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**SOME PROBLEMS CONCERNING ADMISSIBLE
SEQUENCES IN THE THEORY OF ENTIRE
AND MEROMORPHIC FUNCTIONS**

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In the theory of entire and meromorphic functions the notion of λ -admissible sequence plays an essential rôle (see, for example [1–2]). Let $\mathcal{Z} = \{z_\nu\}$ $z_\nu \rightarrow \infty$, as $\nu \rightarrow \infty$ be a sequence of nonzero complex numbers, $n(r, \mathcal{Z}) = \sum_{|z_\nu| \leq r} 1$, $\mathcal{N}(r, \mathcal{Z}) = \int_0^r \frac{n(t, \mathcal{Z})}{t} dt$ and λ a positive nondecreasing on \mathbb{R}_+ function.

The sequence \mathcal{Z} is called λ -admissible, if there exist constants $a > 0$, $b > 0$, such that for all $r > 0$, $r_2 > r_1 > 0$, $k \in \mathbb{N}$ the following inequalities hold $\mathcal{N}(r, \mathcal{Z}) \leq a\lambda(br)$, $\left| \sum_{r_1 < |z_\nu| \leq r_2} z_\nu^{-k} \right| \leq \frac{a\lambda(br_1)}{r_1^k} + \frac{a\lambda(br_2)}{r_2^k}$.

The following problem is formulated together with A. Gol'dberg, B. Vynnytskyi.

Problem 1. Describe the λ -admissible sequences with $\lambda(r) = \mathcal{N}(r, \mathcal{Z})$.

Let \mathcal{K} be the class of positive nondecreasing on \mathbb{R}_+ functions λ , $\lambda(+0) > 0$ such that for every couple of function $\lambda, \tilde{\lambda}$ from \mathcal{K} either the ratio $\lambda(r)/\tilde{\lambda}(r)$ or $\tilde{\lambda}(r)/\lambda(r)$ is bounded.

We shall say that λ is equivalent to $\tilde{\lambda}$, if $\tilde{\lambda} \leq a\lambda(br)$ and $\lambda(r) \leq \tilde{a}\tilde{\lambda}(\tilde{b}r)$ for some positive $a, b, \tilde{a}, \tilde{b}$ and every $r > 0$.

The inequality $\tilde{\lambda}(r) \leq a\lambda(br)$ with some $a > 0$, $b > 0$ and arbitrary $r > 0$ supplies the set $\{K\}$ of equivalence classes with order relation (\leq).

A sequence \mathcal{Z} of nonzero complex numbers is called K -admissible, if one is λ -admissible with respect to some (and, as consequence, with respect to every) function λ from K .

A class K is called *minimal* for \mathcal{Z} if \mathcal{Z} is K_2 -admissible with $K \leq K_2$ and non- K_1 -admissible with $K_1 < K$.

Problem 2. Is there a minimal class for every sequence of nonzero complex numbers?

Problem 3. Find (if exists) a minimal class generated by \mathcal{K} which contains $\mathcal{N}(r, \mathcal{Z}) + 1$ for arbitrary sequence \mathcal{Z} .

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