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ISOMORPHISMS OF SOME ALGEBRAS OF ANALYTIC FUNCTIONS OF BOUNDED TYPE ON BANACH SPACES

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The theory of analytic functions is an important section of nonlinear functional analysis. In many modern investigations topological algebras of analytic functions and spectra of such algebras are studied. In this work we investigate the properties of the topological algebras of entire functions, generated by countable sets of homogeneous polynomials on complex Banach spaces.

Let X and Y be complex Banach spaces. Let $\mathbb{A} = \{A_1, A_2, \ldots, A_n, \ldots\}$ and $\mathbb{P} = \{P_1, P_2, \ldots, P_n, \ldots\}$ be sequences of continuous algebraically independent homogeneous polynomials on spaces X and Y, respectively, such that $\|A_n\|_1 = \|P_n\|_1 = 1$ and $\deg A_n = \deg P_n = n$, $n \in \mathbb{N}$. We consider the subalgebras $H_{b\mathbb{A}}(X)$ and $H_{b\mathbb{P}}(Y)$ of the Fréchet algebras $H_b(X)$ and $H_b(Y)$ of entire functions of bounded type, generated by the sets $\mathbb{A}$ and $\mathbb{P}$, respectively. It is easy to see that $H_{b\mathbb{A}}(X)$ and $H_{b\mathbb{P}}(Y)$ are the Fréchet algebras as well.

In this paper we investigate conditions of isomorphism of the topological algebras $H_{b\mathbb{A}}(X)$ and $H_{b\mathbb{P}}(Y)$. We also present some applications for algebras of symmetric analytic functions of bounded type. In particular, we consider the subalgebra $H_{bs}(L_\infty)$ of entire functions of bounded type on $L_\infty[0,1]$ which are symmetric, i.e. invariant with respect to measurable bijections of $[0,1]$ that preserve the measure. We prove that $H_{bs}(L_\infty)$ is isomorphic to the algebra of all entire functions of bounded type, generated by countable set of homogeneous polynomials on complex Banach space ℓ_∞.

1. Introduction and preliminaries. Let X be a complex locally convex topological vector space. A function $P\colon X \to \mathbb{C}$ is an n-homogeneous polynomial if there exists a symmetric n-linear map B_P defined on the Cartesian power X^n to $\mathbb{C}$ such that $P(x) = B_P(x, \ldots, x)$. The space of all n-homogeneous polynomials on X is denoted by $\mathcal{P}(^nX)$. The direct sum of spaces $\mathcal{P}(^nX)$, $n = 0, 1, 2, \ldots$ forms a unital algebra of continuous polynomials $\mathcal{P}(X)$.

A continuous function $f\colon X \to \mathbb{C}$ is said to be an *entire analytic function* (or just *entire function*) if its restriction on any finite dimensional subspace is analytic. The algebra of all entire functions on X is denoted by $H(X)$. There are a lot of various topologies on $H(X)$. In the paper we assume that $H(X)$ is endowed with the topology of the uniform convergence on compact subsets of X. If f is bounded on all bounded subsets of X, then f is called an *entire function of bounded type*. It is well-known that every function f of bounded type can be represented as a series of homogeneous polynomials f_n, so-called *Taylor polynomials*, such that $f(x) = \sum_{n=0}^{\infty} f_n(x)$, and the series uniformly converges on any bounded subset of X.

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The algebra of all entire functions of bounded type on X is denoted by $H_b(X)$. It is known that if X is a Banach space or a (DF)-space (see [12]), then $H_b(X)$ is a Fréchet algebra. In particular, if X is a Banach space, then the metrizable topology on $H_b(X)$ can be generated by norms

$$\|f\|_r = \sup\{|f(x)|\colon \|x\| \le r\}, \quad r \in \mathbb{Q}_+.$$

If X and Y are locally convex topological vector spaces, then a mapping $F\colon X \to Y$ is *analytic* if $\phi \circ F$ is an analytic function for every continuous linear functional ϕ on Y. A mapping $F\colon X \to X$ is called an *analytic automorphism* if F is analytic, bijective and F^{-1} is analytic. For more detailed information about analytic mappings on locally convex spaces we refer the reader to [10, 17].

