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AN EQUIVARIANT HILBERT CUBE GENERATED BY THE TRANSVERSALITY MAPPING

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The transversality mapping $\perp: GX \to GX$ defines an action of the group $\mathbb{Z}_2$ on the space GX of inclusion hyperspaces of a compactum X. It is proved that GX with this action is equimorphic to the equivariant Hilbert cube for a non-degenerate metrizable continuum X. Equivariant softness of the multiplication mapping $\mu: G\lambda X \to GX$ is proved for openly generated continua.

0. Let H be a compact group. There is a universal object in the category of metrizable compacta with action of the group H, namely, the equivariant Hilbert cube. A characterization theorem for this object was recently obtained by S. Ageev [1]. It turns out that this characterization is a convenient tool for finding realizations of the equivariant Hilbert cube.

Some functorial representations of the Hilbert cube Q are known. In particular, E. Moiseev has proved that the space GX of inclusion hyperspaces of a compactum X is homeomorphic to Q iff X is a non-degenerate metrizable continuum [5].

It was remarked by T. Radul that the transversality operator $\perp$ of J. van Mill and M. van de Vel [6] determines an involution on GX. Therefore, it determines an action of the group $\mathbb{Z}_2$ on GX. The following question arises naturally: is the space GX with this action equimorphic to the equivariant Hilbert cube if X is a non-degenerate metrizable continuum? Here this question is answered affirmatively.

1. Preliminaries. Let X be a metrizable compactum. Denote by $\exp X$ the space of non-void closed subsets of X with Hausdorff's metric [4]. A point $\mathcal{A} \in \exp^2 X$ is called an *inclusion hyperspace* in X if the conditions $A \in \mathcal{A}$ and $A \subset B \in \exp X$ imply $B \in \mathcal{A}$. Denote by GX the set of all inclusion hyperspaces in X endowed with the inherited from $\exp^2 X$ topology.

For a mapping $f: X \to Y$ the mapping $Gf: GX \to GY$ is defined by the formula

$$Gf(\mathcal{A}) = \{B \in \exp X \mid B \supset f(A) \text{ for some } A \in \mathcal{A}\}, \qquad \mathcal{A} \in GX.$$

If Y is a closed subset of X, then GY is naturally identified with the subset of GX by means of the map $Gi: GY \to GX$, where $i: Y \to X$ is the identity embedding. Recall that the *support* $\operatorname{supp} \mathcal{A}$ of a point $\mathcal{A} \in GX$ is defined to be the smallest closed subset $Y \subset X$ such that $\mathcal{A} \in GY$. It is known that the set of inclusion hyperspaces with finite supports is dense in GX [8].

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The *transversality mapping* acts by the formula

$$\perp(\mathcal{A}) = \{B \in \exp X \mid B \cap A \neq \varnothing \text{ for all } A \in \mathcal{A}\}, \qquad \mathcal{A} \in GX.$$

Mark the following properties of this mapping:
1) the mapping $\perp$ is an involution on GX;
2) for every closed subset $\mathcal{C}$ in GX the equality $\perp(\bigcup \mathcal{C}) = \bigcap \perp(\mathcal{C})$ holds.
The item 1) implies that the transversality mapping determines an action of group $\mathbb{Z}_2 = \{0, 1\}$ on GX. Denote by $(GX, \perp)$ the space GX with this action.

Recall the construction of the *superextension* λX of a compactum X [3]. An inclusion hyperspace $\mathcal{A} \in GX$ is said to be *linked system* if $A_1 \cap A_2 \neq \varnothing$ for all $A_1, A_2 \in \mathcal{A}$. The subspace $\lambda X \subset GX$ is formed by all maximal (with respect to inclusion) linked systems.

It was remarked by T. Radul that $\perp(\mathcal{A}) = \mathcal{A}$ iff $\mathcal{A} \in \lambda X$.

One can find more information about the functors G and λ in [2,11].

We shall refer to objects and morphisms of the category of metrizable compacta with an action of the group $\mathbb{Z}_2$ as $\mathbb{Z}_2$-*spaces* and $\mathbb{Z}_2$-*mappings* respectively.

For the group $\mathbb{Z}_2$ let $\mathbb{Q}$ be the product of unit balls of all non-reducible ortogonal real reprezentations of the group $\mathbb{Z}_2$ and, moreover, every factor is countably dublicated in the product. The space $\mathbb{Q}$ with the diagonal action of $\mathbb{Z}_2$ is called the *equivariant Hilbert cube* [10].

