Математичні Студії. Т.7, №1 Matematychni Studii. V.7, No.1

УДК 517.95

D-ADAPTIVE MATHEMATICAL MODEL OF SOLID BODY WITH THIN COATING

I. Dyyak, Ya. Savula

Dedicated to Professor V. Lyantse
on his 75th birthday

I. Dyyak, Ya. Savula. *D-adaptive mathematical model of solid body with thin coating*, Matematychni Studii, **7**(1997) 103–110.

The variation problem of theory of elasticity is formulated for the combined model solid body – Timoshenko plate. The existence of solution is proved.

The problem of construction and justification of mathematical models of elastic bodies with thin coatings have been considered by many authors [1,2,5,11–13]. In papers [1,12,13] an asymptotic approach was used for derivation of principal correlations of thin coating models. The authors of papers [2,11] modelled and investigated the thin coating with special generalized boundary conditions. Such approach is convenient in further investigation of the problem by analytic methods, but entails significant algorithmic complications in application of numerical methods. In this paper, like in [5], the math model of an elastic body with a thin coating is formed by combining the equations of the elasticity theory and Timoshenko's plates theory. Such models are called combined [7,9] or D-adaptive [14] models. They are convenient in methods of numerical computer analysis, such as FEM[8], BEM or their coupling [3], and construction of quality software.

158.07086pt

1. Statement of the problem. Assume that an elastic continuum occupies the domain $\Omega = \Omega_1 \cup \Omega_2^*$, $\Omega_1 \cap \Omega_2^* = \varnothing$, (fig. 1), where Ω_1, Ω_2^*, are arbitrary connected sets of the Euclidean space R^3. We shall assume that the domain Ω_1 is bounded by a Lipschitz boundary Γ_1, and the domain Ω_2^*, is bounded by two parallel planes Ω_2^-, Ω_2^+ and a cylindrical side surface orthogonal

1991 *Mathematics Subject Classification.* 65N12, 65N30, 73C35.

This work was supported, in part, by the International Soros Science Education Program (ISSEP) through grant APU051042.

to these planes. The distance between the planes is h. (Here and in the rest of the paper the terms “plane” and “surface” mean simple parts of planes and surfaces.)

Let us assume that the size h of the domain Ω_2^*, is much smaller than the other characteristic dimensions of Ω_2^* in the sense of [4]. The middle plane Ω_2 is situated between the parallel planes Ω_2^-, Ω_2^+ and it is equidistant from them. The cylindrical side surface intersects the planes Ω_2^-, Ω_2^+ and Ω_2 along the lines $\Gamma_2^-, \Gamma_2^+, \Gamma_2$, i.e. $\Gamma_2^-, \Gamma_2^+, \Gamma_2$, are the boundaries of the plane domains Ω_2^-, Ω_2^+ and Ω_2. Let

$$\Omega_2^- = \Omega_2^{-(1)} \cup \Omega_2^{-(2)},$$

($\Omega_2^{-(1)}$ is a part of the surface Ω_2^- for which the intersection with the boundary Γ_1 is non empty), and

$$\Gamma_1 = \Gamma_1^{(1)} \cup \Gamma_1^{(2)} \cup \Gamma_1^{(3)}, \qquad \Gamma_1^{(1)} \cap \Gamma_1^{(2)} \cap \Gamma_1^{(3)} = \varnothing,$$

where $\Gamma_1^{(1)}$, $\Gamma_1^{(2)}$ are piecewisely smooth surfaces; $\Gamma_1^{(3)}$ is the plane, which coincides with $\Omega_2^{-(1)}$. Assume that

$$\Gamma_2^- = \Gamma_2^{-(1)} \cup \Gamma_2^{-(2)} \quad \text{and} \quad \Gamma_2^{-(1)} \cap \Gamma_2^{-(2)} = \varnothing,$$

($\Gamma_2^{-(1)}$ is a part of the curve for which the intersection with the boundary Γ is non empty).

