УДК 517.95

INVERSE PROBLEM FOR FINDING A MAJOR COEFFICIENT IN A PARABOLIC EQUATION

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M.I. Ivanchov. *Inverse problem for finding a major coefficient in a parabolic equation*, Matematychni Studii, **8**(1997) 212–220.

In the paper we consider an inverse problem for a general parabolic equation with unknown time–dependent major coefficient. We establish existence and uniqueness conditions for the given problem.

The determination of the major coefficient in a parabolic equation is considered usually under assupmtions that unknown coefficients don't depend on all arguments. These coefficients can be constant [1], depend only on the time variable [2]–[4], the space variables [5],[6] or the time variable and a part of the space variables [7]. There appear, however, problems where the dependence on a part of variables is given and on the other ones is unknown [8],[9].

In this paper we consider inverse problems for parabolic equations with unknown major coefficient which is equal to the product of two functions one of them depends on the space variable and is known, and the other one is an unknown function of the time variable. A natural approach to studying such a problem consists of finding of solution of the direct problem by means of the Green function. But in general case it is impossible to find in explicit form the Green function for partial differential equation with variable coefficients. We propose in the paper the method of resolving the mentioned inverse problem which allows to use the Green function for more simple equation.

Denote $\Omega_T = \{(y,t) : 0 < y < l, 0 < t < T\}$. Let us consider an inverse problem for the equation

$$v_t = a(t)c(y)v_{yy} + g(y,t), \quad (y,t) \in \Omega_T, \tag{1}$$

where the coefficient $c(y) > 0$ is given and the function $a(t) > 0$ is to be found. By substitution $x = \beta(y)$, where $\beta(y) = \int_0^y c^{-\frac{1}{2}}(\xi)\, d\xi$, the equation (1) is reduced to the form

$$w_t = a(t)\big(w_{xx} + d(x)w_x\big) + q(x,t), \tag{2}$$

where

$$w(\beta(y),t) = v(y,t), \quad d(\beta(y)) = -\frac{c'(y)}{2\sqrt{c(y)}}, \quad q(\beta(y),t) = g(y,t).$$

1991 *Mathematics Subject Classification.* 35R30.

If we put in the equation (2)

$$w(x,t) = u(x,t)\exp\big(-\tfrac{1}{2}\int_0^x d(\xi)\,d\xi\big)$$

we receive the following equation

$$u_t = a(t)\big(u_{xx} - \big(\tfrac{1}{4}d^2(x) + \tfrac{1}{2}d'(x)\big)u\big) + q(x,t)\exp\big(\tfrac{1}{2}\int_0^x d(\xi)\,d\xi\big). \tag{3}$$

By the corresponding notations we can rewrite the equation (3) in the form

$$u_t = a(t)\big(u_{xx} + b(x)u\big) + f(x,t) \quad (x,t) \in Q_T, \tag{4}$$

where $Q_T = \{(x,t) : 0 < x < h,\ 0 < t < T\}$.

We consider inverse problem for the equation (4) subject to the initial condition

$$u(x,0) = \varphi(x), \quad x \in [0,h], \tag{5}$$

the boundary value conditions

$$u_x(0,t) = \mu_1(t), \quad u(h,t) = \mu_2(t), \quad t \in [0,T], \tag{6}$$

and the overspecified condition

$$u(0,t) = \mu_3(t), \quad t \in [0,T]. \tag{7}$$

We define a solution of the problem (4)–(7) as a pair of functions $\big(a(t), u(x,t)\big) \in C[0,T] \times C^{2,1}(\overline{Q_T})$ [10], $a(t) > 0$, $t \in [0,T]$, satisfying (4)–(7).

Suppose that the following assumptions are fulfiled:

(A1) $\varphi(x) \in C^2[0,h]$, $b(x) \in C^1[0,h]$, $\mu_i(t) \in C^1\,[0T]$, $i = 1,2,3$, $f(x,t) \in C^{2,0}(\overline{Q}_T)$;

(A2) $\varphi''(x) > 0$, $x \in [0,h]$; $b(0)\mu_3(t) \ge 0$, $b(h)\mu_2(t) \le 0$, $\mu_3'(t) - f(0,t) > 0$, $f_x(0,t) - \mu_1'(t) \ge 0$, $\mu_2'(t) - f(h,t) \ge 0$, $t \in [0,T]$; $f_{xx}(0,t) \ge 0$, $(x,t) \in \overline{Q}_T$.

