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P. V. Filevych

ASYMPTOTIC ESTIMATES FOR ENTIRE FUNCTIONS OF MINIMAL GROWTH WITH GIVEN ZEROS

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Let $\zeta=(\zeta_n)$ be an arbitrary complex sequence such that $0<|\zeta_1|\le|\zeta_2|\le\ldots$ and $\zeta_n\to\infty$ as $n\to\infty$, let $n_\zeta(r)$ and $N_\zeta(r)$ be the counting function and the integrated counting function of this sequence, respectively. By $\mathcal{E}_\zeta$ we denote the class of all entire functions whose zeros are precisely the ζ_n, where a complex number that occurs m times in the sequence ζ corresponds to a zero of multiplicity m. Suppose that Φ is a convex function on $\mathbb{R}$ such that $\Phi(\sigma)/\sigma\to+\infty$ as $\sigma\to+\infty$. It is proved that there exists an entire function $f\in\mathcal{E}_\zeta$ such that

$$\overline{\lim_{r\to+\infty}}\frac{\ln\ln M_f(r)}{\Phi(\ln r)}\le\overline{\lim_{r\to+\infty}}\frac{\ln n_\zeta(r)}{\Phi(\ln r)},$$

where $M_f(r)$ denotes the maximum modulus of the function f, and it is shown that the above inequality implies the inequality

$$\overline{\lim_{r\to+\infty}}\frac{\ln\ln M_f(r)}{\Phi(\ln r)}\le\overline{\lim_{r\to+\infty}}\frac{\ln N_\zeta(r)}{\Phi(\ln r)}+\overline{\lim_{\sigma\to+\infty}}\frac{\ln\Phi'_+(\sigma)}{\Phi(\sigma)}.$$

The formulated result is a consequence of the following more general statement: if the right-hand derivative Φ'_+ of the function Φ assumes only integer values and $\sum_{n=1}^{\infty}e^{-\Phi(\ln|\zeta_n|)}<+\infty$, then there exists an entire function $f\in\mathcal{E}_\zeta$ such that $\ln M_f(r)=o(e^{\Phi(\ln r)})$ as $r\to+\infty$.

1. Introduction and main results. We denote by H the class of all non-decreasing functions h on $\mathbb{R}$ such that $h(\sigma)\to+\infty$ as $\sigma\to+\infty$, and let Ω be the class of all non-decreasing convex functions Φ on $\mathbb{R}$ such that $\Phi(\sigma)/\sigma\to+\infty$ as $\sigma\to+\infty$.

Suppose that $\mathcal{Z}$ is the class of all complex sequences $\zeta=(\zeta_n)$ such that $0<|\zeta_1|\le|\zeta_2|\le\ldots$ and $\zeta_n\to\infty$ as $n\to\infty$. For any sequence $\zeta=(\zeta_n)$ from the class $\mathcal{Z}$ by $\mathcal{E}_\zeta$ we denote the class of all entire functions whose zeros are precisely the ζ_n, where a complex number that occurs m times in the sequence ζ corresponds to a zero of multiplicity m. Let $n_\zeta(r)$ and $N_\zeta(r)$ be the counting function and the integrated counting function of the sequence ζ, respectively, that is

$$n_\zeta(r)=\sum_{|\zeta_n|\le r}1,\quad N_\zeta(r)=\int_0^r\frac{n_\zeta(t)}{t}dt,\quad r\ge0.$$

The common value of the orders of the functions $n_\zeta(r)$ and $N_\zeta(r)$ we denote by a_ζ.

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For a non-constant entire function f and each $r \geq 0$, let $M_f(r) = \max\{|f(z)| : |z| = r\}$ be the maximum modulus of the function f. We identify the growth of the function f with the growth of the function $\ln M_f(r)$. By ρ_f we denote the order of the function f.

Well-known classical results about the minimal growth of entire functions from the classes $\mathcal{E}_\zeta$, where $\zeta \in \mathcal{Z}$, are the Borel theorem on the order of a Weierstrass canonical product (see, for example, [1, p. 57]) and the Lindelöf theorem on the description of the zero sets of entire functions whose growth does not exceed normal type of a given order $\rho > 0$ (see, for example, [2, p. 30]). In particular, the Borel theorem implies the following statement.