We note that for given $\Omega_2^{-(2)} = \varnothing$, we have $\Gamma_2^{-(2)} = \varnothing$. We note that the curves Γ_2^-, Γ_2^+ and Γ_2 are congruent and so are the domains Ω_2^-, Ω_2^+ and Ω_2. Thus the curve Γ_2 consists of two parts $\Gamma_2^{(1)}$ and $\Gamma_2^{(2)}$. Let x_1, x_2, x_3 be the Cartesian coordinates system of the elastic body, $\overline{n}_1, \overline{n}_2, \overline{n}_3$ an orthogonal right triple of unit-length vectors defined on Γ_1 , where $\overline{n}_3$ is the outer normal to Γ_1.

Let $\alpha_1, \alpha_2, \alpha_3$ be the Cartesian coordinate system on the middle surface (the direction of α_3 coincides with the normal to the middle plane and normal $\overline{n}_3$ to the boundary of the body $\Gamma_1^{(3)}$). Let $\bar{t}_1, \bar{t}_2$ be a pair of orthogonal unit vectors on Γ_2, where $\bar{t}_1$ is the outer normal to the boundary, $\bar{t}_2$ the tangent vector corresponding to the positive direction Ω_2 along the curve Γ_2.

Let us describe the stress-strain state of the elastic body which occupies the domain Ω_1 by the differential equations of the linear elasticity theory [10] (summing over repeating indices)

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0, \quad i, j = 1, 2, 3 \quad \Omega_1 \subset R^3 \tag{1.1}$$

and the equations of Timoshenko's plate theory [3] for the domain Ω_2^*

$$\begin{aligned} \frac{\partial T_{kl}}{\partial \alpha_l} + p_k &= 0, \\ \frac{\partial Q_k}{\partial \alpha_k} + p_3 &= 0, \\ \frac{\partial M_{kl}}{\partial \alpha_l} - Q_k + m_k &= 0, \qquad k, l = 1, 2, \qquad \Omega_2^* \subset R^2. \end{aligned} \tag{1.2}$$

Here σ_{ij} are the components of the stress tensor; f_i the components of the vector of body forces ; T_{kl} the forces, M_{kl} the moments, Q_k the transverse forces arising in the plate; $p_i; m_k$ the surface loads reduced to the middle plane of the plate.

It is known [4] that

$$p_i = \sigma_{i3}^{+} + \sigma_{i3}^{-} + \rho_i, \qquad i = 1,2,3;$$
$$m_k = \frac{h}{2}(\sigma_{k3}^{+} - \sigma_{k3}^{-}) + \mu_k, \qquad k = 1,2; \tag{1.3}$$

where σ_{i3}^{+} , σ_{i3}^{-} are the components of the vector of surface loading on the surfaces of the plate Ω^{+} and Ω^{-} for $\alpha_3 = \pm\frac{h}{2}$ respectively in the α_i coordinates;
ρ_i, μ_k the components of the reduces to the middle surface of the plate body forces and moments which are evaluated by formulas

$$\rho_i = \int\limits_{-h/2}^{h/2} q_i \quad d\alpha_3, \quad \mu_k = \int\limits_{-h/2}^{h/2} q_k \ \alpha_3 \quad d\alpha_3; \tag{1.4}$$

q_i the components of the body force vector in the coordinate system α_i on the middle surface of the plate.

In linear elasticity theory the components of the stress tensor are expressed in terms of the components of the strain tensor e_{kl} by the physical low

$$\sigma_{ij} = C_{ijkl}\ e_{kl}, \quad i,j,k,l = 1,2,3; \tag{1.5}$$

where C_{ijkl} are elastic constants; the nonzeros among them for the case of isotropic homogeneous body are

$$C_{iiii} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}, \qquad C_{iikk} = \frac{E\nu}{(1+\nu)(1-2\nu)},$$
$$C_{ikik} = \frac{E}{2(1+\nu)} \qquad i,k = 1,2,3; \quad i \neq k; \tag{1.6}$$

Here E is the Young modulus, ν Poisson ratio of the elastic body.

