УДК 515.12

PROBLEMS ON ABSORBING SETS IN THE HYPERSPACE THEORY AND THE THEORY OF FUNCTIONS

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The method of absorbing sets, which has its origin in the works of Anderson, Bessaga and Pełchyński, is now one of the most powerful tools of infinite-dimensioanl topology having numerous applications not only in topology but also in other areas of mathematics. In its modern form, the method of absorbing sets is developed by M. Bestvina and J. Mogilski [BM]. A detailed exposition of the theory of absorbing stes (including some applications) can be found in [BRZ]. See also [BGM] for backgrounds of a related theory of absorbing systems.

1. Hyperspaces.

Given a metric space X (with the metric d), we denote by $\exp X$ the space of all nonempty compact subsets of X endowed with the Hausdorff metric d_H:

$$d_H(A,B) = \inf\{\varepsilon > 0 \mid A \subset O_\varepsilon(B), B \subset O_\varepsilon(A)\}, \qquad A, B \in \exp X.$$

The Curtis-Schori-West Theorem (see, e.g. [CS]) states that the hypespace of each nondegenerate Peano continuum is homeomorphic to the Hilbert cubes Q.

1.1. Question. *Is the subspace* s.c.d.$(Q) = \{X \in \exp Q \mid X$ *is strongly countable-dimensional*$\}$ *homeomorphic to some absorbing set in* Q*?*

R. Pol [P] proved that the set s.c.d.(Q) is coanalytic and we can expect that $(\exp Q, \text{s.c.d.}(Q))$ is homeomorphic to (Q, Π_2). Here Π_2 is the standard absorber for the class of coanalytic spaces.

1.2. Problem. *Let* $\mathcal{D}_\alpha = \{X \in \exp \mathbb{R}^n \mid$ *the Hausdorff dimension of* X *is* $\leq \alpha\}$. *Describe the topology of the system* $\{\mathcal{D}_\alpha \mid \alpha \geq 0\}$.

1.3. Problem. *Describe the topology of the pair* $\big(d(M), t(M)\big)$, *where* $d(M)$ *(respectively,* $t(M)$*) denotes the hyperspace of all dentrites (respectively, trees) in a surface* M.

A subset A of a space X is called strongly homogeneously embedded in X if every autohomeomorphism of A can be extended to a homeomorphism of X.

1.4. Question. *Is the hyperspace of strongly homogeneously embedded Cantor sets in* Q *an absorbing set in* $\exp Q$*?*

1991 *Mathematics Subject Classification.* 54F65, 54B20, 52A20, 26A48.

1.5. Problem. *Describe the topology of the hyperspace of Lipschitz neighborhood retracts in* $\mathbb{R}^n$.

The hyperspace of absolute neighborhood retracts in $\mathbb{R}^n$, $n \geq 3$, is considered by T. Dobrowoloski and L. Rubin [DR]. In particular, they proved that this hyperspace is an absorber for the class of absolute $G_{\delta\sigma\delta}$-spaces.

2. Convexity.

The hyperspace of convex compacta in an open subset $U \subset \mathbb{R}^n$ is denoted by $\mathrm{cc}(U)$. It is known that $\mathrm{cc}(\mathbb{R}^n)$ is homeomorphic to $Q \setminus \{\text{point}\}$ iff $n \geq 2$ (see [M]), so in the sequel we assume that $n \geq 2$.

2.1. Question. *Let* $\mathrm{ccp}(U)$ *denote the hyperspace of convex polyhedra in* U. *Is the pair* $\big(\mathrm{cc}(U), \mathrm{ccp}(U)\big)$ *homeomorphic to the pair* $(Q, \sigma) \times U \times [\,0,1)$?

For $X \in \mathrm{cc}(\mathbb{R}^n)$, let $\xi_X \colon P(X) \to X$ denote the barycenter map defined on the space of probability measures on X. A convex compactum X is called barycentrically open if ξ_X is an open map.

2.2. Problem. *Describe the topology of the hyperspace of barycentrically open compacta in* $\mathbb{R}^n$.

3. Spaces of functions.

Denote by $\mathcal{M}(\mathbb{I})$ the set of all nondecreasing real-valued continuous functions defined on $\mathbb{I} = [0,1]$. Identifying each $f \in \mathcal{M}(\mathbb{I})$ with its graph we can consider $\mathcal{M}(\mathbb{I})$ as a subspace of the hyperspace $\exp(\mathbb{I} \times \mathbb{R})$. Denote by $\widetilde{\mathcal{M}}(\mathbb{I})$ the closure of $\mathcal{M}(\mathbb{I})$ in $\exp(\mathbb{I} \times \mathbb{R})$. In turn, each $f \in \widetilde{\mathcal{M}}(\mathbb{I})$ can be identified with a left-semicontinuous on $[\,0,1)$ nondecreasing function.

Let $\mathcal{H}$ denote the set of functions $f \in \widetilde{\mathcal{M}}(\mathbb{I})$ satisfying the condition $(*)$: the derivative f' vanishes everywhere exepting the set of Jordan measure 0. The following result of the second author [T] is a motivation for formulating some problems concerning the spaces $\mathcal{M}(\mathbb{I})$ and $\widetilde{\mathcal{M}}(\mathbb{I})$.

Theorem. *The pair* $(\widetilde{\mathcal{M}}(\mathbb{I}), \mathcal{H})$ *is homeomorphic to the pair* $\big(Q^\omega \setminus \{*\}, \Sigma^\omega \setminus \{*\}\big)$.

3.1. Problem. *For every* $\mathcal{C}$*-absorbing set* Ω *in* Q, *where* $\mathcal{C}$ *is one of the absolute Borel or projective classes, find a subspace* $X(\mathcal{C})$ *in* $\widetilde{\mathcal{M}}(\mathbb{I})$ *such that* $\big(\widetilde{\mathcal{M}}(\mathbb{I}), X(\mathcal{C})\big) \cong \big(Q \setminus \{*\}, \Omega \setminus \{*\}\big)$.

Not that the above theorem can be considered as a solution of Problem 3.1 for the class of absolute $F_{\sigma\delta}$-classes.

3.2. Problem. *Describe the topology of the subspace* $\mathcal{H} \cap \mathcal{M}(\mathbb{I})$.

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Received 7.05.97