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IDENTITIES ON ADDITIVE MAPPINGS IN SEMIPRIME RINGS

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Consider a ring R, which is semiprime and also having k-torsion freeness. If $F, d : R \to R$ are two additive maps fulfilling the algebraic identity

$$F(x^{n+m}) = F(x^m)x^n + x^m d(x^n)$$

for each x in R. Then F will be a generalized derivation having d as an associated derivation on R. On the other hand, in this article, it is also derived that f is a generalized left derivation having a linked left derivation δ on R if they satisfy the algebraic identity

$$f(x^{n+m}) = x^n f(x^m) + x^m \delta(x^n)$$

for each x in R and $k \in \{2, m, n, (n+m-1)!\}$ and at last an application on Banach algebra is presented.

1. Introduction. The ring R will be appraises as an associative with identity and $Z(R)$ noted as the center of this ring throughout. $Q_l(R_C)$ treated as left Martindale quotients ring and $\mathcal{C}$ will be treated as extended centroid. A ring R is called n-torsion free for an integer $n > 1$ and for each $x \in R$ if $nx = 0$ pointed out for each $x \in R$ that $x = 0$. $[x, y]$ denotes the commutator $xy - yx$. Recollect the definition of a ring R will be called as prime ring when the expression $aRb = \{0\}$ signify that either $a = 0$ or $b = 0$, and is called semiprime ring if the expression $aRa = \{0\}$ pointed out $a = 0$. A mapping $d : R \to R$ is said to be a derivation if d is additive and fulfill the condition

$$d(zy) = d(z)y + zd(y)$$

for every z, y in R and is called a Jordan derivation if for every w in R its fulfill the condition

$$d(w^2) = d(w)w + wd(w).$$

If d is a derivation, then it will be obviously a Jordan derivation, but generally the confer statement can not be consider true. A well known result due to Herstein [5], asserts that a Jordan derivation will be same as derivation for a ring, which is prime and holding characteristic is different from two. Cusack [4] revive the last statement of Herstein for a semiprime ring having 2-torsion freeness.

A mapping $F\colon R \to R$ which is additive and satisfying the expression

$$F(vy) = F(v)y + vd(y)$$

for all v, y in R is termed as a generalized derivation linked with a derivation d on R. Particularly, if $v = y$, then F is called a generalization of Jordan derivation, we say that if

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there exists a Jordan derivation d on R. It is easy to verify that all generalized derivation is generalized Jordan derivation but the converse statement generally does not hold good. If F is a generalized derivation (correspondingly generalized Jordan derivation) associated with a derivation (correspondingly Jordan derivation) d on R, then the algebraic identity

$$F(w^{2n}) = F(w^n)w^n + w^n d(w^n)$$

exist for each w inside R, but what about the converse? In [9], we have studied the converse of the this statement. Specifically, we planned under what condition on R, a generalized derivation F linked with a derivation d if it satisfies the algebraic identity

$$F(w^{2n}) = F(w^n)w^n + w^n d(w^n)$$

for all $w \in R$. Inspired by the afore said result, inside the current paper we generalize the above result by taking the case when F and d satisfying the algebraic expression

$$F(w^{n+m}) = F(w^m)w^n + w^m d(w^n)$$

for all w inside R.

Next, a mapping $\delta : R \to R$ is called a *left derivation* (respectively *Jordan left derivation*) if it is additive and satisfying

$$\delta(wu) = w\delta(u) + u\delta(w) \quad \big(\text{respectively} \;\; \delta(w^2) = 2w\delta(w)\big)$$

for all $w, u \in R$. A mapping $\delta : R \to R$, we say a *right derivation* (respectively *Jordan right derivation*) if δ is additive and fulfilling the expression

$$\delta(uy) = \delta(u)y + \delta(y)u \quad \big(\text{respectively} \;\; \delta(w^2) = 2\delta(w)w\big)$$

