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Ihor Guran (in honour of the 70th anniversary of birth)

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Some pages of biography and main scientific achievements of Ihor Guran, a famous Ukrainian mathematician, are presented. This is a modified and expanded version of the anniversary text published in *Math. Bulletin of Shevchenko Scientific Society*, **12** (2015).



February 23, 2025 marked the 70th anniversary of the birth of the famous Lviv mathematician Ihor Yosypovych Huran, founder of the modern Lviv topological school, member of the Shevchenko Scientific Society, member of the Editorial Boards of the journals “*Matematychni Studii*” and “*In the World of Mathematics*”, and Dean of the Faculty of Mechanics and Mathematics of Ivan Franko National University of Lviv.

Ihor Guran was born in the Galician city of Sambir. During 1962–1972 he studied at secondary school No. 1 of this city. In 1972, he entered the Faculty of Mechanics and Mathematics of the Ivan Franko Lviv State University. In 1975, the faculty was divided into two faculties: the Faculty of Mathematics and the Faculty of Applied Mathematics and Mechanics. Student Ihor Guran was enrolled in the Applied Mathematics

Department, but in 1976 (on the advice of his colleague Yaroslav Mykytyuk) he was transferred to the Faculty of Mathematics. In 1977 he defended his Master thesis “*Tensor Products of Linear Topological Spaces*” under the supervision of Professor Vladyslav Lyantse.

In the autumn of the same year, Ihor Guran entered the postgraduate course of the Faculty of Mechanics and Mathematics of the Moscow State University. At that time, that Faculty was one of the largest mathematical centers in the world. I. Guran actively worked under the guidance of the Professor Alexander Arhangel’skii.

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In 1981, I. Guran successfully defended his PhD thesis “Topological groups and properties of their spaces” and returned to Lviv as Assistant Professor at the Department of Geometry of Lviv University. In 1982, the Department of Geometry was renamed the Department of Algebra and Topology, and Guran created a topology circle, which eventually grew to the city seminar “Topology and Applications”. Around this seminar, the modern Lviv topological school emerged, which enjoys well-deserved authority in the world. In particular, in 2003, a special issue of the journal “Topology and its Applications” was published, dedicated to the works of the Lviv topological seminar; also, one of the sections of the famous mathematical problem collection “Open Problems in Topology, II”, published by the transnational publishing house Elsevier in 2007, was titled “Problems from the Lviv topological seminar”.

Ihor Guran is the author of important results in topological algebra. Already in his postgraduate publication, he introduced the class of ω -bounded topological groups and obtained a characterization of such groups as subgroups of products of metrizable separable groups. A topological group is called ω -bounded if it is covered by a countable number of shifts of an arbitrary neighborhood of the unit. In a certain sense, this concept is an algebraic-topological analogue of the topological concept of a Lindelöf space, but it has much better hereditary properties: unlike Lindelöf space, ω -boundedness is preserved by subgroups and arbitrary Tikhonov products. The concept of a ω -bounded topological group very aptly combines the topological and algebraic properties of a topological group, reflecting the intertwining of two structures, from which a new quality arises. This concept and the results associated with it have been included in standard textbooks and monographs on the theory of topological groups (in particular, in the recent monograph by Arhangel'skii and Tkachenko “Topological Groups and Related Structures”, published in 2009; in the book by S. Todorčević “Topics in topology”, published in 1997 as volume 1652 of the Springer series “Lecture notes in mathematics”, the term “Guran group” is used). The corresponding article by Guran [1] is well known and widely cited in scientific publications by Ukrainian and foreign scientists; according to various scientometric databases (in particular, Google Scholar), it is one of the most cited modern publications of the Faculty of Mechanics and Mathematics at the Lviv University.

