UDC 512.579

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SOME CLASS OF NUMERICAL RADIUS PEAK n-LINEAR MAPPINGS ON l_p-SPACES

S. G. Kim. *Some class of numerical radius peak n-linear mappings on l_p-spaces*, Mat. Stud. **57** (2022), 10–15.

For $n \geq 2$ and a real Banach space E, $\mathcal{L}(^nE : E)$ denotes the space of all continuous n-linear mappings from E to itself. Let

$$\Pi(E) = \Big\{[x^*, (x_1, \dots, x_n)] : x^*(x_j) = \|x^*\| = \|x_j\| = 1 \text{ for } j = 1, \dots, n\Big\}.$$

For $T \in \mathcal{L}(^nE : E)$, we define

$$\mathrm{Nr}(T) = \Big\{[x^*, (x_1, \dots, x_n)] \in \Pi(E) : |x^*(T(x_1, \dots, x_n))| = v(T)\Big\},$$

where $v(T)$ denotes the numerical radius of T. T is called *numerical radius peak mapping* if there is $[x^*, (x_1, \dots, x_n)] \in \Pi(E)$ such that $\mathrm{Nr}(T) = \{\pm[x^*, (x_1, \dots, x_n)]\}$. In this paper, we investigate some class of numerical radius peak mappings in $\mathcal{L}(^nl_p : l_p)$ for $1 \leq p < \infty$. Let $(a_j)_{j\in\mathbb{N}}$ be a bounded sequence in $\mathbb{R}$ such that $\sup_{j\in\mathbb{N}} |a_j| > 0$. Define $T \in \mathcal{L}(^nl_p : l_p)$ by

$$T\Big(\sum_{i\in\mathbb{N}} x_i^{(1)} e_i, \cdots, \sum_{i\in\mathbb{N}} x_i^{(n)} e_i\Big) = \sum_{j\in\mathbb{N}} a_j\ x_j^{(1)} \cdots x_j^{(n)}\ e_j. \qquad (*)$$

In particular, it is proved the following statements: 1. If $1 < p < +\infty$ then T is a numerical radius peak mapping if and only if there is $j_0 \in \mathbb{N}$ such that

$$|a_{j_0}| > |a_j| \text{ for every } j \in \mathbb{N}\backslash\{j_0\}.$$

2. If $p = 1$ then T is not a numerical radius peak mapping in $\mathcal{L}(^nl_1 : l_1)$.

1. Introduction. Let us sketch a brief history of norm or numerical radius attaining multilinear forms and polynomials on Banach spaces. In 1961 Bishop and Phelps [2] initiated and showed that the set of norm attaining functionals on a Banach space is dense in the dual space. Shortly after, attention was paid to possible extensions of this result to more general settings, specially bounded linear operators between Banach spaces. The problem of denseness of norm attaining functions has moved to other types of mappings like multilinear forms or polynomials. The first result about norm attaining multilinear forms appeared in a joint work of Aron, Finet and Werner [1], where they showed that the Radon-Nikodym property is sufficient for the denseness of norm attaining multilinear forms. Choi and Kim [3] showed that the Radon-Nikodym property is also sufficient for the denseness of norm or

2010 *Mathematics Subject Classification:* 46A22.
Keywords: numerical radius points; numerical radius peak mappings.
doi:10.30970/ms.57.1.10-15

numerical radius attaining polynomials. Jiménez-Sevilla and Payá [5] studied the denseness of norm attaining multilinear forms and polynomials on preduals of Lorentz sequence spaces. Choi, Domingo, Kim and Maestre [6] showed that for a scattered compact Hausdorff space K, every continuous n-homogeneous polynomial on $\mathcal{C}(K:\mathbb{C})$ can be approximated by norm attaining ones at extreme points and also that the set of all extreme points of the unit ball of $\mathcal{C}(K:\mathbb{C})$ is a norming set for every continuous complex polynomial. The authors obtained similar results if "norm" is replaced by "numerical radius".