Теорема A. *Let $\zeta \in \mathcal{Z}$. Then there exists an entire function $f \in \mathcal{E}_\zeta$ such that $\rho_f = a_\zeta$.*

Suppose that $\lambda \in H$ is a continuous function on $\mathbb{R}$. Generalizing the Lindelöf theorem, L.A. Rubel and B.A. Taylor [3] found necessary and sufficient conditions on a sequence $\zeta \in \mathcal{Z}$, under which there exist an entire function $f \in \mathcal{E}_\zeta$ and positive constants A and B such that $\ln M_f(r) \leq A\lambda(Br)$ for all $r > 0$.

Let $\zeta \in \mathcal{Z}$. A.A. Gol'dberg [4] proposed another approach to describing the minimal growth of entire functions from the class $\mathcal{E}_\zeta$. The essence of this approach is to establish the best possible estimates from above on the growth of functions $f \in \mathcal{E}_\zeta$ by $n_\zeta(r)$ or $N_\zeta(r)$. This approach is used in many works. In particular, estimates from above for $\ln M_f(r)$ by $n_\zeta(r)$, which describe the minimal growth of entire functions $f \in \mathcal{E}_\zeta$ and hold along some increasing to $+\infty$ sequences of values of r, were established in [5, 6, 7, 8, 9]. Similar estimates from above for $\ln M_f(r)$ by $N_\zeta(r)$ are found in [10, 11]. Best possible, in a certain sense, estimates from above on the growth of functions $f \in \mathcal{E}_\zeta$ by $n_\zeta(r)$ or $N_\zeta(r)$, that hold outside small exceptional sets of values of r, were obtained in [1, 6, 10, 12, 13]. M.M. Sheremeta [14] found upper estimates for $\ln M_f(r)$, where $f \in \mathcal{E}_\zeta$, which hold for all sufficiently large values of r (see Theorems B and C below).

For any function $\Phi \in H$, a non-constant entire function f, and a sequence $\zeta \in \mathcal{Z}$, we put

$$\rho_{\Phi,f} = \varlimsup_{r\to+\infty} \frac{\ln\ln M_f(r)}{\Phi(\ln r)}, \quad a_{\Phi,\zeta} = \varlimsup_{r\to+\infty} \frac{\ln n_\zeta(r)}{\Phi(\ln r)}, \quad A_{\Phi,\zeta} = \varlimsup_{r\to+\infty} \frac{\ln N_\zeta(r)}{\Phi(\ln r)}.$$

The main results of this article are the following two theorems, the first of which follows from the second and is an analogue of Theorem A for $\rho_{\Phi,f}$ and $a_{\Phi,\zeta}$ instead of ρ_f and a_ζ, respectively.

Theorem 1. *Let $\zeta \in \mathcal{Z}$ and $\Phi \in \Omega$. Then there exists an entire function $f \in \mathcal{E}_\zeta$ such that $\rho_{\Phi,f} \leq a_{\Phi,\zeta}$.*

Theorem 2. *Let $\zeta = (\zeta_n)$ be a sequence from the class $\mathcal{Z}$ and let $\Psi \in \Omega$ be a function such that Ψ'_+ assumes only integer values. If*

$$\sum_{n=1}^{\infty} \frac{1}{e^{\Psi(\ln|\zeta_n|)}} < +\infty, \tag{1}$$

then there exists an entire function $f \in \mathcal{E}_\zeta$ such that

$$\ln M_f(r) = o(e^{\Psi(\ln r)}), \quad r \to +\infty. \tag{2}$$

2. Proof of main results. For each $z \in \mathbb{C}$ and an arbitrary integer $p \geq 0$, we denote by $E(z,p)$ the Weierstrass primary factor, i.e.

$$E(z,p) = \begin{cases} 1-z, & \text{if } p=0; \\ (1-z)\exp\left(\sum\limits_{n=1}^{p} \dfrac{z^n}{n}\right), & \text{if } p \geq 1. \end{cases}$$

The following statement is well known (see, for example, [4]).

Lemma 1. *Let $\zeta = (\zeta_n)$ be a sequence from the class $\mathcal{Z}$ and let (p_n) be a sequence of non-negative integers. If the series*

$$\sum_{n=1}^{\infty} \left(\frac{r}{|\zeta_n|}\right)^{p_n+1} \tag{3}$$

converges for every $r > 0$, then the product

$$\prod_{n=1}^{\infty} E\left(\frac{z}{\zeta_n}, p_n\right) \tag{4}$$

converges absolutely and uniformly in any boundend subset of $\mathbb{C}$ to an entire function $f(z)$ such that $f \in \mathcal{E}_\zeta$ and $\ln M_f(r) \leq G(r)$ for all $r \geq 0$, where $G(r)$ is the sum of series (3).