for each w, u, y belongs to R. If δ is both left as well as right derivation, then it is a *derivation*. Clearly, every left (respectively right) derivation on a ring R is a Jordan left (respectively Jordan right) derivation but the converse statement will not work generally (look Example 1.1 in [12]). Following [3], a mapping f from R to itself, which is additive will be called a *generalized left derivation* (correspondingly *generalized Jordan left derivation*) if for a corresponding Jordan left deviation δ from R to itself the statement

$$f(uy) = uf(y) + y\delta(u) \quad \big(\text{respectively} \;\; f(u^2) = uf(u) + u\delta(u)\big)$$

holds good for every u, y belongs to R. Think of a result by Zalar [13], an additive mapping $T\colon R \to R$ is said to be a *left centralizer* for every u, y inside R if $T(uy) = T(u)y$ holds good. We say T a *right centralizer* if $T(xy) = xT(y)$ is true for all x, y inside in R. Particularly, T is Jordan left and respectively Jordan right centralizer of R if $x = y$. It is from observation that f is a generalized left derivation on R if and only if $f = \delta + T$, where T is a right centralizer of R and δ a left derivation on R. The theory of generalized left derivations covering the theory of left derivations. On the other hand, if we take $\delta = 0$, a generalized left derivation covers the theory of right centralizer on R. One can easily see that $f_1 + f_2$ will be also a generalized left derivations, if f_1 and f_2 are generalized left derivations on R. For any fixed element a in R, every map $f(x) = xa + \delta(x)$ is a generalized left derivation, where δ is any left derivation on R. Now, if f is a generalized left derivation and δ is associated left derivation of f on R, then

$$f(x^n y^n) = x^n f(y^n) + y^n \delta(x^n)$$

holds for all $x, y \in R$. The confer statement of aforesaid statement is true with some restrictions on R (see [2]). In [9], we arrive at the same conclusion by taking a weaker condition. More precisely, it is prove that a generalized left derivation f and δ is an associated left derivation of f on R if

$$f(x^{2n}) = x^n f(x^n) + x^n \delta(x^n)$$

satisfies for all x inside in R with some restrictions on R. Inspired by above results, we prove here a more general case (see Theorem 4).

Let us start with Theorem 1.

2. Main Theorems.

Theorem 1. *Suppose that $m, n \geq 1$ are any two fixed integers and R is a semiprime ring having k-torsion freeness. If F and d are two additive mappings from R to itself which satisfy the algebraic equation*

$$F(x^{n+m}) = F(x^m)x^n + x^m d(x^n)$$

for every x in R, where $k \in \{2, m, n, (n+m-1)!\}$. Then we say F a generalized derivation linked with a derivation d on R.

Proof. We have given that

$$F(x^{n+m}) = F(x^m)x^n + x^m d(x^n) \text{ for all } x \in R. \tag{1}$$

Notice that $d(e) = 0$ and if we putting $x + ky$ for x in (1), we find

$$F\Big(x^{n+m} + \binom{n+m}{1}x^{n+m-1}ky + \binom{n+m}{2}x^{n+m-2}k^2y^2 + ... + k^{n+m}y^{n+m}\Big) =$$
$$= F\Big(x^m + \binom{m}{1}x^{m-1}ky + \binom{m}{2}x^{m-2}k^2y^2 + ... + k^m y^m\Big)\Big(x^n + \binom{n}{1}x^{n-1}ky +$$
$$+\binom{n}{2}x^{n-2}k^2y^2 + ... + k^n y^n\Big) + \Big(x^m + \binom{m}{1}x^{m-1}ky + \binom{m}{2}x^{m-2}k^2y^2 + ... + k^m y^m\Big)\times$$
$$\times d\Big(x^n + \binom{n}{1}x^{n-1}ky + \binom{n}{2}x^{n-2}k^2y^2 + ... + k^n y^n\Big).$$

Rewrite the above expression by using (1) as

$$kf_1(x,y) + k^2 f_2(x,y) + ... + k^{n+m-1}f_{n+m-1}(x,y) = 0,$$

where $f_i(x,y)$ stand for the coefficients of k^i's for each $i \in \{1, 2, ..., n+m-1\}$. If we replace k by $1, 2, ..., n+m-1$, then we find a system of $n+m-1$ homogeneous equations. It gives us a Vandermonde matrix

$$\begin{pmatrix} 1 & 1 & ... & 1 \\ 2 & 2^2 & ... & 2^{n+m-1} \\ ... & & & \\ ... & & & \\ n+m-1 & (n+m-1)^2 & ... & (n+m-1)^{n+m-1} \end{pmatrix}.$$