Let $n\in\mathbb{N}, n\geq 2$. We write S_E for the unit sphere of a Banach space E. We denote by $\mathcal{L}(^nE:E)$ the Banach space of all continuous n-linear mappings from E into itself endowed with the norm $\|T\|=\sup_{(x_1,\cdots,x_n)\in S_E\times\cdots\times S_E}\|T(x_1,\cdots,x_n)\|$. $\mathcal{L}_s(^nE:E)$ denotes the closed subspace of all continuous symmetric n-linear mappings on E. We let

$$\Pi(E)=\Big\{[x^*,x_1,\ldots,x_n]:x^*(x_j)=\|x^*\|=\|x_j\|=1 \text{ for } j=1,\ldots,n\ \Big\}.$$

An element $[x^*,x_1,\ldots,x_n]\in\Pi(E)$ is called a *numerical radius point* of $T\in\mathcal{L}(^nE:E)$ if $|x^*(T(x_1,\ldots,x_n))|=v(T)$, where the numerical radius

$$v(T)=\sup_{[y^*,y_1,\ldots,y_n]\in\Pi(E)}\Big|y^*\Big(T(y_1,\ldots,y_n)\Big)\Big|.$$

We define

$$\mathrm{Nr}(T)=\Big\{[x^*,x_1,\ldots,x_n]\in\Pi(E):[x^*,x_1,\ldots,x_n] \text{ is a numerical radius point of } T\Big\}.$$

Notice that $[x^*,x_1,\ldots,x_n]\in\mathrm{Nr}(T)$ if and only if $[-x^*,-x_1,\ldots,-x_n]\in\mathrm{Nr}(T)$.

Kim [12] classified $\mathrm{Nr}(T)$ for every $T\in\mathcal{L}(^2l_1^2:l_1^2)$, where $l_1^2=\mathbb{R}^2$ with the l_1-norm. Kim [11] also studied $\mathrm{Nr}(T)$ for every $T\in\mathcal{L}(^nl_\infty^m:l_\infty^m)$ $(m\in\mathbb{N})$ and classified $\mathrm{Nr}(T)$ for every $T\in\mathcal{L}(^2l_\infty^2:l_\infty^2)$, where $l_\infty^m=\mathbb{R}^m$ with the sup-norm.

T is called *numerical radius peak mapping* if there is $[x^*,(x_1,\ldots,x_n)]\in\Pi(E)$ such that $\mathrm{Nr}(T)=\Big\{\ \pm[x^*,(x_1,\ldots,x_n)]\ \Big\}$. An element $(x_1,\ldots,x_n)\in E^n$ is called a *norming point* of $T\in\mathcal{L}(^nE)$ or $\mathcal{L}(^nE:E)$ if $\|x_1\|=\cdots=\|x_n\|=1$ and $\|T\|=\|T(x_1,\ldots,x_n)\|$. We define

$$\mathrm{Norm}(T)=\Big\{(x_1,\ldots,x_n)\in E^n:(x_1,\ldots,x_n) \text{ is a norming point of } T\Big\}.$$

$\mathrm{Norm}(T)$ is called the *norming set* of T.

In papers [9], [7], [10] $\mathrm{Norm}(T)$ is classified for every $T\in\mathcal{L}_s(^2l_\infty^2)$, $\mathcal{L}(^2l_\infty^2)$ or $\mathcal{L}_s(^3l_1^2)$, respectively.

A mapping $P:E\to\mathbb{C}$ is a continuous n-homogeneous polynomial if there exists a continuous n-linear form L on the product $E\times\cdots\times E$ such that $P(x)=L(x,\ldots,x)$ for every $x\in E$. We denote by $\mathcal{P}(^nE)$ the Banach space of all continuous n-homogeneous polynomials from E into $\mathbb{R}$ endowed with the norm $\|P\|=\sup_{\|x\|=1}|P(x)|$.

For more details about the theory of multilinear mappings and polynomials on a Banach space, we refer to [4].