We shall prove the following result.

Theorem 1. *For every nondegenerate metrizable continuum X the $\mathbb{Z}_2$-space $(GX,\perp)$ is equimorphic to $\mathbb{Q}$.*

In particular, this theorem answers affirmatively the question of M. Zarichnyi (1989) whether the $\mathbb{Z}_2$-space $(GX/\lambda X, \perp)$ is $\mathbb{Z}_2$-contractible.

A $\mathbb{Z}_2$-space Y is said to be a $\mathbb{Z}_2$-*absolute extensor* ($Y \in \mathbb{Z}_2$-AE) if every $\mathbb{Z}_2$-mapping $f: A \to Y$ of an invariant closed subset A of a $\mathbb{Z}_2$-space B extends to a $\mathbb{Z}_2$-mapping $\bar{f}: B \to Y$. A $\mathbb{Z}_2$-space Y satisfies the $\mathbb{Z}_2$-*disjoint approximation property* ($\mathbb{Z}_2$-DAP) if for any $\varepsilon > 0$ there exist $\mathbb{Z}_2$-maps $f_1, f_2: Y \to Y$ with $f_1(Y) \cap f_2(Y) = \varnothing$ which are ε-close to the identity $1: Y \to Y$.

The following result is a partial case of the Ageev characterization of equimorphic Hilbert cubes [1].

Characterization theorem. *A $\mathbb{Z}_2$-space Z is equimorphic to $\mathbb{Q}$ iff*
1) $(GX, \perp) \in \mathbb{Z}_2$-AE;
2) $(GX, \perp)$ has $\mathbb{Z}_2$-DAP;
3) there exist $x, y \in Z$ with $1 \cdot x = x$, $1 \cdot y \neq y$;
4) any $\mathbb{Z}_2$-map $\mathbb{Z}_2 \to Z$ is approximated with $\mathbb{Z}_2$-embeddings $\mathbb{Z}_2 \to Z$.

Remark that for $Z = (GX, \perp)$ conditions 3) and 4) are satisfied trivially.

2. Construction of an "averaging" operator. Let X be a nondegenerate metrizable continuum. Since GX is homeomorphic to the Hilbert cube, Toruńczyk's characterization of Q [9] implies that GX is an absolute extensor and has the disjoint approximation property. Therefore, it is natural to search for every $\mathbb{Z}_2$-compactum Y a continuous operator $\Delta: C(Y, GX) \to \mathbb{Z}_2\text{-}C(Y, GX)$, which transforms every continuous mapping $f: Y \to GX$ onto a $\mathbb{Z}_2$-mapping $\Delta f: Y \to (GX, \perp)$ and has the property $\Delta f \equiv f$ on the domain of equivariantness of f.

In this section we shall construct such an operator.

For this, we need several lemmas. Let Y be a $\mathbb{Z}_2$-compactum.

Since the transversality operator is a natural transformation of the functor G, we obtain the following result at once.

Lemma 1. *For every $f\colon X \to Y$ the map $Gf\colon GX \to GY$ is a $\mathbb{Z}_2$-mapping.* $\square$

For two compacta X and Y the *tensor product mapping* $\otimes\colon GX\times GY \to G(X\times Y)$ is defined as follows. Given $\mathcal{A} \in GX$, $\mathcal{B} \in GY$ we set

$$\mathcal{A} \otimes \mathcal{B} = \Big\{C \in \exp(X \times Y) \mid C \supset \bigcup_{x\in A} \{x\} \times B_x, \text{ where } A \in \mathcal{A}, B_x \in \mathcal{B}\Big\}.$$

According to [12], the map $\otimes$ is continuous.

Lemma 2. *The equality $\perp(\mathcal{A}\otimes\mathcal{B}) = \perp\mathcal{A}\otimes\perp\mathcal{B}$ holds for every $\mathcal{A} \in GX$, $\mathcal{B} \in GY$.*