(A3) $\varphi(0) = \mu_3(0)$, $\varphi(h) = \mu_2(0)$, $\varphi'(0) = \mu_1(0)$, $\big(\mu_2'(0) - f(h,0)\big)\big(\varphi''(0) + b(0)\varphi(0)\big) = \big(\mu_3'(0) - f(0,0)\big)\big(\varphi''(h) + b(h)\varphi(h)\big)$.

Supposing for a moment that the continuous function $a(t) > 0$ is known, we reduce the problem (4)–(6) to the equivalent integral equation

$$u(x,t) = u_0(x,t) + \int_0^t \int_0^h G_0(x,t,\xi,\tau)a(\tau)b(\xi)u(\xi,\tau)\,d\xi d\tau, \quad (x,t) \in \overline{Q}_T, \tag{8}$$

where

$$u_0(x,t) = \int_0^h G_0(x,t,\xi,0)\varphi(\xi)\,d\xi - \int_0^t G_0(x,t,0,\tau)a(\tau)\mu_1(\tau)\,d\tau -$$
$$- \int_0^t G_{0\xi}(x,t,h,\tau)a(\tau)\mu_2(\tau)\,d\tau + \int_0^t \int_0^h G_0(x,t,\xi,\tau)f(\xi,\tau)\,d\xi d\tau,$$

$$G_k(x,t,\xi,\tau) = \frac{1}{2\sqrt{\pi(\theta(t)-\theta(\tau))}} \sum_{n=-\infty}^{\infty} (-1)^n \Big(\exp\Big(-\frac{(x-\xi+2nh)^2}{4(\theta(t)-\theta(\tau))}\Big) +$$
$$+ (-1)^k \exp\Big(-\frac{(x+\xi+2nh)^2}{4(\theta(t)-\theta(\tau))}\Big)\Big), \quad k = 0,1, \quad \theta(t) = \int_0^t a(\tau)\,d\tau. \tag{9}$$

It is easy to verify that $G_0(x,t,\xi,\tau)$ is the Green function of problem (5),(6) for the equation $u_t = a(t)u_{xx}$. To find another equation for functions $\big(a(t), u(x,t)\big)$, we differentiate the condition (7) with respect to t and use it in the equation (4) at the point $x = 0$. Therefore, we obtain

$$a(t)\big(u_{xx}(0,t) + b(0)\mu_3(t)\big) = \mu_3'(t) - f(0,t). \tag{10}$$

Note that from the assumptions (A1)–(A3) we have $u(x,t) \in C^{2,1}(\overline{Q}_T)$, i.e. the transformation of the equation (4) was correct.

Let calculate the derivatives $u_{0x}(x,t)$, $u_{0xx}(x,t)$, using the equalities

$$\begin{aligned} G_{0x}(x,t,\xi,\tau) &= -G_{1\xi}(x,t,\xi,\tau), \\ G_{0xx}(x,t,\xi,\tau) &= G_{0\xi\xi}(x,t,\xi,\tau), \\ a(\tau)G_{0\xi\xi}(x,t,\xi,\tau) &= -G_{0\tau}(x,t,\xi,\tau) \end{aligned} \tag{11}$$

which can be easily verified. Integrating by parts and using the assumptions (A1),(A2) (for details see [11]) we find