The forces and moments acting in plate are also expressed in terms of deformation characteristics ε_{ij} and κ_{ij} by means of the relations of the physical law of Timoshenko's plate theory

$$T_{kk} = B(\varepsilon_{kk} + \nu\varepsilon_{ll}), \quad T_{kl} = B\frac{1-\nu}{2}\varepsilon_{kl}, \quad Q_k = G\varepsilon_{k3},$$
$$M_{kk} = D(\kappa_{kk} + \nu\kappa_{ll}), \quad M_{kl} = D\frac{1-\nu}{2}\kappa_{kl}, \qquad k,l = 1,2; k \neq l. \tag{1.7}$$

Note that we do not sum over k,l in (1.7). Here constants B, D, G (in the case of isotropic material) are given by

$$B = \frac{Eh}{1-\nu^2}, \quad D = \frac{Eh^3}{12\,(1-\nu^2)}, \quad G = \frac{5Eh(1+\nu)}{12}. \tag{1.8}$$

Let u_i $(i = 1, 2, 3)$ be the components of the displacement vector of the elastic body in the x_1, x_2, x_3 coordinate system; v_i displacements of the points of the middle surface in the direction of axes α_i $(i = 1, 2, 3)$, and γ_l $(l = 1, 2)$ rotation angles of the normal to the middle surface in the direction of axes α_l.

The following Cauchy relations hold

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad i, j = 1, 2, 3, \tag{1.9}$$

$$\begin{gathered} \varepsilon_{kl} = \frac{1}{2}\left(\frac{\partial v_k}{\partial \alpha_l} + \frac{\partial v_l}{\partial \alpha_k}\right), \quad \varepsilon_{k3} = \frac{\partial v_3}{\partial \alpha_k} + \gamma_k, \\ \kappa_{kl} = \frac{1}{2}\left(\frac{\partial \gamma_k}{\partial \alpha_l} + \frac{\partial \gamma_l}{\partial \alpha_k}\right), \qquad k, l = 1, 2. \end{gathered} \tag{1.10}$$

To equations (1.1), (1.2) we add boundary conditions and conditions of continuity of the medium and equilibrium of the body and the plate. Boundary conditions on the boundary of Ω_1 are:

$$u_{ni} = 0, \quad i = 1, 2, 3 \text{ on } \Gamma_1^{(1)}; \tag{1.11}$$

$$\sigma_{ni,n3} = p, \quad i = 1, 2, 3 \text{ on } \Gamma_1^{(2)}; \tag{1.12}$$

where

$$u_{ni} = u_k n_{ik}, \quad \sigma_{ni,n3} = \sigma_{kl}\, n_{ik}\, n_{3l}, \quad k, l = 1, 2, 3.$$

$n_{ij} = \cos(n_i, x_j)$ are the direction cosines of the triple $\overline{n}_j$. Boundary conditions on the boundary of Ω_2 are:

$$v_{tk} = 0, \quad v_3 = 0, \quad \gamma_{tk} = 0, \qquad k = 1, 2 \text{ on } \Gamma_2^{(1)}, \tag{1.13}$$

$$T_{tk,tl} = 0, \quad Q_{ti} = 0, \quad M_{tk,tl} = 0, \qquad i = 1, k = 1, 2 \text{ on } \Gamma_2^{(2)}. \tag{1.14}$$

Here

$$\begin{gathered} v_{tk} = v_l t_{kl}, \quad \gamma_{tk} = \gamma_l t_{kl}, \qquad l = 1, 2 \\ T_{tk,tl} = T_{ij}\, t_{ki}\, t_{lj}, \quad Q_{tk} = Q_l t_{il}, \qquad M_{tk,tl} = M_{ij}\, t_{ki}\, t_{lj}, \quad i, j = 1, 2, \end{gathered}$$

where t_{kl} are the direction cosines of the vectors $\bar{t}_k$ in the coordinate system α_l : $t_{kl} = \cos(t_k, \alpha_l)$.

Conditions of continuity of the mediums and equilibrium of the body and the plate on $\Gamma_1^{(3)} = \Omega_2^{-(1)}$ are

$$u_{ni} = v_i - \frac{h}{2}\gamma_i, \quad u_3 = v_3, \qquad i = 1, 2; \tag{1.15}$$

$$\sigma_{j3}^{-} = \sigma_{nj,n3}(u_1, u_2, u_3) \quad \text{ on } \Gamma_1^3 = \Omega_2^{-(1)}. \tag{1.16}$$

Thus, the D-adaptive mathematical model of the elastic body and Timoshenko's plate in terms of displacements and rotation angles that describes the stress-strain state of the system a massive body and thin coating is made up of the equilibrium equations (1.1), (1.2) written in terms of unknowns u_i, v_i, $i = 1,2,3$ and γ_k, $k = 1,2$ by means of formulas (1.5), (1.7), (1.9), (1.10). These equations form a closed system by means of relations (1.15), (1.16) and boundary conditions (1.11)–(1.14).