Which yields that $f_i(x,y) = 0$ for every x, y in R and for $i \in \{1, 2, .., n+m-1\}$. In particular, for all $x, y \in R$, we have the following

$$f_1(x,y) = \binom{n+m}{1}F(x^{n+m-1}y) - \binom{n}{1}F(x^m)x^{n-1}y - \binom{m}{1}F(x^{m-1}y)x^n -$$
$$-\binom{n}{1}x^m d(x^{n-1}y) - \binom{m}{1}x^{m-1}yd(x^n) = 0.$$

Let us put $x = e$ and making use of $d(e) = 0$ to appear

$$(n+m)F(y) = nF(e)y + mF(y) + nd(y).$$

The n-torsion freeness of R gives that

$$F(y) = F(e)y + d(y) \text{ for all } y \text{ in } R. \tag{2}$$

Next we observe

$$f_2(x,y) = \binom{n+m}{2}F(x^{n+m-2}y^2) - \binom{n}{2}F(x^m)x^{n-2}y^2 - \binom{m}{1}\binom{n}{1}F(x^{m-1}y)x^{n-1}y- \\ -\binom{m}{2}F(x^{m-2}y^2)x^n - \binom{n}{2}x^m d(x^{n-2}y^2) - \binom{m}{1}\binom{n}{1}x^{m-1}yd(x^{n-1}y)- \\ -\binom{m}{2}x^{m-2}y^2d(x^n) = 0$$

for all x, y in R. Rewrite the above expression by substituting e for x to obtain

$$\binom{n+m}{2}F(y^2) = \binom{n}{2}F(e)y^2 + \binom{m}{1}\binom{n}{1}F(y)y+ \\ +\binom{m}{2}F(y^2) + \binom{n}{2}d(y^2) + \binom{m}{1}\binom{n}{1}yd(y)$$

for all $y \in R$. This implies that

$$\frac{(n+m)(n+m-1)}{2}F(y^2) = \frac{n(n-1)}{2}F(e)y^2 + mnF(y)y + \frac{m(m-1)}{2}F(y^2)+ \\ +\frac{n(n-1)}{2}d(y^2) + mnyd(y).$$

A simple manipulation give us

$$n(2n+m-1)F(y^2) = (n^2-n)F(e)y^2 + 2mnF(y)y + (n^2-n)d(y^2) + 2mnyd(y).$$

Using torsion restriction on R, we find

$$(2n+m-1)F(y^2) = (n-1)F(e)y^2 + 2mF(y)y + (n-1)d(y^2) + 2myd(y).$$

An application of (2) yields that

$$(2n+m-1)\big[F(e)y^2 + d(y^2)\big] = (n-1)F(e)y^2 + 2m\big[F(e)y + d(y)\big]y+ \\ +(n-1)d(y^2) + 2myd(y).$$

On simplifying the above expression, we obtain

$$(2n+m-1-n+1-2m)F(e)y^2 + (3m-1-m+1)d(y^2) = 2md(y)y + 2myd(y).$$

This implicit that for all $y \in R$,

$$2md(y^2) = 2md(y)y + 2myd(y).$$

$2m$-torsion freeness of R allow us to write last expression as $d(y^2) = d(y)y + yd(y)$. That is nothing but the definition of Jordan derivation. As the ring R is semiprime having 2-torsion freeness, then use [4] to get that d is a derivation on R. Consider (2) once again, so that

$$F(y^2) = F(e)y^2 + d(y^2) = \big[F(e)y + d(y)\big]y + yd(y) = F(y)y + yd(y)$$

Hence F is generalized Jordan derivation on R with d, a associated derivation. We conclude the required result by theorem [14]. □

There are prompt consequences of the above theorem:

Theorem 2. *Suppose that $m, n \geq 1$ are any two fixed integers and R is a semiprime ring having k-torsion freeness. If F and d are two additive mappings from R to itself which satisfy the algebraic equation*

$$F(x^{n+m}) = F(x^m)x^n \quad \forall x \in R,$$

where $k \in \{2, m, n, (n+m-1)!\}$. Then, F will be a centralizer on R.