$$\begin{aligned} u_{0x}(x,t) &= \int_0^h G_1(x,t,\xi,0)\varphi'(\xi)\,d\xi - \int_0^t G_{0x}(x,t,0,\tau)a(\tau)\mu_1(\tau)\,d\tau + \\ &+ \int_0^t G_1(x,t,h,\tau)\big(\mu_2'(\tau) - f(h,\tau)\big)\,d\tau + \int_0^t\int_0^h G_1(x,t,\xi,\tau)f_\xi(\xi,\tau)\,d\xi d\tau, \\ u_{0xx}(x,t) &= \int_0^h G_0(x,t,\xi,0)\varphi''(\xi)\,d\xi + \int_0^t G_0(x,t,0,\tau)\big(f_x(0,\tau) - \mu_1'(\tau)\big)\,d\tau + \\ &+ \int_0^t G_{1x}(x,t,h,\tau)\big(\mu_2'(\tau) - f(h,\tau)\big)\,d\tau + \int_0^t\int_0^h G_0(x,t,\xi,\tau)f_{\xi\xi}(\xi,\tau)\,d\xi d\tau. \end{aligned} \tag{12}$$

We obtain from (12) and the assumption (A3) the estimate

$$u_{0xx}(0,t) \ge \int_0^h G_0(0,t,\xi,0)\varphi''(\xi)\,d\xi > 0, \quad t \in [0,T]. \tag{13}$$

Taking into consideration that $u_{xx}(0,0) = u_{0xx}(0,0) > 0$, we may conclude existence of the number T_0, $0 < T_0 \le T$ such that $u_{xx}(0,t) > 0$, $t \in [0,T_0]$. Hence, the equation (10) can be transformed to the form

$$a(t) = \frac{\mu_3'(t) - f(0,t)}{u_{xx}(0,t) + b(0)\mu_3(t)}, \quad t \in [0,T_0]. \tag{14}$$

To establish existence of solution of the system (8),(14) we shall apply the Schauder fixed-point theorem for the equicontinuous operators. First, we evaluate the functions $u_0(x,t)$, $u(x,t)$ and its derivatives with respect to x. Applying the inequalities

$$0 \le \int_0^h G_0(x,t,\xi,\tau)\,d\xi \le 1, \quad \sum_{n=1}^\infty e^{-n^2x^2} \le \frac{\sqrt{\pi}}{2x}, \quad x \in (0,\infty),$$

we receive from (9)

$$|u_0(x,t)| \leq M_1 < \infty, \quad (x,t) \in \overline{Q}_T, \tag{15}$$

where the constant $M_1 > 0$ depends only on the initial data of the problem (4)–(7). Denote $m(t) = \max_{x\in[0,h]} |u(x,t)|$. We obtain from (8)

$$m(t) \leq M_1 + \big(\max_{x\in[0,h]} |b(x)|\big) \int_0^t a(\tau)m(\tau)\,d\tau \int_0^h G_0(x,t,\xi,\tau)\,d\xi \leq M_1 + C_1 \int_0^t a(\tau)m(\tau)\,d\tau.$$

Applying the Gronwall lemma, we evaluate $m(t)$ and $u(x,t)$:

$$m(t) \leq M_1 e^{C_1\theta(t)}, \quad |u(x,t)| \leq M_1 e^{C_1\theta(t)}, \quad (x,t) \in \overline{Q}_T. \tag{16}$$

We find from (12) the estimate

$$|u_{0x}(x,t)| \leq C_2 + C_3 \int_0^t \frac{d\tau}{\sqrt{\theta(t)-\theta(\tau)}}. \tag{17}$$

By differentiating the equation (8) we obtain

$$u_x(x,t) = u_{0x}(x,t) + \int_0^t \int_0^h G_{0x}(x,t,\xi,\tau)a(\tau)\,b(\xi)u(\xi,\tau)\,d\xi d\tau \tag{18}$$

that leads to the estimate

$$\begin{aligned}|u_x(x,t)| &\leq C_2 + C_3 \int_0^t \frac{d\tau}{\sqrt{\theta(t)-\theta(\tau)}} + C_4 \int_0^t \frac{a(\tau)e^{C_1\theta(\tau)}\,d\tau}{\sqrt{\theta(t)-\theta(\tau)}} \leq \\ &\leq C_2 + C_3 \int_0^t \frac{d\tau}{\sqrt{\theta(t)-\theta(\tau)}} + 2C_4 e^{C_1\theta(t)}\sqrt{\theta(t)}.\end{aligned} \tag{19}$$