An element $[x^*,x]\in\Pi(E)$ is called a *numerical radius point* of $P\in\mathcal{P}(^nE:E)$ if $|x^*(P(x))|=v(P)$, where the numerical radius

$$v(P)=\sup_{[y^*,y]\in\Pi(E)}\Big|y^*(P(y))\Big|.$$

We define

$$\mathrm{Nr}(P) = \Big\{[x^*, x] \in \Pi(E) : [x^*, x] \text{ is a numerical radius point of } P\Big\}.$$

$\mathrm{Nr}(P)$ is called the *numerical radius points set* of P. Notice that $[x^*, x] \in \mathrm{Nr}(P)$ if and only if $[-x^*, -x] \in \mathrm{Nr}(P)$.

An element $x \in E$ is called a *norming point* of $P \in \mathcal{P}(^nE)$ or $\mathcal{P}(^nE : E)$ if $\|x\| = 1$ and $\|P\| = \|P(x)\|$. We define

$$\mathrm{Norm}(P) = \Big\{x \in E : x \text{ is a norming point of } P\Big\}.$$

$\mathrm{Norm}(P)$ is called the *norming set* of P.

In [8], $\mathrm{Norm}(P)$ is classified for every $\mathcal{P}(^2l_\infty^2)$. If $T \in \mathcal{L}(^nE)$ or $\mathcal{L}(^nE : E)$ and $\mathrm{Norm}(T) \neq \varnothing$, T is called a *norm attaining* and if $T \in \mathcal{L}(^nE : E)$ and $\mathrm{Nr}(T) \neq \varnothing$, T is called a *numerical radius attaining*. Similarly, If $P \in \mathcal{P}(^nE)$ or $\mathcal{P}(^nE : E)$ and $\mathrm{Norm}(P) \neq \varnothing$, P is called a *norm attaining* and if $P \in \mathcal{P}(^nE : E)$ and $\mathrm{Nr}(P) \neq \varnothing$, P is called a *numerical radius attaining* (See [3]).

In was shown in [6] that for a scattered compact Hausdorff space K and $n \in \mathbb{N}$, $P \in \mathcal{P}(^n\mathcal{C}(K : \mathbb{C}) : \mathcal{C}(K : \mathbb{C}))$ is norm attaining if and only if it is numerical radius attaining.

Let

$$\mathrm{NA}(\mathcal{L}(^nE : E)) = \{T \in \mathcal{L}(^nE : E) : T \text{ is norm attaining}\}$$

and

$$\mathrm{NRA}(\mathcal{L}(^nE : E)) = \{T \in \mathcal{L}(^nE : E) : T \text{ is numerical radius attaining}\}.$$

It seems to be interesting to characterize a Banach space E such that $\mathrm{NA}(\mathcal{L}(^nE : E)) = \mathrm{NRA}(\mathcal{L}(^nE : E))$.

In this paper, we investigate some class of numerical radius peak mappings in $\mathcal{L}(^nl_p : l_p)$ for $1 \leq p < \infty$.

2. Main results. Let $n \geq 2$ and $1 < p < \infty$. Let $\{e_n\}_{n\in\mathbb{N}}$ be the canonical basis of real or complex space l_p and $\{e_n^*\}_{n\in\mathbb{N}}$ the biorthogonal functionals associated to $\{e_n\}_{n\in\mathbb{N}}$.

Theorem 1. *Let $n \geq 2$, $1 < p < \infty$ and $(a_j)_{j\in\mathbb{N}}$ be a bounded sequence in $\mathbb{R}$ such that $\sup_{j\in\mathbb{N}} |a_j| > 0$. Define $T \in \mathcal{L}(^nl_p : l_p)$ by*

$$T\Big(\sum_{i\in\mathbb{N}} x_i^{(1)} e_i, \cdots, \sum_{i\in\mathbb{N}} x_i^{(n)} e_i\Big) = \sum_{j\in\mathbb{N}} a_j\, x_j^{(1)} \cdots x_j^{(n)}\, e_j.$$

Then T is a numerical radius peak mapping if and only if there is $j_0 \in \mathbb{N}$ such that

$$|a_{j_0}| > |a_j| \text{ for every } j \in \mathbb{N}\backslash\{j_0\}.$$

Proof. Let $M = \sup_{j\in\mathbb{N}} |a_j|$.

Claim 1. $\|T\| = v(T) = M$.