Proof. We wind up by substituting $D = 0$ in Theorem 1 and conclusion is straight forward. □

Theorem 3. *Let $n \geq 1$ and $m \geq 1$ be two fixed integers and R be a semiprime ring having k-torsion freeness, where $k \in \{2m, n, (n+m-1)!\}$. Suppose an additive mapping $d : R \to R$ which satisfies the identity*

$$d(x^{n+m}) = d(x^m)x^n + x^m d(x^n)$$

for every x in R, then d will be a derivation on R.

Proof. Considering d as F and using similar steps as we did in Theorem 1, we conclude the result. □

Corollary 1. *If $F : R \to R$ is an additive mapping which satisfies $F(x^{2n}) = F(x^n)x^n$ for each x in R, where R is any 2, n and $(2n-1)!$ torsion free semiprime ring and $n \geq 1$ be an integer that is fixed and arbitrary, then F will be a centralizer on R.*

Proof. Putting $m = n$ in Theorem 2, one can find the required conclusion. □

Corollary 2. *Let $n \geq 1$ be any fixed integer and R be a semiprime ring having 2, n and $(2n-1)!$ torsion freeness. Suppose an additive mapping $d : R \to R$ which satisfies the identity $d(x^{2n}) = d(x^n)x^n + x^n d(x^n)$ for each x in R. Then we say d is a derivation on R.*

Proof. Considering $m = n$ in Theorem 3, we will arrive at the conclusion. □

Now, move towards the next main theorem of this article:

Theorem 4. *Let $n, m \geq 1$ be any two fixed integers and R be k-torsion free semiprime ring, where $k \in \{2, m, n, (n+m-1)!\}$. If two mapping $f, \delta : R \longrightarrow R$ are additive and fulfilling the algebraic identity*

$$f(x^{n+m}) = x^n f(x^m) + x^m \delta(x^n) \quad \forall x \in R,$$

then f will be generalized left derivation associated with a left derivation δ on R.

Proof. Since

$$f(x^{n+m}) = x^n f(x^m) + x^m \delta(x^n) \; \forall x \in R, \tag{3}$$

then, we put $x + ky$ in place of x to get

$$f\Big(x^{n+m} + \binom{n+m}{1} x^{n+m-1}ky + \binom{n+m}{2} x^{n+m-2}k^2y^2 + ... + k^{n+m}y^{n+m}\Big) =$$
$$= \Big(x^n + \binom{n}{1} x^{n-1}ky + \binom{n}{2} x^{n-2}k^2y^2 + ... + k^n y^n\Big) \times$$
$$\times f\Big(x^m + \binom{m}{1} x^{m-1}ky + \binom{m}{2} x^{m-2}k^2y^2 + ... + k^m y^m\Big) + \Big(x^m + \binom{m}{1} x^{m-1}ky +$$
$$+ \binom{m}{2} x^{m-2}k^2y^2 + ... + k^m y^m\Big)\delta\Big(x^n + \binom{n}{1} x^{n-1}ky + \binom{n}{2} x^{n-2}k^2y^2 + ... + k^n y^n\Big).$$

Rewrite the above expression by using (3) as

$$kP_1(x,y) + k^2P_2(x,y) + ... + k^{n+m-1}P_{n+m-1}(x,y) = 0,$$

where $P_i(x,y)$ stand for the coefficients of k^i's for $i \in \{1,2,...,n+m-1\}$. If we replace k by $1,2,...,n+m-1$, then we find a system of $n+m-1$ homogeneous equations. It gives us a Vandermonde matrix

$$\begin{bmatrix} 1 & 1 & ... & 1 \\ 2 & 2^2 & ... & 2^{n+m-1} \\ ... & & & \\ ... & & & \\ n+m-1 & (n+m-1)^2 & ... & (n+m-1)^{n+m-1} \end{bmatrix}.$$