Proof. It is sufficient to prove this equality if $\mathcal{A}$ and $\mathcal{B}$ have finite supports.
($\supset$) Let $C \in \perp\mathcal{A} \otimes \perp\mathcal{B}$; $A \in \mathcal{A}$ and $\{B_x \mid x \in A\} \subset \mathcal{B}$ be arbitrary. Then C contains $\bigcup_{x'\in M}\{x'\} \times N_{x'}$ for some $M \in \perp\mathcal{A}$, $\{N_{x'} \mid x' \in M\} \subset \perp\mathcal{B}$. Let $x_0 \in M \cap A$ and $y_0 \in N_{x_0} \cap B_{x_0}$. We obtain that $(x_0, y_0) \in C \cap \big(\bigcup_{x\in A}\{x\} \times B_x\big)$ and thus $C \in \perp(\mathcal{A}\otimes\mathcal{B})$.
($\subset$) Consider $C \in \perp(\mathcal{A} \otimes \mathcal{B})$. Let $\widetilde{M} = \mathrm{pr}_X(C)$, where $\mathrm{pr}_X\colon X \times Y \to X$ is the projection onto the factor X. Clearly, $\widetilde{M} \in \perp\mathcal{A}$. Denote by N_x the section $C \cap (\{x\} \times Y)$ of C by the "vertical" axe $\{x\} \times Y$, $x \in \widetilde{M}$. Set $M = \{x \in \widetilde{M} \mid N_x \in \perp\mathcal{B}\}$. Then M is closed in X. Indeed, suppose that $x \in \overline{M}\backslash M \subset \widetilde{M}\backslash M$ and $N_x \cap B = \varnothing$ for some $B \in \mathcal{B}$. There exists a subset $D \subset C$ with $\overline{\mathrm{pr}_X D} \ni x$ and $\mathrm{pr}_Y D \subset B$ (here pr_Y is the projection onto the factor Y). Then $\overline{D} \subset C$ and $\mathrm{pr}_Y \overline{D} \subset B$, $\mathrm{pr}_X \overline{D} \ni x$. Hence $N_x \cap B \neq \varnothing$. Contradiction.

Remark that $M \neq \varnothing$. Indeed, otherwise we may find a finite $A \in \mathcal{A}$ and a family $\{B_x \in \mathcal{B} \mid x \in \widetilde{M} \cap A\}$ with $B_x \cap N_x = \varnothing$. Then the closed set

$$T = \Big(\bigcup_{x\in A\cap\widetilde{M}} \{x\} \times B_x\Big) \cup \Big(\bigcup_{x\in A\cap(X\backslash\widetilde{M})} \{x\} \times Y\Big)$$

belongs to $\mathcal{A} \otimes \mathcal{B}$ but $C \cap T = \varnothing$. Contradiction.

We have also $M \in \perp\mathcal{A}$. Otherwise, there exists a finite $A \in \mathcal{A}$ with $M \cap A = \varnothing$. Taking for this A a family $\{B_x \in \mathcal{B} \mid x \in \widetilde{M} \cap A\}$ and the set T as above, we obtain a contradiction again.

Finally, $C \supset \bigcup_{x\in M}\{x\} \times N_x$ and thus $C \in \perp\mathcal{A} \otimes \perp\mathcal{B}$. $\square$

Recall the construction of the *multiplication mapping* $\mu\colon G^2X \to GX$ [7]. It is defined by the formula $\mu(\mathcal{A}) = \bigcup\{\bigcap A \mid A \in \mathcal{A}\}$, $\mathcal{A} \in G^2X$. There is a simple example which shows that this mapping is not equivariant. But the following fact holds.

Lemma 3. *The restriction μ_* of μ on $G\lambda X$ is a $\mathbb{Z}_2$-mapping, i.e., $\mu_*(\perp\mathcal{A}) = \perp\mu_*(\mathcal{A})$ for every $\mathcal{A} \in G\lambda X$.*

Proof. It is sufficient to prove this equality for $\mathcal{A} \in G\lambda X$ with $|\mathrm{supp}(\mathcal{A})| < \infty$. Consider the hyperspaces $\mathcal{B}_1, \mathcal{B}_2 \in GX$,

$$\mathcal{B}_1 = \mu_*(\perp\mathcal{A}) = \bigcup\Big\{\bigcap B \mid B \in \exp\lambda X,\ B \cap A \neq \varnothing \text{ for all } A \in \mathcal{A}\Big\},$$

$$\mathcal{B}_2 = \perp\mu_*(\mathcal{A}) = \{C \in \exp X \mid C \cap M \neq \varnothing \text{ for any } A \in \mathcal{A} \text{ and } M \in \bigcap A\}.$$

At first prove the inclusion $\mathcal{B}_1 \subset \mathcal{B}_2$. Let $C \in \mathcal{B}_1$. Take $B \in \perp\mathcal{A}$ with $C \in \bigcap B$. Let $A \in \mathcal{A}$ and $M \in \bigcap A$ be arbitrary. Then for some $\xi \in B \cap A$ we have $M \in \xi$ and $C \in \xi$. Thus $M \cap C \neq \varnothing$ and $C \in \mathcal{B}_2$.

