Математичні студії
випуск 5

PROBLEM SECTION

1. It is easy to construct a function f analytic in the unit disk $\mathbb{D}$ with no zeroes which is such that $\rho[f] > \lambda[f]$. We recall that

$$\rho[f] = \varlimsup_{r\to 1} \frac{\log T(r,f)}{\log \frac{1}{1-r}}, \quad \lambda[f] = \varliminf_{r\to 1} \frac{\log T(r,f)}{\log \frac{1}{1-r}}.$$

For instance, we can take the composition $F \circ \mu$ of a linear-fractional map $\mu : \mathbb{D} \to \{w : \operatorname{Re} w > 0\}$ with F where F is a canonical Weierstrass product having negative zeroes and the counting function $n(t) = t^{l(t)}$, $t \to \infty$. Here an oscilate order l [1, p.91] is such that

$$\varliminf_{t\to\infty} l(r) = \lambda + 1, \ \varlimsup_{t\to\infty} l(r) = \rho + 1.$$

The problem is to describe functions of regular growth in $\mathbb{D}$ (i.e. $\rho[f] = \lambda[f]$) without zeroes. An integral representation given by M.M. Dzhrbashyan in [2] may be useful for this purpose.

1. Гольдберг А.А., Островский И.В. Распределение значений мероморфных функций. – М.:Наука, 1970. 592с.
2. Джрбашян М.М. Интегральные преобразования и представления функций в комплексной области. – М.:Наука, 1966, 671с.

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2. Let the counting function of a subharmonic function be of the form

$$n(t) = \int_{\lambda}^{\rho} t^s \, d\Delta(s) + \varphi(t).$$

Here real numbers $\rho, \lambda, \varkappa$ satisfy the inequality $\rho > \lambda \geq \varkappa > [\rho]$, a proximate order $\varkappa \to \varkappa$ as $t \to \infty$, a function φ satisfies the condition (a real number $q \geq 1$)

$$\int_{T}^{2T} |\varphi(t)|^q \, dt = o(T^{\varkappa(T)q+1}), \, T \to \infty,$$

a real-valued function Δ has the at most countable set of discontinuities , and $\operatorname{supp}\Delta \cap \{\rho\} \neq \varnothing$. Is it possible the inequality $\rho[n] < \rho$?

In the case $d\Delta(s) = \cos \frac{1}{\rho - s} \, ds$ I know $\rho[n] = \rho$.

This problem is of interest for the so called integral asymptotic representation of a subharmonic function.

M. Ghirnyk