Further, we find $u_{xx}(x,t)$ from (8):

$$u_{xx}(x,t) = u_{0xx}(x,t) + \int_0^t a(\tau)\ d\tau \int_0^h G_{0xx}(x,t,\xi,\tau)b(\xi)u(\xi,\tau)\,d\xi.$$

We apply (11) for integrating by parts in the above equality and we obtain

$$\begin{aligned}u_{xx}(x,t) = u_{0xx}(x,t) + b(h)\int_0^t G_{0\xi}(x,t,h,\tau)a(\tau)\,\mu_2(\tau)\,d\tau - \\ - \int_0^t a(\tau)\ d\tau \int_0^h G_{0\xi}(x,t,\xi,\tau)(b(\xi)u(\xi,\tau))_\xi\,d\xi.\end{aligned} \tag{20}$$

We substitute (20) into the equation (14) and transform it to the form

$$\begin{aligned}a(t) = (\mu_3'(t) - f(0,t))\Big(b(0)\mu_3(t) + u_{0xx}(0,t) + b(h)\int_0^t G_{0\xi}(0,t,h,\tau)\times \\ \times a(\tau)\,\mu_2(\tau)\,d\tau - \int_0^t a(\tau)\ d\tau \int_0^h G_{0\xi}(0,t,\xi,\tau)(b(\xi)u(\xi,\tau))_\xi\,d\xi\Big)^{-1}, \ t \in [0,T_0].\end{aligned} \tag{21}$$

Now we shall evaluate $a(t)$ using (21). We find from (20),(13),(16),(19) and the assumption (A3)

$$a(t) \leq \big(\max_{[0,T]}(\mu_3'(t) - f(0,t))\big)\Big(\int_0^h G_0(0,t,\xi,0)\varphi''(\xi)\,d\xi -$$
$$- \int_0^t \frac{a(\tau)}{\sqrt{\theta(t)-\theta(\tau)}}\Big(C_5 + C_6\int_0^\tau \frac{d\sigma}{\sqrt{\theta(\tau)-\theta(\sigma)}} + C_7 e^{C_1\theta(\tau)}(1+\sqrt{\theta(\tau)})\Big)\,d\tau\Big)^{-1}$$

that can be rewritten in the form

$$a(t) \leq C_8\Big(\int_0^h G_0(0,t,\xi,0)\varphi''(\xi)\,d\xi - C_6\pi t - F(\theta(t))\Big)^{-1}, \tag{22}$$

where

$$F(z) = 2C_5\sqrt{z} + C_7 e^{C_1 z}\Big(2\sqrt{z} + \frac{\pi}{2}z\Big). \tag{23}$$

Let $\sigma_0 > 0$ be a number such that the following inequality is valid on $[0,\sigma_0]$:

$$\frac{1}{3\sqrt{\pi\sigma}}\int_0^h \varphi''(\xi)\sum_{n=-\infty}^{\infty}(-1)^n \exp\Big(-\frac{(\xi+2nh)^2}{4\sigma}\Big)\,d\xi \geq F(\sigma). \tag{24}$$

We shall suppose that the number T_0, $0 < T_0 \leq T$ satisfies the condition

$$C_6\pi T_0 \leq \min_{[0,\sigma_0]}\frac{1}{3\sqrt{\pi\sigma}}\int_0^h \varphi''(\xi)\sum_{n=-\infty}^{\infty}(-1)^n \exp\Big(-\frac{(\xi+2nh)^2}{4\sigma}\Big)\,d\xi. \tag{25}$$

We multiply both sides of inequality (22) by the denominator and integrate from 0 to t:

$$\int_0^t\Big(\int_0^h G_0(0,\tau,\xi,0)\varphi'\prime(\xi)\,d\xi - C_6\pi T_0 - F(\theta(\tau))\Big)a(\tau)\,d\tau \leq C_8 t. \tag{26}$$