Proof of Theorem 2. Let $\zeta = (\zeta_n)$ be a sequence from the class $\mathcal{Z}$ and let $\Psi \in \Omega$ be a function such that Ψ'_+ assumes only integer values and (1) holds. Note that $\Psi'_+ \in H$. We put $n_0 = \min\{n \in \mathbb{N} : \Psi'_+(\ln|\zeta_n|) \geq 1\}$. Let $p_n = \Psi'_+(\ln|\zeta_n|) - 1$ for all integers $n \geq n_0$ and let $p_n = 0$ if $n \in \mathbb{N}$ and $n < n_0$. Consider series (3). Since $\Psi(\sigma)/\sigma \to +\infty$ as $\sigma \to +\infty$, there exists a function $l \in H$ such that

$$\sum_{n \leq l(r)} \left(\frac{r}{|\zeta_n|}\right)^{p_n+1} = o(e^{\Psi(\ln r)}), \quad r \to +\infty. \tag{5}$$

In addition, for any $r > 0$ and every integer $n \geq 1$, we have

$$\Psi(\ln r) - \Psi(\ln|\zeta_n|) = \int_{|\zeta_n|}^{r} \frac{\Psi'_+(\ln t)}{t} dt \geq \Psi'_+(\ln|\zeta_n|)(\ln r - \ln|\zeta_n|).$$

Taking into account this fact, for any $r > 0$ and every integer $m \geq n_0$, we obtain

$$\sum_{n \geq m} \left(\frac{r}{|\zeta_n|}\right)^{p_n+1} = e^{\Psi(\ln r)} \sum_{n \geq m} \frac{1}{e^{\Psi(\ln r) - \Psi'_+(\ln|\zeta_n|)(\ln r - \ln|\zeta_n|)}} \leq e^{\Psi(\ln r)} \sum_{n \geq m} \frac{1}{e^{\Psi(\ln|\zeta_n|)}}. \tag{6}$$

From (6) and (1) we see that series (3) converges for every $r > 0$, and therefore by Lemma 1 product (4) converges absolutely and uniformly in any boundend subset of $\mathbb{C}$ to an entire function $f(z)$ such that $f \in \mathcal{E}_\zeta$ and $\ln M_f(r) \leq G(r)$ for all $r \geq 0$, where $G(r)$ is the sum of series (3). In addition, from (6) and (1) we obtain

$$\sum_{n > l(r)} \left(\frac{r}{|\zeta_n|}\right)^{p_n+1} = o(e^{\Psi(\ln r)}), \quad r \to +\infty. \tag{7}$$

It follows from (5) and (7) that $G(r) = o(e^{\Psi(\ln r)})$ as $r \to +\infty$, and hence (2) holds. $\square$

Proof of Theorem 1. Let $\zeta = (\zeta_n)$ be a sequence from the class $\mathcal{Z}$, $\Phi \in \Omega$ and $a = a_{\Phi,\zeta}$. We suppose that $a \in [0, +\infty)$, since otherwise there is nothing to prove. From the definition of the quantity $a_{\Phi,\zeta}$ it follows that there exist a function ε decreasing to a on $\mathbb{R}$ and a number $r_0 \geq |\zeta_0|$ such that

$$\ln(n_\zeta(r) \ln^2 n_\zeta(r)) \leq \varepsilon(\ln r)\Phi(\ln r), \quad r \geq r_0. \tag{8}$$

Without loss in generality we can assume that $\Phi'_+(\ln r_0) > 0$ and, in the case when $a = 0$, that for all $r \geq r_0$ the inequality $\varepsilon(\ln r) \geq 1/\sqrt{\Phi'_+(\ln r)}$ holds.