It follows that

$$v(T) \leq \|T\| = \sup_{\|x_k\|_p=1,\ 1\leq k\leq n} \|T(x_1, \ldots, x_1)\|_p =$$

$$= \sup_{\|x_k\|_p=1,\ 1\le k\le n} \Big(\sum_{j\in\mathbb{N}} \Big(|a_j|\ \Big|x_j^{(1)}\Big|\cdots\Big|x_j^{(n)}\Big|\Big)^p\Big)^{1/p} \le M \sup_{\|x_1\|_p=1} \Big(\sum_{j\in\mathbb{N}} \Big|x_j^{(1)}\Big|^p\Big)^{1/p} =$$

$$= M = \sup_{j\in\mathbb{N}} |a_j| = \sup_{j\in\mathbb{N}} \Big|e_j^*(T(e_j,\dots,e_j))\Big| \le v(T),$$

which shows the claim 1.

($\Rightarrow$). Assume the contrary. We consider two cases:

Case 1. $M > |a_j|$ for all $j \in \mathbb{N}$.

Claim 1.1. $\mathrm{Nr}(T) = \varnothing$.

Assume that there is $[z^*,\ x_1,\ \dots,\ x_n] \in \mathrm{Nr}(T)$. Let $q \in \mathbb{R}$ be such that $1/p + 1/q = 1$. Write $z^* = \sum_{j\in\mathbb{N}} z_j e_j^* \in S_{l_q}$. Notice that $x_j = x_1$ for every $j = 2,\dots,n$. Write $x_1 = \sum_{j\in\mathbb{N}} x_j^{(1)} e_j$. It follows that

$$M = v(T) = |z^*(T(x_1,\dots,x_1))| = \Big|\sum_{j\in\mathbb{N}} z_j\ a_j\ \Big(x_j^{(1)}\Big)^n\Big| \le \sum_{j\in\mathbb{N}} |z_j|\ |a_j|\ \Big|x_j^{(1)}\Big|^n \le$$

$$\le M\Big(|z_1|\ \Big|x_1^{(1)}\Big| + \sum_{j\ge 2} |z_j|\ \Big|x_j^{(1)}\Big|\Big) \le M\ \|z^*\|_q\ \Big\|\Big(x_j^{(1)}\Big)_{j\in\mathbb{N}}\Big\|_p = M \text{ (by the Hölder inequality).}$$

Hence,

$$M = \sum_{j\in\mathbb{N}} |z_j|\ |a_j|\ \Big|x_j^{(1)}\Big|^n. \tag{1}$$

We will show that $|z_j|\ |x_j^{(1)}|^n = 0$ for all $j \in \mathbb{N}$. Assume that $|z_{\tilde{j}}|\ |x_{\tilde{j}}^{(1)}|^n \neq 0$ for some $\tilde{j} \in \mathbb{N}$. By (1), it follows that

$$M = \sum_{j\in\mathbb{N}} |z_j|\ |a_j|\ \Big|x_j^{(1)}\Big|^n < M\ |z_{\tilde{j}}|\ \Big|x_{\tilde{j}}^{(1)}\Big|^n + \sum_{j\in\mathbb{N}\setminus\{\tilde{j}\}} |z_j|\ |a_j|\ \Big|x_j^{(1)}\Big|^n \le$$

$$\le M\Big(\sum_{j\in\mathbb{N}} |z_j|\ \Big|x_j^{(1)}\Big|\Big) \le M\ \|z^*\|_q\ \Big\|\Big(x_j^{(1)}\Big)_{j\in\mathbb{N}}\Big\|_p = M,$$

which is impossible. Hence, $M = 0$. This is a contradiction. Hence, the claim 1.1 holds. Therefore, T is not a numerical radius peak mapping in $\mathcal{L}(^n l_p : l_p)$. This is a contradiction.

Case 2. There are $j_1 \neq j_2 \in \mathbb{N}$ such that $|a_{j_k}| = M$ for $k = 1, 2$.

Notice that $\pm[e_{j_k}^*,\ e_{j_k},\ \dots,\ e_{j_k}] \in \mathrm{Nr}(T)$ for $k = 1, 2$. Hence, T is not a numerical radius peak mapping in $\mathcal{L}(^n l_p : l_p)$. This is a contradiction.

