HM^*-group. If G' is a p'-group and $G/G' \cong \mathbb{C}_{p^\infty} \times \cdots \times \mathbb{C}_{p^\infty}$ (s factors) then*

$$G = N_1 \cdot \ldots \cdot N_k \times L,$$

where $L \cong \mathbb{C}_{p^\infty} \times \cdots \times \mathbb{C}_{p^\infty}$ (n factors), $k + n = s$, $[N_j, G] = G'$, $N_j = G' \rtimes T_j$, $T_j \cong \mathbb{C}_{p^\infty}$, $G' = {N_1}' \cdot \ldots \cdot {N_k}'$ $(j = 1, \ldots, k)$

Proof. By definition $\overline{G} = G/G' = \overline{N_1} \times \cdots \times \overline{N_s}$, where $\overline{N_i} \cong \mathbb{C}_{p^\infty}$ $(i = 1, \ldots, s)$. Let N_i be the inverse image of $\overline{N_i}$ in G. Then $G' \leq N_i$. Since G is non-hypercentral, we conclude that, for example, N_1 is non-hypercentral. If N_t is a hypercentral subgroup for some integer t $(2 \leq t \leq s)$ then $N_t = G' \times K$, where $K \cong \mathbb{C}_{p^\infty}$, and consequently $K \leq Z(G)$. Therefore $G = N_1 \cdot \cdots \cdot N_k \times L$, where L is a divisible Černikov p-group of rank n for an integer n with $n + k = s$ and N_j is a non-hypercentral group $(j = 1, \ldots, k)$. Since $[G', G] \leq [N_j, G]$, we conclude that $G' = [N_j, G]$ $(j = 1, \ldots, k)$. As a consequence of Proposition 2.3 (*ii*) we obtain that $G' = {N_1}' \cdot \ldots \cdot {N_k}'$ and the proposition is proved.

Remark 2.6. Example 1.10 yields that there is an HM^*-group G such that $G = N_1 \cdot \ldots \cdot N_k$ (N_i as in Proposition 2.4) and ${N_i}' \neq G'$ $(i = 1, \ldots, k)$.

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Received 19.02.1998
Revised 26.01.1999