Now consider the function $r(\sigma)$ defined by the formula

$$r(\sigma) = \int_0^\sigma\Big(\frac{1}{\sqrt{\pi z}}\int_0^h \varphi''(\xi)\sum_{n=-\infty}^{\infty}(-1)^n \exp\Big(-\frac{(\xi+2nh)^2}{4z}\Big)\,d\xi - C_6\pi T_0 - F(z)\Big)dz. \tag{27}$$

It is easy to see that the substitution $\theta(\tau) = z$ reduces inequality (26) to the form

$$r(\theta(t)) \leq C_8 t. \tag{28}$$

It follows from (24),(25),(27) that the function $r(\sigma)$ is continuous and $r(\sigma) \geq 0$, $r'(\sigma) > 0$ on $[0,\sigma_0]$. Thus there exists an inverse function $s = r^{-1}(\sigma)$ defined on the segment $[0,R_0]$ where $R_0 = r(\sigma_0)$. Moreover, it is positive and monotonically increasing. Hence, we obtain from (28)

$$\theta(t) \leq r^{-1}(C_8 t), \quad t \in [0,T_0] \tag{29}$$

under the assumption that

$$C_8 T_0 \le R_0. \tag{30}$$

We return to inequality (22) and evaluate $a(t)$ using (24),(25),(29),(30)

$$a(t) \le a_{\max} < \infty, \quad t \in [0, T_0], \tag{31}$$

where

$$a_{\max} = C_8 \Big[\min_{[0,\sigma_0]} \frac{1}{3\sqrt{\pi\sigma}} \int_0^h \varphi''(\xi) \sum_{n=-\infty}^{\infty} (-1)^n \exp\Big(-\frac{(\xi + 2nh)^2}{4\sigma}\Big)\, d\xi \Big]^{-1}$$

and the number T_0 is defined by means of inequalities (25),(30) and does not depend on $a(t)$.

Using (20),(12),(31) we obtain from (14)

$$a(t) \ge C_9 \Big[C_{10} + \frac{C_{11}}{\sqrt{\min_{[0,T_0]} a(t)}} \Big]^{-1}, \quad t \in [0, T_0]$$

where C_i, $i = 9, 10, 11$, are positive constants depending only on the problem data. This inequality holds true at the point of minimum of $a(t)$, thus we have

$$\min_{[0,T_0]} a(t) \ge C_9 \Big[C_{10} + \frac{C_{11}}{\sqrt{\min_{[0,T_0]} a(t)}} \Big]^{-1}.$$

Resolving this inequality with respect to $\sqrt{\min_{[0,T_0]} a(t)}$, we find the estimation of $a(t)$ from below

$$a(t) \ge a_{\min} > 0, \quad t \in [0, T_0], \tag{32}$$

where

$$a_{\min} = \Big(\frac{\sqrt{C_{11}^2 + 4C_9 C_{10}} - C_{11}}{2C_{10}} \Big)^2.$$

Now it is possible to evaluate u, u_x, u_{xx} from (16),(19),(20) using (31),(32)

$$|u(x,t)| \le M_2 < \infty, \ \ |u_x(x,t)| \le M_3 < \infty, \ \ |u_{xx}(x,t)| \le M_4 < \infty, \ (x,t) \in \overline{Q}_{T_0}. \tag{33}$$

Taking into account the presence of $u_{xx}(0,t)$ in the equation (14), we introduce two additional equations with respect to u_x and u_{xx} and we change equation (14). Denoting $v = u_x$, $w = u_{xx}$ we obtain from (18),(20) and (14)

$$\begin{aligned} v(x,t) &= u_{0x}(x,t) + \int_0^t \int_0^h G_{0x}(x,t,\xi,\tau) a(\tau)\, b(\xi) u(\xi,\tau)\, d\xi d\tau, \quad (x,t) \in \overline{Q}_{T_0}, \\ w(x,t) &= u_{0xx}(x,t) + b(h) \int_0^t G_{0\xi}(x,t,h,\tau) a(\tau)\, \mu_2(\tau)\, d\tau - \\ &\quad - \int_0^t \int_0^h G_{0\xi}(x,t,\xi,\tau) a(\tau) \big(b(\xi) v(\xi,\tau) + b'(\xi) u(\xi,\tau) \big)\, d\xi d\tau, \quad (x,t) \in \overline{Q}_{T_0}, \\ a(t) &= \frac{\mu_3'(t) - f(0,t)}{w(0,t) + b(0)\mu_3(t)}, \quad t \in [0, T_0]. \end{aligned} \tag{34}$$