2. Variational statement of the problem. Let us write the BVP described by equations (1.1) and (1.2) using operator notation as

$$\mathbf{A}\ Z = f, \qquad f \in H, \tag{2.1}$$

where

$$\mathbf{H} = [L_2(\Omega_1)]^3 \times [L_2(\Omega_2)]^5,$$
$$\mathbf{Z} = (u_1, u_2, u_3, v_1, v_2, v_3, \gamma_1, \gamma_2),$$
$$\mathbf{f} = (f_1, f_2, f_3, \sigma_{13}^{+} + \rho_1, \sigma_{23}^{+} + \rho_2, \sigma_{33}^{+} + \rho_3, \frac{h}{2}\sigma_{13}^{+} + \mu_1, \frac{h}{2}\sigma_{23}^{+} + \mu_2).$$

The operator of problem (2.1) is defined on

$$\mathbf{D}_A = \{u_i, v_i, \gamma_k : i = 1,2,3; k = 1,2;\ u_i \in [W_2^{(2)}(\Omega_1)]^3;$$
$$v_i \in [W_2^{(2)}(\Omega_2)]^3;\ \gamma_k \in [W_2^{(2)}(\Omega_2)]^2\};\ \text{conditions (1.11)–(1.15).}\};$$

Let us define a scalar product of the vector functions over the lineal $\mathbf{D}_A$ as

$$(u, \widehat{u}) = \int\limits_{\Omega_1} u_i \widehat{u}_i\ d\Omega_1\ +\ \int\limits_{\Omega_2} (u_i \widehat{u}_i + \gamma_k \widehat{\gamma}_k)\ d\Omega_2.$$

The following lemma holds.

Lemma. *The operator of problem (2.1) is symmetrical in the space* $\mathbf{H}$.

Proof. Note that since $C_0^{(\infty)} \subset \mathbf{D}_A$, the lineal D_A is a dense set in the space $\mathbf{H}$ [6]. Consider the bilinear form $(\mathbf{A}\ Z\ , \widehat{Z})$, where Z, $\widehat{Z}$ are are arbitrary elements of D_A. Applying the Ostrogradsky formula and counting the boundary conditions (1.11)–(1.14) we will obtain after simple but unwieldy transformations

$$(\mathbf{A}\ Z,\ \widehat{Z}) = \int\limits_{\Omega_1} \sigma_{ij}(u_1, u_2, u_3)\ e_{ij}(\widehat{u}_1, \widehat{u}_2, \widehat{u}_3)\ d\Omega_1\ +$$
$$+ \int\limits_{\Omega_2} (T_{kl}(v_1, v_2)\ \varepsilon_{kl}(\widehat{v}_1, \widehat{v}_2) + Q_k(v_3, \gamma_1, \gamma_2)\ \varepsilon_{k3}(\widehat{v}_3, \widehat{\gamma}_1, \widehat{\gamma}_2) +$$
$$+\ M_{kl}(\gamma_1, \gamma_2)\ \kappa_{kl}(\widehat{\gamma}_1, \widehat{\gamma}_2))\ d\Omega_2 - \int\limits_{\Gamma_1^{(3)}} \sigma_{ni,n3}\ \widehat{u}_{ni}\ d\Gamma +$$
$$+ \int\limits_{\Omega_2^{-\ (1)}} (\sigma_{i3}^{-}\ \widehat{v}_i - \frac{h}{2}\sigma_{k3}^{-}\ \widehat{\gamma}_k)\ d\Gamma, \qquad i,j = 1,2,3;\ k,l = 1,2.$$

Note that the sum of the last two integrals in the previous formula is equal to zero because they are evaluated on the other sides of the plane $\Gamma_1^{(3)} = \Omega_2^{-(1)}$, according to (1.15), (1.16).

