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SOME OPEN PROBLEMS IN DIRICHLET SERIES

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Let $\Lambda = (\lambda_n)$ be an increasing to infinity sequence of nonnegative numbers and $H(\Lambda)$ the class of entire (absolutely convergent in $\mathbb{C}$) Dirichlet series

$$F(z) = \sum_{n=0}^{+\infty} a_n e^{z\lambda_n}, \qquad z = x + iy.$$

Put

$$M(x,F) = \sup\{|F(x+iy) \mid y \in \mathbb{R}\},$$
$$\mu(x,F) = \max\{|a_n|e^{x\lambda_n} \mid n \ge 0\}.$$

Let $\omega(x)$ be a continuously differentiable positive increasing to $+\infty$ on $[0,+\infty)$ function such that $\omega'(x)\downarrow$. Denote by $h(x)$ an inverse to $\frac{1}{\omega'(x)}$ function. If $h'(t)\nearrow$, $\ln\bigl(1+\frac{h'(t)}{t}\bigr) = o(t)$ $(t \to +\infty)$ then [1] implies that for every function $F \in H(\Lambda)$ outside some set of bounded measure

$$\omega\bigl(\ln M(x,F)\bigr) - \omega\bigl(\ln \mu(x,F)\bigr) \to 0, \qquad x \to +\infty, \tag{1}$$

holds iff

$$\int^{+\infty} \frac{h\bigl(\ln n(t)\bigr)}{t^2}\,dt < +\infty, \tag{2}$$

where $n(t) = \sum_{\lambda_n \le t} 1$ is the counting function of the sequence (λ_n). Putting $\omega(x) = \sqrt{x}$, we obtain a complete solution of Problem 5 from [2].

In the mentioned above theorem from [1] we have $\omega(x) \ge x$. Therefore, we have the following conjectures.

Conjecture 1. *If $\omega(x) \le x$ (probably, under some additional conditions on the growth of $\omega(x)$ to $+\infty$), then for every function $F \in H(\Lambda)$ condition (2) is necessary and sufficient for (1) whenever $x \to +\infty$ outside some set of bounded measure.*

Let $\Phi(x) \uparrow +\infty$ $(x \to +\infty)$. Denote by $H(\Lambda, \Phi)$ the class of functions $F \in H(\Lambda)$ such that

$$\ln \mu(x,F) = O\bigl(x\Phi(x)\bigr) \qquad (x \to +\infty).$$

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Conjecture 2. *If $\omega(x)$ is s either as in the mentioned above theorem from [1] or as in Conjecture 1, then (1) holds for every function $F \in H(\Lambda, \Phi)$ whenever $x \to +\infty$ outside some set of zero density iff*

$$(\forall b > 0): \qquad \frac{1}{x}\int_{b\Phi(x)}^{+\infty} t^{-2} h\big(\ln n(t)\big)\, dt \to 0 \qquad (x \to +\infty). \tag{3}$$

Conjecture 3. *The assertion of Conjecture 2 holds in the class $\underline{H}(\Lambda, \Phi)$ with replacing condition (3) by*

$$(\forall b > 0): \qquad \varliminf_{x\to+\infty} \frac{1}{x}\int_{b\overline{\Phi}(x)}^{+\infty} t^{-2} h\big(\ln n(t)\big)\, dt = 0.$$

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