Equations (8),(34) form a system of equations with respect to unknown functions $\big(a(t), u(x,t), v(x,t), w(x,t)\big)$. We write the system (8),(34) as an operator equation

$$\omega = P\omega,$$

where $\omega = \big(a(t), u(x,t), v(x,t), w(x,t)\big)$. We denote by $\mathcal{N}$ the set defined in the following manner

$$\mathcal{N} = \Big\{\big(a(t), u(x,t), v(x,t), w(x,t)\big) \in C[0,T_0] \times C(\overline{Q}_{T_0}) \times C(\overline{Q}_{T_0}) \times C(\overline{Q}_{T_0}) : \\ a_{\min} \le a(t) \le a_{\max},\ |u(x,t)| \le M_2,\ |v(x,t)| \le M_3,\ |w(x,t)| \le M_4\Big\}.$$

The set $\mathcal{N}$ is closed and convex in the space $C[0,T_0] \times C(\overline{Q}_{T_0}) \times C(\overline{Q}_{T_0}) \times C(\overline{Q}_{T_0})$ and the operator P maps $\mathcal{N}$ into $\mathcal{N}$. To verify the equicontinunuity of P on $\mathcal{N}$ it is sufficient to apply the Arzela-Ascoli theorem (for details see [3],[11]). Using the Schauder theorem we establish existence of solution of the system (8),(34) that gives existence of solution $(a(t), u(x,t))$ of the problem (4)–(7) in $C[0,T_0] \times C^{2,1}(Q_T) \cap C^{1,0}(\overline{Q_{T_0}})$. It follows from condition (A3) and equation (10) that the concordance conditions at the point $x = 0, t = 0$ are fulfiled. Then the substitution of $a(0)$ from (10) into the equality

$$\mu_2'(0) = a(0)(\varphi''(h) + b(h)\varphi(h)) + f(h,0)$$

and condition $(A3)$ give the concordance condition of the first order at the point $x = h,\ t = 0$. It implies $u(x,t) \in C^{2,1}(\overline{Q_{T_0}})$.

Theorem 1. *Suppose that the assumptions (A1)–(A3) are fulfiled. Then there exists a solution of the problem (4)–(7) defined in the domain $\overline{Q}_{T_0}$, where the number T_0, $0 < T_0 \le T$ satisfies conditions (25),(30).*

It follows from the proof of Theorem 1 that assumption (A3) may be replaced by a weaker one.

Theorem 2. *The assertion of Theorem 1 remains true if assumption (A3) is replaced by the following one:*

(A4) $\varphi''(x) \ge 0,\ x \in [0,h],\ \varphi''(0) > 0;\ b(0)\mu_3(t) \ge 0,\ \mu_3'(t) - f(0,t) > 0,\ f_x(0,t) - \mu_1'(t) \ge 0,\ t \in [0,T]$.

The proof of this theorem differs from that of Theorem 1 only by the estimate of $u_{0xx}(0,t)$ from below:

$$u_{0xx}(0,t) \ge \int_0^h G_0(0,t,\xi,0)\varphi''(\xi)\,d\xi - \int_0^t \big|G_{1x}(0,t,h,\tau)(\mu_2'(\tau) - f(h,\tau))\big|\,d\tau - \\ - \int_0^t\int_0^h G_0(0,t,\xi,\tau)|f_{\xi\xi}(\xi,\tau)|\,d\xi d\tau \ge \int_0^h G_0(0,t,\xi,0)\varphi''(\xi)\,d\xi - C_{12}t,$$

where the constant $C_{12} > 0$ depends only on the problem initial data. This estimate implies replacement of the constant C_6 in (22) by another one that does not change the proof of the theorem.

Uniqueness conditions for the problem (4)–(7) are given by the following theorem.