Now obtain the inclusion $\mathcal{B}_2 \subset \mathcal{B}_1$. Let $C \in \mathcal{B}_2$. For every $A \in \mathcal{A}$ there exists $\xi_A \in A$ which contains C. Indeed, $C \in \perp(\bigcap A) = \bigcup(\perp(A)) = \bigcup A$, because $A \subset \lambda X$. Consider the closed in λX set $B = \{\xi_A \mid A \in \mathcal{A}, A \subset \operatorname{supp} \mathcal{A}\}$ and obtain that $C \in \bigcap B$ and $B \in \perp\mathcal{A}$. Thus $C \in \mathcal{B}_1$. □

Consider the continuous mapping $t: X^3 \to \lambda X$,

$$t(x,y,z) = \{A \in \exp X \mid |A \cap \{x,y,z\}| \geq 2\}, \qquad x,y,z \in X.$$

Clearly, t is commutative. Let $x \in X$. Denote by t_x the mapping $X \times X \to \lambda X$, $t_x(y,z) = t(x,y,z)$. Let $\Delta: X \times C(X,Y) \to C(X,Y)$ act by the formula

$$\Delta(x,f)(y) = \mu_* \circ Gt_x \circ \Big(f(y) \otimes \perp f(1 \cdot y)\Big), \qquad f: Y \to GX, y \in Y. \tag{1}$$

Lemmas 1–3 and the commutativity of t_x yield the equivariantness of the mapping $\Delta(x,f): Y \to GX$.

Let the mapping $j: X \times G(X \times X) \to G(X \times X \times X)$ is defined by the formula $j(x,a) = Gi_x(a)$ (here $i_x(y) = (x,y)$, $y \in X \times X$), $x \in X$, $a \in G(X \times X)$. It is known that j is continuous [2]. Since the mappings μ_* and $\otimes$ are continuous and $Gt_x(a) = Gt \circ j(x,a)$, the operator Δ is also continuous.

Remark that for every $x \in X$ we have $\Delta(x,f)(y) = f(y)$ if $f(1 \cdot y) = \perp f(y)$, $y \in Y$. For, we have to show the equality $\mu_* \circ Gt_x(\mathcal{A} \otimes \mathcal{A}) = \mathcal{A}$, $x \in X$, $\mathcal{A} \in GX$. Denote by $\mathcal{B}$ the left part of this equality and prove that $\mathcal{A} \subset \mathcal{B}$. Indeed, let $A \in \mathcal{A}$. Then $A \times A \in \mathcal{A} \times \mathcal{A}$ and $A \in \bigcap t_x(A \times A)$. Hence, $A \in \mathcal{B}$.

On the other hand, let $B \in \mathcal{B}$. Then $B \in \bigcap t_x(M)$ for some $M \in \mathcal{A} \otimes \mathcal{A}$. There exists $A \in \mathcal{A}$, $\{A_a \mid a \in A\} \subset \mathcal{A}$ with $M \supset \bigcup_{a \in A}\{a\} \times A_a$. If $A \subset B$, then $B \in \mathcal{A}$. If $A \backslash B$ contains some $a \in X$, then $A_a \cup \{x\} \subset B$. So always $B \in \mathcal{A}$ and therefore $\mathcal{B} \subset \mathcal{A}$.

Hence, we proved the following

Proposition 1. *Let Y be a $\mathbb{Z}_2$-compactum. Then the equality (1) determines the continuous operator $\Delta: X \times C(Y, GX) \to \mathbb{Z}_2\text{-}C\big(Y, (GX, \perp)\big)$, such that for every $x \in X$ we have $\Delta(x,f)(y) = f(y)$ if $f(1 \cdot y) = \perp f(y)$, $y \in Y$.*

In particular, we have also

Corollary 1. *The $\mathbb{Z}_2$-space $(GX, \perp)$ is a $\mathbb{Z}_2$-absolute extensor.* □

3. $\mathbb{Z}_2$-disjoint approximation property for $(GX, \perp)$. We shall prove the following proposition and obtain Theorem 1 as a consequence.