Which yields that $P_i(x,y) = 0$ for all $x,y \in R$ and for $i \in \{1,2,..,n+m-1\}$. Particularly, $i = 1$ give us

$$P_1(x,y) = \binom{n+m}{1} f(x^{n+m-1}y) - \binom{n}{1} x^{n-1}yf(x^n) - \binom{n}{1} x^n f(x^{n-1}y) - \\ - \binom{n}{1} x^n \delta(x^{n-1}y) - \binom{n}{1} x^{n-1}y\delta(x^n) = 0$$

$\forall x,y \in R$. Putting $x = e$ and making use of $d(e) = 0$ and n-torsion freeness of R, we arrive at

$$f(y) = yf(e) + \delta(y) \tag{4}$$

for every $y \in R$. Next,

$$P_2(x,y) = \binom{n+m}{2} f(x^{n+m-2}y^2) - \binom{m}{2} x^n f(x^{m-2}y^2) - \binom{m}{1}\binom{n}{1} x^{n-1}yf(x^{m-1}y) - \\ - \binom{n}{2} x^{n-2}y^2 f(x^m) - \binom{n}{2} x^m \delta(x^{n-2}y^2) - \binom{m}{1}\binom{n}{1} x^{m-1}y\delta(x^{n-1}y) - \\ - \binom{m}{2} x^{m-2}y^2\delta(x^n) = 0$$

for every $x,y \in R$. Rewrite the above expression by substituting e for x to obtain

$$\frac{(n+m)(n+m-1)}{2} f(y^2) = \frac{n(n-1)}{2} y^2 f(e) + mnyf(y) + \\ + \frac{m(m-1)}{2} f(y^2) + \frac{n(n-1)}{2} \delta(y^2) + mny\delta(y).$$

That is,

$$(n+m)(n+m-1)f(y^2) = n(n-1)y^2f(e) + 2mnyf(y) + m(m-1)f(y^2) + \\ + n(n-1)\delta(y^2) + 2mny\delta(y).$$

After simple manipulation, we arrive at

$$(2mn + n^2 - n)f(y^2) = n(n-1)y^2f(e) + 2mnyf(y) + n(n-1)\delta(y^2) + 2mny\delta d(y).$$

Using (4) to get the following

$$(2mn + n^2 - n)\Big[y^2f(e) + d(y^2)\Big] = n(n-1)y^2f(e) + 2mny\Big[yf(e) + d(y)\Big] + \\ + n(n-1)\delta(y^2) + 2mny\delta d(y).$$

Simplify the above expression and making use of $2mn$-torsion freeness of R, we have

$$\delta(y^2) = 2y\delta(y) \text{ for all } y \in R.$$

Therefore, d is a Jordan left derivation of R. Now, from (4), we get

$$f(y^2) = y^2 f(e) + \delta(y^2) = y\big[f(e)y + \delta(y)\big] + y\delta(y) = yf(y) + y\delta(y)$$

Hence F is generalized Jordan left derivation on R having a linked left derivation d. Using theorem from [1], we find the required conclusion. □

The next result is a consequence of Theorem 4.

Theorem 5. *Let two integers $n \geq 1$ and $m \geq 1$ be fixed and R be a semiprime ring having k-torsion freeness and $k \in \{2, m, n, (n+m-1)!\}$. If two mappings $f, \delta : R \to R$ are additive and satisfying*

$$f(x^{n+m}) = x^n f(x^m) + x^m \delta(x^n)$$

for every x in R. Then

1. *we say δ a derivation of R and for each x, y in R, $[\delta(x), y] = 0$.*
2. *$\delta(R) = Z(R)$,*
3. *one give R commutative or the other give $\delta = 0$ on R,*
4. *f will be a generalized derivation of R,*
5. *$f(x) = xq$ for some $q \in Q_l(R_C)$ $\forall x \in R$.*

Proof. (1) Since $f(x^{n+m}) = x^n f(x^m) + x^m \delta(x^n)$ for each x belong to R, then, making use of Theorem 4 and [1, Theorem 3.1], we get that δ is derivation on R and $[\delta(x), y] = 0$ $\forall x, y \in R$.