Theorem 3. *Suppose that besides assumptions (A1),(A2) the following condition*

$$\mu_3'(t) - f(0,t) > 0, \quad t \in [0,T]$$

is satisfied. Then the solution of the problem (4)–(7) is unique in the domain $\overline{Q}_T$.

Proof. If there are two solutions $(a_i(t), u_i(x,t))$, $i = 1,2$ of the problem (4)–(7) then their difference $a_0(t) = a_1(t) - a_2(t)$, $\omega(x,t) = u_1(x,t) - u_2(x,t)$ satisfies the equation

$$\omega_t = a_1(t)\big(\omega_{xx} + b(x)\omega\big) + a_0(t)\big(u_{2xx}(x,t) + b(x)u_2(x,t)\big), \quad (x,t) \in Q_T, \quad (35)$$

the homogeneous initial and boundary conditions

$$\omega(x,0) = 0, \quad x \in [0,h], \quad \omega_x(0,t) = \omega(h,t) = 0, \quad t \in [0,T] \quad (36)$$

and the homogeneous overspecified condition

$$\omega(0,t) = 0, \quad t \in [0,T]. \quad (37)$$

Denote by $G(x,t,\xi,\tau)$ the Green function of the problem (35),(36). We find the solution of the problem (35),(36) by means of Green function and put it into condition (37):

$$\int_0^t a_0(\tau)\,d\tau \int_0^h G(0,t,\xi,\tau))\big(u_{2\xi\xi}(\xi,\tau) + b(\xi)u_2(\xi,\tau)\big)\,d\xi = 0. \quad (38)$$

It follows from the assumptions (A1),(A2) that the function $u_{2xx}(x,t) + b(x)u_2(x,t)$ is continuous in $\overline{Q}_T$ and its derivative with respect to x is continuous in Q_T. Using the properties of heat potentials [12] we differentiate (38) with respect to t and obtain

$$a_0(t)\big(u_{2xx}(0,t) + b(0)u_2(0,t)\big) + a_1(t)\int_0^t a_0(\tau)\,d\tau \int_0^h G_{\xi\xi}\,(0,t,\xi,\tau) \times$$
$$\times\big(u_{2\xi\xi}(\xi,\tau) + b(\xi)u_2(\xi,\tau)\big)\,d\xi = 0. \quad (39)$$

Since $u_2(x,t)$ is a solution of the problem (4)–(7), we find from condition (7) and from equation (4) at the point $x = 0$ that

$$a_2(t)\big(u_{2xx}(0,t) + b(0)u_2(0,t)\big) = \mu_3'(t) - f(0,t) > 0.$$

Consequently, equation (39) is a homogeneous Volterra integral equation of the second kind with integrable kernel and, hence, $a_0(t) \equiv 0$, $t \in [0,T]$. Then we put $a_0(t) \equiv 0$ in equation (35) and we obtain $\omega(x,t) \equiv 0$ in $\overline{Q}_T$ as a solution of homogeneous direct problem (35),(36) [10]. The proof is complete.

Remarks. 1. The method that we use above can be applied to inverse problems for equation (4) and consequently, for equation (1) with different boundary and overspecified conditions.

2. To show the compatibility of conditions of Theorem 1 we consider the equation

$$u_t = a(t)(u_{xx} - xu) + x^3 - 1 + (x^3 + x - 2)t + xt^2,\ 0 < x < 1/2,\ 0 < t < 1 \quad (40)$$

subject to conditions

$$u(x,0) = x^2, \quad 0 \le x \le 1/2, \quad (41)$$

$$u_x(0,t) = 0, \quad u(h,t) = h^2 + t, \quad u(0,t) = t, \quad 0 \le t \le 1. \quad (42)$$

It is easy to verify that the data of the problem (40)–(42) satisfy the assumptions of Theorem 1 and the problem (40)–(42) has a solution $a(t) = 1 + t$, $u(x,t) = x^2 + t$. We remark that the method proposed above can be applied to inverse problems for equation (4) and consequently, for equation (1) with different boundary and overspecified conditions.

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Received 15.12.1996