Proposition 2. *The $\mathbb{Z}_2$-space $(GX, \perp)$ has $\mathbb{Z}_2$-DAP.*

Proof. Fix points $x_1, x_2 \in X$, $x_1 \neq x_2$. Let $\varepsilon > 0$ be arbitrary and $S \subset X \backslash \{x_1\}$ a finite ε-net with $x_2 \in S$. Denote by $O_\delta S$ the open δ-neighbourhood of S. Because of connectness of X, for sufficiently small $\delta > 0$ the set $T = X \backslash O_\delta S$ is a 2ε-net in X.

Let $\mathcal{R}, \mathcal{P} \in GX$ be defined by the formulae

$$\mathcal{R} = \{A \in \exp X \mid A \supset T\}, \quad \mathcal{P} = \{A \in \exp X \mid A \cap S \neq \varnothing\}.$$

Consider the continuous mappings $f_1, f_2: GX \to GX$,

$$f_1(\mathcal{A}) = \mathcal{A} \cup \mathcal{R}, \quad f_2(\mathcal{A}) = \mathcal{A} \cap \mathcal{P}, \qquad \mathcal{A} \in GX.$$

Remark that these mappings are 2ε-close to the identity $1\colon GX \to GX$. Indeed, since $\mathcal{A} \subset f_1(\mathcal{A})$ and $f_2(\mathcal{A}) \subset \mathcal{A}$ for every $\mathcal{A} \in GX$, it is sufficient to prove for $\mathcal{A} \in GX$ that

a) the 2ε-neighbourhood in $\exp X$ of $\mathcal{A}$ contains $\mathcal{A} \cup \mathcal{R}$ and

b) the 2ε-neighbourhood in $\exp X$ of $\mathcal{A} \cap \mathcal{P}$ contains $\mathcal{A}$.

For a) remark that the 2ε-neighbourhood of $X \in \mathcal{A}$ contains $\mathcal{A} \cup \mathcal{R}$, because T is 2ε-net in X and is contained in every element of $\mathcal{A} \cup \mathcal{R}$.

For b) let $A \in \mathcal{A}$. Choose $s \in S$ such that the ε-neighbourhood of s intersects A. Then the element $A \cup \{s\} \in \mathcal{A} \cap \mathcal{P}$ and is contained in the ε-neighbourhood of A.

Let $g_i = \Delta(x_i, f_i) = \mu_* \circ Gt_{x_i}(f_i \otimes \perp f_i \perp)$, $i = 1, 2$. The $\mathbb{Z}_2$-mappings $g_1, g_2\colon GX \to GX$ can be chosen arbitrarily close to $1\colon GX \to GX$, because Δ is continuous.

We state that $g_1(GX) \cap g_2(GX) = \varnothing$. Indeed, let $\mathcal{A}, \mathcal{B} \in GX$. Since $T \in f_1(\mathcal{A})$, we have $T \cup \{x_1\} \in g_1(\mathcal{A})$. Consider any $B \in g_2(\mathcal{B})$. It belongs to $\bigcap t_{x_2}\big(\bigcup_{x \in F}\{x\} \times F_x\big)$ for some $F \in f_2(\mathcal{B})$, $F_x \in \perp f_2 \perp(\mathcal{B})$. There exists $s \in F \cap S$, and therefore $s \in B$ or $x_2 \in B$. So $B \cap S \neq \varnothing$ and thus $T \cup \{x_1\} \notin g_2(\mathcal{B})$. □

4. The equivariant softness of μ_*.

Definition 1. Let H be a compact group. A H-mapping $f\colon X \to Y$ is said to be *H-soft* if for any compact H-space B, any closed invariant subspace $A \subset B$ and any H-mappings $\varphi\colon A \to X$ and $\psi\colon B \to Y$ with $\psi|A = f \circ \varphi$ there exists a H-mapping $\Psi\colon B \to X$ such that $\Psi|A = \varphi$ and $f \circ \Psi = \psi$.

Recall that the space X is said to be *openly generated* if it is the limit of a σ-spectrum of metrizable compacta with open projections. M. Zarichnyi in [12] proved that $\mu\colon G^2X \to GX$ is H-soft for an openly generated H-continuum X (on GX we consider the extended action of H). For the mapping μ_* and the transversality operator we now obtain a counterpart of this result.