Among other publications by I. Guran, one should mention the article [4], which contains a solution to the Arhangel'skii problem on the existence of a non-metrizable minimal topological group of countable pseudocharacter. In this article, Guran constructs an example of such a group as a subgroup of the wreath product $A_5 \wr H$ of the alternating group A_5 and a certain discrete group H . In a certain sense, this result is developed in the joint article with T. Banach and I. Protasov [9], where it is proved the strict minimality of an arbitrary topological group G which is contained in the group of permutations $S(X)$ of an infinite set X and contains a subgroup $S_\omega(X)$ of permutations with finite supports. This result solves a number of problems of D. Dikranjan and D. Shakhmatov and is a simultaneous generalization of Gauguin's theorem of 1967 (on the strict minimality of the permutation group $S(X)$) and the Dierolf-Schwanengel theorem of 1977 (on the minimality of topological groups G with $S_\omega(X) \subset G \subset S(X)$). In the same paper [9] the non-discrete topologizability of the quotient group $S(X)/S_\omega(X)$ is proved, which solves another problem of D. Dikranjan and A. Giordano-Bruno.

His joint publication with B. Bokalo [3] is also well-known. There was proved that sequentially compact topological semigroups with reduction are topological groups, which provides an answer to the problem of Robby and Svetlichny in 1996. In the joint article [8] with T. Banach and O. Ravsky, an interesting characterization of thin paratopological groups

was obtained, namely, it was proved that a Hausdorff paratopological group is thin (i.e., it is a space of the first Baire category) if and only if it is covered by a countable number of shifts of some nowhere dense set.

In the joint paper [2], M. Zarichnyi and I. Guran solved M. Hirsch's problem on the normality of spaces of smooth mappings in the strong Whitney topology. A development of results on the normality of functional spaces is published in [6].

Ihor Guran also conducts research on the history of mathematics. These are reflected in the articles: [7] on János Bolyai's stay in Lviv, [11] on the Lviv period of Stanisław Ulam's scientific work, [10] on the scientific legacy of Józef Schreier, one of the founders of topological algebra, who belonged to the famous Lviv mathematical school of the interwar period, and [5] on the American mathematician Alexander Wallace, one of the creators of the theory of topological semigroups.



M.E. Rudin I. Guran M. Zarichnyi
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With his broad erudition and subtle intuition, Ihor Guran managed to predict (and, in a sense, to shape) the directions of development of topological algebra for the future. In particular, when in the 90s I. Guran proposed studying topological semigroups and paratopological groups (at a time when everyone was focusing only on topological groups), few among the then mathematical luminaries expected that this particular direction would become a trend two decades later. This strategic vision of Ihor Guran and the ability to foresee events, looking beyond the horizon, became the key to the fact that the Lviv school of topological algebra (which, in addition to I. Guran, is represented by his students and colleagues

T. Banakh, B. Bokalo, O. Gutik, M. Zarichnyi, N. Pyrch, O. Ravskiy) retains priority in a number of areas of modern topological algebra.

I. Guran has two defended students, Oleg Gutik (PhD thesis “Immersion of topological inverse semigroups and the structure of their connections with constraints on shifts”, defended in 1997) and Oleksandr Ravskiy (“Topological-algebraic properties of paratopological groups”, 2004). After defending their Ph.D. theses, both of his students continue to actively engage in topological algebra.

The scientific work of I. Guran consists of about forty scientific articles, textbooks and scientific and methodological manuals. His near-mathematical activity is also very significant. Guran is an active organizer of schoolchildren and students’ mathematical Olympiads, starting from the district or faculty level, and ending with the All-Ukrainian tour. He is also the head of advanced training courses for mathematics teachers at the Institute of Postgraduate Education.

Ihor Guran is a multitalented person. He loves music, is well versed in it, and from time to time returns to violin, his favorite instrument. He is also interested in literature.

Ihor Guran is characterized by optimism in life, friendliness towards people, and a willingness to help them.

The entire creative life of Ihor Guran is connected with the Faculty of Mechanics and Mathematics of the Ivan Franko National University of Lviv. Here he began his mathematical education, here he returned after defending his candidate’s thesis, worked as an assistant, and then as an associate professor. Today he is the Dean of the Faculty of Mechanics and Mathematics. Under the leadership of Ihor Guran, the faculty survived the pandemic with all its negative consequences. In the last war years, together with the entire academic community of the university, the faculty provides a high level of education and scientific research.

We wish Ihor Guran good health, strength, and inspiration for his further scientific, educational, and administrative activities, innovative mathematical ideas, and fruitful scientific achievements!

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