($\Leftarrow$). Notice that $\pm[e_{j_0}^*,\ e_{j_0},\ \dots,\ e_{j_0}] \in \mathrm{Nr}(T)$. Let $[z^*,\ x_1,\ \dots,\ x_n] \in \mathrm{Nr}(T)$. Write $z^* = \sum_{j\in\mathbb{N}} z_j e_j^*$. Notice that $x_j = x_1$ for every $j = 2,\dots,n$. Write $x_1 = \sum_{j\in\mathbb{N}} x_j^{(1)} e_j$. By (1),

$$M = \sum_{j\in\mathbb{N}} |z_j|\ |a_j|\ \Big|x_j^{(1)}\Big|^n.$$

Claim 2.1. $z_j\ (x_j^{(1)})^n = 0$ for every $j \in \mathbb{N}\setminus\{j_0\}$.

Assume that there is $j' \in \mathbb{N}\setminus\{j_0\}$ such that $z_{j'}\ (x_{j'}^{(1)})^n \neq 0$. It follows that

$$M = \sum_{j\in\mathbb{N}} |z_j|\ |a_j|\ \Big|x_j^{(1)}\Big|^n < |z_{j_0}|\ |a_{j_0}|\ \Big|x_{j_0}^{(1)}\Big|^n + M\ |z_{j'}|\ \Big|x_{j'}^{(1)}\Big|^n + \sum_{j\in\mathbb{N}\setminus\{j_0,\ j'\}} |z_j|\ |a_j|\ \Big|x_j^{(1)}\Big|^n \le$$

$$\leq M\Big(\sum_{j\in\mathbb{N}} |z_j|\ \Big|x_j^{(1)}\Big|\Big) \leq\leq M\ \|z^*\|_q\ \Big\|\Big(x_j^{(1)}\Big)_{j\in\mathbb{N}}\Big\|_p = M,$$

which is impossible. Hence, the claim 3 holds and $M = |z_{j_0}|\ |a_{j_0}|\ |x_{j_0}^{(1)}|$. Hence, $|z_{j_0}| = 1 = |x_{j_0}^{(1)}|$. Thus, $[z^*,\ x_1,\ \ldots,\ x_n] = \pm[e_{j_0}^*,\ e_{j_0},\ \ldots,\ e_{j_0}]$. Therefore, T is a numerical radius peak mapping in $\mathcal{L}(^n l_p : l_p)$. □

Corollary 1. *Let $n \geq 2$ and $1 < p < \infty$. Let $(a_j)_{j\in\mathbb{N}}$ be a bounded sequence in $\mathbb{R}$ such that $M := \sup_{j\in\mathbb{N}} |a_j| > 0$. Let $T \in \mathcal{L}(^n l_p : l_p)$ be the same as in Theorem 2.1. Then the following hold:*

(a) If $S := \{j \in \mathbb{N} : |a_j| = M\} = \varnothing$, then $\mathrm{Nr}(T) = \varnothing$; (b) If $S \neq \varnothing$, then $|\,\mathrm{Nr}(T)| = 2^{|S|}$.

Proof. It follows from analogous arguments as in the proof of Theorem 2.1. □

Theorem 2. *Let $n \geq 2$. Let $(a_j)_{j\in\mathbb{N}}$ be a bounded sequence in $\mathbb{R}$ such that $\sup_{j\in\mathbb{N}} |a_j| > 0$. Define $T \in \mathcal{L}(^n l_1 : l_1)$ by*

$$T\Big(\sum_{i\in\mathbb{N}} x_i^{(1)} e_i, \cdots, \sum_{i\in\mathbb{N}} x_i^{(n)} e_i\Big) = \sum_{j\in\mathbb{N}} a_j\ x_j^{(1)} \cdots x_j^{(n)}\ e_j.$$

Then T is not a numerical radius peak mapping in $\mathcal{L}(^n l_1 : l_1)$.