Theorem 2. *Let X be an openly generated continuum. Then the mapping $\mu_*\colon(G\lambda X, \perp) \to (GX, \perp)$ is $\mathbb{Z}_2$-soft.*

Proof. Consider a commutative diagram

$$\begin{array}{ccc} A & \xrightarrow{\ \varphi\ } & G\lambda X \\ \cap\!\downarrow & & \downarrow \mu_* \\ B & \xrightarrow{\ \psi\ } & GX, \end{array}$$

where A is a closed invariant subspace of a $\mathbb{Z}_2$-compactum B, φ and ψ are equivariant.

Remark that the results of section 2 hold also for nonmetrizable X. In particular, for a continuum X we have $(G\lambda X, \perp)$ is $\mathbb{Z}_2$-AE. Let $\Phi\colon B \to G\lambda X$ be an equivariant extension of the mapping φ on B.

We shall think analogously to the arguments of M. Zarichnyi [12].

Recall some definitions. Let $\mathcal{A}, \mathcal{B} \in GX$, $\mathcal{A} \subset \perp\mathcal{B}$. Sets of the form

$$\mathcal{H}(\mathcal{A}, \mathcal{B}) = \{\mathcal{C} \in GX \mid \mathcal{A} \subset \mathcal{C} \subset \perp\mathcal{B}\} \in \exp GX$$

will be called *convex* in GX. Denote by KGX the subspace of $\exp X$ consisting of all nonempty convex subsets of GX. It is known that the "nearest point" mapping $\xi_G\colon GX \times KGX \to GX$, $\xi_G(\mathcal{C}, \mathcal{H}(\mathcal{A}, \mathcal{B})) = \mathcal{A} \cup \big(\mathcal{C} \cap \perp\mathcal{B}\big)$, $\mathcal{A}, \mathcal{B}, \mathcal{C} \in GX$, is continuous, and $\xi_G(\mathcal{C},\mathcal{H}(\mathcal{A},\mathcal{B})) \in \mathcal{H}(\mathcal{A},\mathcal{B})$ [12]. This mapping is equivariant in the following sense $\xi_G\big(\perp\mathcal{C},\perp(\mathcal{H}(\mathcal{A},\mathcal{B}))\big) = \perp\xi_G(\mathcal{C},\mathcal{H}(\mathcal{A},\mathcal{B}))$. Indeed,

$$\xi_G\big(\perp\mathcal{C},\perp(\mathcal{H}(\mathcal{A},\mathcal{B}))\big) = \xi_G(\perp\mathcal{C},\mathcal{H}(\mathcal{B},\mathcal{A})) = \mathcal{B}\cup(\perp\mathcal{C}\cap\perp\mathcal{A}) =$$
$$(\perp\mathcal{A}\cap\perp\mathcal{C})\cup(\perp\mathcal{A}\cap\mathcal{B}) = \perp\mathcal{A}\cap(\perp\mathcal{C}\cup\mathcal{B}) = \perp(\mathcal{A}\cup(\mathcal{C}\cap\perp\mathcal{B})) = \perp\xi_G(\mathcal{C},\mathcal{H}(\mathcal{A},\mathcal{B})).$$

Remark also that
a) the set $G\lambda X = \mathcal{H}(\{\lambda X\},\perp(\exp^2\lambda X))$ is convex;
b) the intersection of convex sets is convex:

$$\mathcal{H}(\mathcal{A},\mathcal{B})\cap\mathcal{H}(\mathcal{A}',\mathcal{B}') = \mathcal{H}(\mathcal{A}\cap\mathcal{A}',\mathcal{B}\cup\mathcal{B}').$$

Since fibers of the mapping $\mu\colon G^2X\to GX$ are convex [12], the items a) and b) imply that fibres of μ_* are also convex.

Since X is openly generated, we have μ is open [12] and thus $\mu^{-1}\colon GX\to\exp G^2X$ is continuous. The mapping $\cap\colon KG\times KG\to KG$ is continuous and $\mu_*^{-1}(\mathcal{A}) = \mu^{-1}(\mathcal{A})\cap G\lambda X$, $\mathcal{A}\in GX$. Therefore the mapping $\mu_*^{-1}\colon GX\to\exp G\lambda X$ is also continuous.

Finally, the mapping $\Psi\colon B\to G\lambda X$, $\Psi(b) = \xi_G\big(\Phi(b),\mu_*^{-1}(\psi(b))\big)$, is as required in Definition 1. □

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