Let $\sigma_0 = \ln r_0$. We put $\gamma(x) = 0$ for all $x < \sigma_0$ and let

$$\gamma(x) = \sup\{\varepsilon(t)\Phi'_+(t) : t \in [\sigma_0, x]\}, \quad x \geq \sigma_0.$$

It is easy to show that $\gamma \in H$, $\gamma(x)/\Phi'_+(x) \to a$ as $x \to +\infty$, and $\gamma(x) \geq \varepsilon(x)\Phi'_+(x)$ for all $x \geq \sigma_0$. Therefore, taking $\Gamma(\sigma) = \int_{\sigma_0}^{\sigma} \gamma(x)dx + \varepsilon(\sigma_0)\Phi(\sigma_0)$ for all $\sigma \in \mathbb{R}$, we see that $\Gamma \in \Omega$, $\Gamma(\sigma)/\Phi(\sigma) \to a$ as $\sigma \to +\infty$, and

$$\Gamma(\sigma) \geq \varepsilon(\sigma) \int_{\sigma_0}^{\sigma} \Phi'_+(x)dx + \varepsilon(\sigma_0)\Phi(\sigma_0) \geq \varepsilon(\sigma)\Phi(\sigma), \quad \sigma \geq \sigma_0. \tag{9}$$

We put $\Psi(\sigma) = \int_{\sigma_0}^{\sigma}([\Gamma'_+(x)] + 1)dx + \Gamma(\sigma_0)$ for all $\sigma \in \mathbb{R}$. Then $\Psi \in \Omega$, Ψ'_+ assumes only integer values, $\Psi(\sigma) \sim \Gamma(\sigma)$ as $\sigma \to +\infty$, and $\Psi(\sigma) \geq \Gamma(\sigma)$ for all $\sigma \geq \sigma_0$. From (8) and (9) we have $\ln(n \ln^2 n) \leq \Psi(\ln |\zeta_n|)$ for all integers $n \geq n_0$, and therefore condition (1) holds. By Theorem 2, there exists an entire function $f \in \mathcal{E}_\zeta$ such that (2) is satisfied. Taking into account the above estimates and using (2), we obtain $\rho_{\Phi,f} \leq a$. □

3. Some consequences. Let $\zeta = (\zeta_n)$ be a sequence from the class $\mathcal{Z}$. We put $\Gamma_\zeta(\ln r) = 0$ for all $r \in (0, |\zeta_0|)$ and let

$$\Gamma_\zeta(\ln r) = \int_{|\zeta_0|}^{r} \frac{\ln n_\zeta(t)}{t} dt, \quad r \geq |\zeta_0|.$$

It is clear that $\Gamma_\zeta \in \Omega$.

We denote by Ω' the class of all continuously differentiable on $\mathbb{R}$ functions $\Phi \in \Omega$ such that Φ' is a positive function on $\mathbb{R}$. For each $\Phi \in \Omega'$ we put

$$\Phi_0(\sigma) = \sigma - \frac{\Phi(\sigma)}{\Phi'(\sigma)}, \quad \sigma \in \mathbb{R}.$$

Note that Φ_0 is a continuously function on $\mathbb{R}$ increasing to $+\infty$ (see, for example, [15]).

As we have already noted above, M.M. Sheremeta [14] proved the following two theorems.

Теорема B ([14])**.** *Let $\zeta \in \mathcal{Z}$. Then for every $q > 1$ there exists an entire function $f \in \mathcal{E}_\zeta$ such that*

$$\ln \ln M_f(r) = O(\Gamma_\zeta(\ln(qr))), \quad r \to +\infty.$$

Теорема C ([14])**.** *Let $\zeta = (\zeta_n)$ be a sequence from the class $\mathcal{Z}$, $\Phi \in \Omega'$, and $\delta \in (0, 1)$. If*

$$\ln n \leq \Phi'(\Phi_0^{-1}((\delta + o(1)) \ln |\zeta_n|)) \ln |\zeta_n|, \quad n \to \infty, \tag{10}$$

then there exists an entire function $f \in \mathcal{E}_\zeta$ such that $\rho_{\Phi,f} \leq 1/(1 - \delta)$.

In [14] it is noted that the number $q>1$ in Theorem B, generally speaking, cannot be replaced by 1. This fact also follows from the following result.

Теорема D ([12])**.** *Let $l\in H$ be a continuously function on $\mathbb{R}$. Then there exists a sequence $\zeta\in\mathcal{Z}$ such that $\ln n_\zeta(r)\ge l(\ln r)$ for all $r\ge r_0$, $\ln n_\zeta(r-0)=l(\ln r)$ on an unbounded from above set of values r, and for any entire function $f\in\mathcal{E}_\zeta$ we have*

$$l^{-1}(\ln n_\zeta(r))\ln n_\zeta(r)=o(\ln\ln M_f(r)),\quad r\in F_f,\ r\to+\infty,$$

where $F_f\subset[1,+\infty)$ is a set of infinite logarithmic measure.

We show that Theorems B and C follow from Theorem 1.