Proof. **Claim.** $\|T\| = v(T) = M.$

It follows that

$$v(T) \leq \|T\| = \sup_{\|x_k\|_1=1,\ 1\leq k\leq n} \|T(x_1, \ldots, x_1)\|_1 =$$

$$= \sup_{\|x_k\|_1=1,\ 1\leq k\leq n} \sum_{j\in\mathbb{N}} |a_j|\ \Big|x_j^{(1)}\Big| \cdots \Big|x_j^{(n)}\Big| \leq M \sup_{\|x_k\|_1=1,\ 1\leq k\leq n} \Big(\sum_{j\in\mathbb{N}} \Big|x_j^{(1)}\Big|\Big) \cdots \Big(\sum_{j\in\mathbb{N}} \Big|x_j^{(n)}\Big|\Big) =$$

$$= M = \sup_{j\in\mathbb{N}} |a_j| = \sup_{j\in\mathbb{N}} \Big|e_j^*(T(e_j, \ldots, e_j))\Big| \leq v(T),$$

which shows the claim.

We consider two cases:

Case 1. $M > |a_j|$ for all $j \in \mathbb{N}$.

Claim. $\mathrm{Nr}(T) = \varnothing$.

Assume that there is $[z^*,\ x_1,\ \ldots,\ x_n] \in \mathrm{Nr}(T)$. Write $z^* = \sum_{j\in\mathbb{N}} z_j e_j^* \in S_{l_\infty}$. Write $x_k = \sum_{j\in\mathbb{N}} x_j^{(k)} e_j \in S_{l_1}$ for $k = 1, \ldots, n$. It follows that

$$M = v(T) = |z^*(T(x_1, \ldots, x_1))| = \Big|\sum_{j\in\mathbb{N}} z_j\ a_j\ x_j^{(1)} \cdots x_j^{(n)}\Big| \leq$$

$$\leq \sum_{j\in\mathbb{N}} |z_j|\ |a_j|\ \Big|x_j^{(1)}\Big| \cdots \Big|x_j^{(n)}\Big| \leq M\Big(\sum_{j\in\mathbb{N}} \Big|x_j^{(1)}\Big|\Big) = M.$$

Hence,

$$M = \sum_{j\in\mathbb{N}} |z_j|\ |a_j|\ \Big|x_j^{(1)}\Big| \cdots \Big|x_j^{(n)}\Big|. \tag{2}$$

We will show that $|z_j|\,|x_j^{(1)}|\cdots|x_j^{(n)}|=0$ for all $j\in\mathbb{N}$. Assume that $|z_{\tilde{j}}|\,|x_{\tilde{j}}^{(1)}|\cdots|x_{\tilde{j}}^{(n)}|\neq 0$ for some $\tilde{j}\in\mathbb{N}$. By (2), it follows that

$$M=\sum_{j\in\mathbb{N}}|z_j|\,|a_j|\,\left|x_j^{(1)}\right|\cdots\left|x_j^{(n)}\right|<M\,|z_{\tilde{j}}|\,\left|x_{\tilde{j}}^{(1)}\right|\cdots\left|x_{\tilde{j}}^{(n)}\right|+\sum_{j\in\mathbb{N}\setminus\{\tilde{j}\}}|z_j|\,|a_j|\,\left|x_j^{(1)}\right|\cdots\left|x_j^{(n)}\right|\leq$$
$$\leq M\Big(\sum_{j\in\mathbb{N}}\left|x_j^{(1)}\right|\Big)=M,$$

which is impossible. By (2), $M=0$, which is a contradiction. Hence the claim holds. Therefore, T is not a numerical radius peak mapping in $\mathcal{L}(^n l_1:l_1)$.

Case 2. There are $j_1\in\mathbb{N}$ such that $|a_{j_1}|=M$.

Let $j_2\neq j_1\in\mathbb{N}$. Notice that $\pm[e_{j_1}^*+e_{j_2}^*,\ e_{j_1},\ \ldots,\ e_{j_1}]\in \mathrm{Nr}(T)$. Hence, T is not a numerical radius peak mapping in $\mathcal{L}(^n l_1:l_1)$. $\square$

Question. Characterize all numerical radius peak mappings in $\mathcal{L}(^n l_p:l_p)$ for $n\geq 2$ and $1\leq p\leq\infty$.

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Received 30.11.2021
Revised 29.03.2022