УДК 513.88

WHEN A PERTURBATION OF A DENSE MANIFOLD IS DENSE?

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Problem. *Let H, H_+ be Hilbert spaces endowed with the norms $\|\cdot\|$, $\|\cdot\|_+$ respectively. Suppose that H_+ is continuously and densely embedded into H: $H_+ \subset H$, $\overline{H}_+ = H$ (the closure with respect to the norm $\|\cdot\|$), $\|y\| \le \|y\|_+$ for all $y \in H_+$. Further, suppose that $\mathfrak{H}$ is a Hilbert space, $U \in \mathcal{B}(H_+, \mathfrak{H})$ $R(U) = \mathfrak{H}$, $\ker U$ is dense in $\mathfrak{H}$, $\Phi \in \mathcal{B}(H, \mathfrak{H})$. Prove (or disprove) the following statement: $\ker(U - \Phi)$ is dense in $\mathfrak{H}$.*

Remark. The results of [1,2] make the above statement plausible provided at least one of the following conditions holds:
(a) U is normally Φ-solvable (and hence $R(U - \Phi) = \mathfrak{H}$);
(b) $\forall \varepsilon > 0\ \exists k(\varepsilon)\ \forall y \in H_+$

$$\|U_y\|_{\mathfrak{H}+} \le \varepsilon \|y\|_+ + k(\varepsilon)\|y\|_+.$$

Note that (a) holds if Φ is either compact or sufficiently small by norm.

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Received 15.04.97

1991 *Mathematics Subject Classification.* 46C20, 47A55.