Let us express stresses, forces, and moments in terms of strains in the previous formula using (1.5), (1.7). We will obtain

$$(\mathbf{A}\ Z,\ \widehat{Z}) = \int\limits_{\Omega_1} C_{ijmn}\ e_{mn}\ \widehat{e}_{ij}\ d\Omega_1\ +\ \int\limits_{\Omega_2} (B(\varepsilon_{kk} + \nu\varepsilon_{ll})\ \widehat{\varepsilon}_{kk} +$$

$$+B\frac{1-\nu}{2}\varepsilon_{kl}\widehat{\varepsilon}_{kl} + +G\varepsilon_{k3}\widehat{\varepsilon}_{k3} + D(\kappa_{kk} + \nu\kappa_{ll})\widehat{\kappa}_{kk} + D\frac{1-\nu}{2}\kappa_{kl}\widehat{\kappa}_{kl})\ d\Omega_2, \quad (2.2)$$

where $i, j, m, n = 1, 2, 3$; $k, l = 1, 2$; $k \neq l$; $\widehat{e}_{ij}$, $\widehat{\varepsilon}_{kl}$, $\widehat{\varepsilon}_{k3}$, $\widehat{\kappa}_{kk}$ are deformations calculated on the based of functions from $\widehat{\mathbf{Z}}$.

Taking into account the symmetry of the matrix C_{ijmn} it is clear that the expression (2.2) is symmetrical with respect to functions $Z, \widehat{Z}$. Hence, $(\mathbf{A}Z, \widehat{Z}) = (Z, A\widehat{Z})$.■

Theorem 1. *The operator of problem (2.1) is positive.*

Proof. In view of the lemma it is sufficient to show that

$$(\mathbf{A}\ Z,\ Z) \geq 0; \quad (2.3)$$

$$\text{and, if } (\mathbf{A}\ Z,\ Z) = 0, \text{ then } \quad Z \equiv 0.$$

Using (2.2) we write the expression for $(\mathbf{A}\ Z,\ Z)$. It is known in the theory of elasticity that the integrand of the first integral is a positively definite quadratic form of the strain tensor components (it can be verified by direct calculations in the case of an isotropic material). The integrand of the second integral in the formula for $(\mathbf{A}\ Z,\ Z)$ is also a positively definite quadratic form of the plate deformations characteristics (in can be verified by simple calculations).

Thus we can write the inequality

$$(\mathbf{A}\ Z,\ Z) \geq 0.$$

Next, if $(\mathbf{A}\ Z,\ Z) = 0$, then each summand must necessarily be zero, because $(\mathbf{A}\ Z,\ Z)$ is a sum of positive definite quadratic forms of the deformations of the body and plate. Besides, they will equal zero too. Then the functions $u_i, v_i, i = 1, 2, 3; \gamma_k, k = 1, 2$ are constants. But taking into account the boundary conditions (1.11) and (1.13) we obtain $Z = 0$.■

Uniqueness of the weak solution of problem (2.1) and possibility to give its variational statement as the problem of minimization of quadratic functional follow from the proved theorem.

To write this functional let us introduce the spaces

$$U = \{u_1, u_2, u_3 :\ u_i \in \mathbf{W}_2^{(1)}(\Omega_1),\ u_i = 0,\ \text{on} \quad \Gamma_1^{(1)}\};$$

$$V = \{v_1, v_2, v_3, \gamma_1, \gamma_2 : v_i,\ \gamma_k \in \mathbf{W}_2^{(1)}(\Omega_2),\ v_i = 0, \gamma_k = 0 \text{ on} \quad \Gamma_2^{(1)}\};$$

$$Y = \{U \times V : u_{ni} = v_i - \frac{h}{2}\gamma_i, \quad u_3 = v_3, \quad i = 1, 2 \text{ on} \quad \Gamma_1^{(3)}\}.$$

According to the energy functional theorem [5] the problem (2.1) is equivalent to functional minimization problem

$$\mathbf{F}(\ Z) = \int\limits_{\Omega_1} \sigma_{ij}\ e_{ij}\ d\Omega_1\ + \int\limits_{\Omega_2} (T_{kl}\ \varepsilon_{kl} + Q_k\ \varepsilon_{k3} +\ M_{kl}\ \kappa_{kl})\ d\Omega_2 -$$
$$-2\int\limits_{\Omega_1} f_i u_i\ d\Omega_1 -\ 2\int\limits_{\Omega_2} (p_i v_i + m_k \gamma_k)\ d\Omega_2, \qquad Z \in Y, \tag{2.4}$$

where $i, j = 1, 2, 3;\ k, l = 1, 2;\ p_i, m_k$ are given by formulas (1.3), where we set $\sigma_{i3}^{-} = 0$. Note, that conditions (1.12), (1.14), and (1.16) are natural for the given functional.