Let $\zeta=(\zeta_n)$ be a sequence from the class $\mathcal{Z}$ and $q>1$. Then for all $r\ge|\zeta_0|$ we have

$$\ln n_\zeta(r)\le\frac{1}{\ln q}\int_r^{qr}\frac{\ln n_\zeta(t)}{t}dt\le\frac{1}{\ln q}\Gamma_\zeta(\ln(qr)),$$

and therefore, by Theorem 1, the following more precise version of Theorem A is true.

Theorem 3. *Let $\zeta\in\mathcal{Z}$. Then for every $q>1$ there exists an entire function $f\in\mathcal{E}_\zeta$ such that*

$$\varlimsup_{r\to+\infty}\frac{\ln\ln M_f(r)}{\Gamma_\zeta(\ln(qr))}\le\frac{1}{\ln q}.$$

Let $\Phi\in\Omega'$, $\eta\in(0,1)$, and $p=1/\eta$. Then for all $\sigma\in\mathbb{R}$ we have

$$\Phi(p\Phi_0(\sigma))=\int_\sigma^{p\Phi_0(\sigma)}\Phi'(x)dx+\Phi(\sigma)\ge(p\Phi_0(\sigma)-\sigma)\Phi'(\sigma)+\Phi(\sigma)=(p-1)\Phi'(\sigma)\Phi_0(\sigma).$$

Making the substitution $y=p\Phi_0(\sigma)$ and noting that $\sigma=\Phi_0^{-1}(\eta y)$, for all $y\ge y_0$ we get $\Phi(y)\ge(p-1)\Phi'(\Phi_0^{-1}(\eta y))\eta y=(1-\eta)\Phi'(\Phi_0^{-1}(\eta y))y$. Now it is easy to see that condition (10) implies the condition $a_{\Phi,\zeta}\le1/(1-\delta)$, and therefore Theorem 1 implies Theorem C.

In view of Theorem 1, the following question arises: is it possible to replace the quantity $a_{\Phi,\zeta}$ in this theorem with the quantity $A_{\Phi,\zeta}$? The negative answer to this question follows from the following result.

Теорема E ([10])**.** *Let $h\in H$. Then there exists a sequence $\zeta\in\mathcal{Z}$ such that for every entire function $f\in\mathcal{E}_\zeta$ we have*

$$\varlimsup_{r\to+\infty}\frac{\ln\ln M_f(r)}{h(\ln N_\zeta(r))}=+\infty.$$

Suppose, for example, that $h(\sigma)=e^{2\sigma}$ for all $\sigma\in\mathbb{R}$. Let $\zeta\in\mathcal{Z}$ be a sequence whose existence for a given h is guaranteed by Theorem E. Put $\Phi(\ln r)=N_\zeta(r)$ for all $r>0$. Then $\Phi\in\Omega$, $A_{\Phi,\zeta}=0$, but $\rho_{\Phi,f}=+\infty$ for every entire function $f\in\mathcal{E}_\zeta$.

For any function $\Phi\in\Omega$ we put

$$\Delta_\Phi=\varlimsup_{\sigma\to+\infty}\frac{\ln\Phi'_+(\sigma)}{\Phi(\sigma)}.$$

The following result shows that in the case when $\Delta_\Phi=0$ the answer to the above question is positive, that is in this case the inequality $\rho_{\Phi,f}\le a_{\Phi,\zeta}$ in Theorem 1 can be replaced by the equality $\rho_{\Phi,f}=A_{\Phi,\zeta}$.

Theorem 4. *Let $\zeta \in \mathcal{Z}$ and $\Phi \in \Omega$. Then there exists an entire function $f \in \mathcal{E}_\zeta$ such that $\rho_{\Phi,f} \le A_{\Phi,\zeta} + \Delta_\Phi$.*

Theorem 4 is a direct consequence of Theorem 1 and the following lemma applied with $\Psi(\sigma) = N_\zeta(e^\sigma)$ for all $\sigma \in \mathbb{R}$.

Lemma 2 ([16]). *Let $\Psi, \Phi \in \Omega$. Then*

$$\varlimsup_{\sigma\to+\infty} \frac{\ln \Psi'_+(\sigma)}{\Phi(\sigma)} \le \varlimsup_{\sigma\to+\infty} \frac{\ln \Psi(\sigma)}{\Phi(\sigma)} + \Delta_\Phi.$$

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Lviv Polytechnic National University
Lviv, Ukraine
p.v.filevych@gmail.com
petro.v.filevych@lpnu.ua

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