(2) Given that $f(x^{n+m}) = x^n f(x^m) + x^m \delta(x^n)$ for every x, y in R. Then use of Theorem 4, f will be a generalized left derivation linked with the Jordan left derivation δ of R. Therefore, using [11, Theorem 2], we conclude that $\delta(R) = Z(R)$.

(3) Suppose that $\delta \neq 0$. From (1) δ will be noted as a derivation and $[\delta(w), y] = 0$ for every w and y in R. For instance $[\delta(w), w] = 0$ for every w in R, As $\delta \neq 0$, therefore we say R, a commutative ring by utilyzing [7, Theorem 2] .

(4) Since $f(x^{n+m}) = x^n f(x^m) + x^m \delta(x^n)$ for all $x \in R$, then from Theorem 4, f will be a generalized left derivation on R. Again, if R is a noncommutative semiprime ring possess 2-torsion freeness, then from (3), we have $\delta = 0$. Therefore, f will be a right centralizer of R. Hence, using Proposition 2.10 of [1], there exists $q \in Q_l(R_C)$ such that $f(x) = xq$ for each x inside R.

(5) Considering $f(x^{n+m}) = x^n f(x^m) + x^m \delta(x^n)$ $\forall x \in R$. In perspective of part (3) and Theorem 4, ring R noted as commutative and δ, a derivation of R. Hence, f will be mark as generalized derivation of R.

□

Particularly, if we take $m = n$, we will arrive at Theorem 2.5 of [9]. Next, consider the algebraic condition

$$\mathfrak{F}(x^{n+m}) = x^n \mathfrak{F}(x^m) + x^m \Delta(x^n)$$

for all $x \in \mathfrak{A}$ on a semisimple Banach algebra $\mathfrak{A}$. To prove Theorem 6, we required the following results:

Result 1 ([6]). *Every linear derivation is continuous on a semi-simple Banach algebra.*

Result 2 ([8]). *Any continuous linear derivation maps algebra into its radical on a commutative Banach algebra.*

Result 3 ([10]). *On commutative semi-simple Banach algebras, every linear derivation is zero.*

In perspective of the above theorems, we conclude the following theorem:

Theorem 6. *If $n, m \geq 1$ are two fixed integers and $\mathfrak{A}$ is a semi-simple Banach algebra. Assuming that $\mathfrak{F}, \Delta \colon \mathfrak{A} \to \mathfrak{A}$ are two additive mappings which satisfies*

$$\mathfrak{F}(x^{n+m}) = x^n \mathfrak{F}(x^m) + x^m \Delta(x^n)$$

for all $x \in \mathfrak{A}$, then $\Delta = 0$ on $\mathfrak{A}$.

Proof. Recall that every semi-simple Banach algebra is semiprime, then all assumptions of first part of Theorem 5 are satisfied, therefore we find a derivation on semi-simple Banach algebra $\mathfrak{A}$, which is also linear. Thus $\Delta = 0$ from Theorem 4 of the reference [11]. □

Example 1 demonstrates that the main results of this article are not superfluous.

Example 1. Consider a ring

$$R = \left\{ \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} : m_1, m_2 \in 2\mathbb{Z}_8 \right\},$$

$\mathbb{Z}_8$ has its usual meaning. Define mappings $F, d, f, \delta : R \to R$ by

$$F \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & m_2 \end{pmatrix}, \qquad d \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$f \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & m_2 \end{pmatrix}, \qquad \delta \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & 0 \end{pmatrix},$$

It is obvious that F and f are not a generalized derivation and generalized left derivation on R respectively but F, d, f, δ follow the algebraic conditions

$$F(x^6) = x^2 F(x^4) + x^2 D(x^4)$$

and

$$f(x^6) = f(x^2)x^4 + x^4 \delta(x^2) \text{ for all } x \in R.$$

Which shows that semiprimess and torsion restriction on R are essential conditions in Theorem 1 and Theorem 4.

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