<u>3. Existence of solution of problem (2.1).</u> It is well known [6] that the weak solution of problem (2.1) exists provided $\mathbf{A}$ is a positively definite operator. Thus, the following theorem answers the question of existence of the weak solution of problem (2.1).

Theorem 2. *The operator of problem (2.1) is positively definite.*

Proof. In order to estimate from below bilinear form (2.2) we will use positive definiteness of the bilinear form of displacement functions corresponding to the first integral [6], positive definiteness of the bilinear form of the deformations characteristics of the plate, Korn's inequality for both deformations characteristics $\widehat{\varepsilon}_{kl},\ \widehat{\kappa}_{kk}$ $k, l = 1, 2$, and the inequalities

$$2v_3'\gamma_k \geq -(\lambda v_3'^2 + \frac{1}{\lambda}\gamma_k^2), \quad \forall\lambda > 0,$$

and also Friedrichs' inequality [6]. We will obtain

$$(\mathbf{A}\ Z,\ Z)\ \geq C_1^2\ \|\ Z\|^2, \tag{3.2}$$

where $C_1 \in R, C_1 > 0$,

$$\|\ Z\|^2 = \int\limits_{\Omega_1} u_i u_i\ d\Omega_1 + \int\limits_{\Omega_2} (v_i v_i + \gamma_k\gamma_k)\ d\Omega_2, \quad i = 1,2,3,\ k = 1,2.$$

The last inequality means that the operator of problem (2.1) is positively definite.■

Thus, problem (2.1) has a unique solution $Z \in Y$.

REFERENCES

1. Гольденвейзер А.Л. *Общая теория упругих тел (оболочки, покрытия, прокладки)* // Изв. РАН Мех. твер.тела, 1992, №3, С.5–17.

2. Лянце В.Е., Сторож О.Г. *Про одну некласичну крайову задачу теорії пластин* // ДАН УРСР, Сер. А, 1989, №2, С.15–17.

3. Паук Н.М. *Застосування об'їднаного гранично – скінченно елементного аналізу для чисельного розв'язування двовимірної задачі теорії пружності.* // Вісник ЛДУ, сер. мех-мат., вип.39, 1993, С.55–60.

4. Пелех Б.Л. Обобщенная теория оболочек. – Львов: Вища школа, 1978, 159с.

5. Пелех Б.Л., Максимук А.В., Коровайчук. Контактные задачи для сложных элементов конструкций и тел с покрытиями. – К.: Наукова думка, 1988.

6. Ректорис К. Вариационные методы в математической физике и технике. – М.: Наука, 1985–590с.

7. Савула Я.Г., Дыяк И.И., Дубовик А.В. *Применение комбинированной модели для расчета напряженно-деформированного состояния пространственных конструкций* // Прикладная механика, Т.25, №9, 1989, С.62–65.

8. Савула Я.Г., Флейшман Н.П. Расчет и оптимизация оболочек с резными срединными поверхностями. – Львов: Вища школа, 1989. С.172.

9 Савула Я.Г. *Краевые и вариационные задачи для одной комбинированной математической модели теории упругости. Математ.методы и физ.-мех.поля* // 1990, вып. 32, С.92–95.

10. Тимошенко С.П. Курс теории упругости. – Киев: Наукова думка, 1972, 583с.

11. Флейшман Н.П. *Нелінійна модель спряження деформівного середовища з тонким прошарком*// Вісник ЛДУ, сер. мех-мат., вип.35, 1991, С.42–47.

12. CiarletP.G. Plates and Junctions in Elastic Multi-Structures. – Paris, 1990. 216p.

13. Ciarlet Ph.G., Je Dret H., Nzengwa R. *Junctions between treee-dimensional and two-dimensional linearly elastic structures* // J. Math. pures et appl, 1989, №3, P.261–295.

14. E. Stein, S. Ohnimus. *Concept and realization of integrated adaptive finite element methods in solid and structural-mechanics* // Num. Meth. in Eng.'92 (Ed. Ch. Hirsch) 1992, P.163–170.

Department of Applied Math. & Informatics, L'viv University,
Universitetska 1, 290602, Ukraine.

Received 12.07.94.