For a given Banach space X we denote by $M_b(X)$ the spectrum of $H_b(X)$. In other words, $M_b(X)$ consists of all nonzero continuous complex valued homomorphisms (*characters*) of $H_b(X)$. A point evaluation functional $\delta_x\colon f \mapsto f(x)$ for a fixed $x \in X$ is a typical example of a character of $H_b(X)$. A *radius function* $R(\varphi)$ of a character φ is defined as the infimum of all $r > 0$ such that φ is continuous on the normed space $(H_b(X), \|\cdot\|_r)$ and can be computed (see [2]) by

$$R(\varphi) = \limsup_{n\to\infty} \|\varphi_n\|^{1/n} < \infty, \tag{1}$$

where φ_n is the restriction of φ to the Banach space $(\mathcal{P}(^nX), \|\cdot\|_1)$. According to [2], $R(\delta_x) = \|x\|$. Algebras of entire functions of bounded type on Banach spaces and their spectra were studied by many authors (see e. g. [2, 5, 23]).

Let $\mathbb{P} = \{P_1, P_2, \ldots, P_n, \ldots\}$ be a sequence of polynomials on a Banach space X. We denote by $\mathcal{P}_{\mathbb{P}}(X)$ the minimal unital algebra containing polynomials in $\mathbb{P}$. Let $H_{b\mathbb{P}}(X)$ be the closure of $\mathcal{P}_{\mathbb{P}}(X)$ in $H_b(X)$. Throughout in the paper we assume that the sequence $\mathbb{P}$ is algebraically independent and $\|P_n\|_1 = 1$, $n \in \mathbb{N}$. Recall that a sequence of elements in an algebra is *algebraically independent* if every nontrivial algebraic combination of element of this sequence is not equal to zero. Clearly that $\mathbb{P}$ forms an algebraic basis in $\mathcal{P}_{\mathbb{P}}(X)$, that is, every polynomial in $\mathcal{P}_{\mathbb{P}}(X)$ can be uniquely represented as an algebraic combination of elements in $\mathcal{P}_{\mathbb{P}}(X)$. It is easy to see that the basis $\mathbb{P}$ is not unique.

Algebras $H_{b\mathbb{P}}(X)$ for various sequences of polynomials were considered in [9, 15, 22, 18]. We denote by $M_{b\mathbb{P}} = M_{b\mathbb{P}}(X)$ the spectrum of $H_{b\mathbb{P}}(X)$. It is known (see [9, 15]) that the spectrum $M_{b\mathbb{P}}$ can be described as the set of sequences

$$\big\{(\varphi(P_1), \varphi(P_2), \ldots, \varphi(P_n), \ldots)\colon \varphi \in M_{b\mathbb{P}}\big\}.$$

In [15] it is proved that the radius function of any character $\varphi \in M_{b\mathbb{P}}$ can be computed by the same formula (1), where φ_n is the restriction of φ to $\mathcal{P}_{\mathbb{P}}(^nX)$. Typical examples of $H_{b\mathbb{P}}(X)$ can be obtained as algebras of symmetric analytic functions with respect to appropriate symmetry groups of isometric operators on X. Algebras of symmetric analytic functions on Banach spaces were studied in [1, 3, 4, 6, 7, 8, 11, 13, 14, 20, 21].

In Section 2 we consider conditions under which two algebras $H_{b\mathbb{A}}(X)$ and $H_{b\mathbb{P}}(Y)$ are isomorphic via a mapping Θ such that $\Theta(A_n) = P_n$, $n \in \mathbb{N}$. In Section 3 we propose some applications for algebras of symmetric analytic functions of bounded type.

2. Conditions of continuity.

Proposition 1. *Let X be a complex Banach space. Then the radius function of any point evaluation functional δ_x on $H_{b\mathbb{P}}(X)$ is less or equal than $\|x\|$.*

Proof. Since the space of homogeneous polynomials $\mathcal{P}_{\mathbb{P}}({}^nX)$ is a subspace in the space of all n-homogeneous continuous polynomials $\mathcal{P}({}^nX)$, the norm of the restriction of δ_x to $\mathcal{P}_{\mathbb{P}}({}^nX)$ is less or equal than the norm of the restriction of δ_x to $\mathcal{P}({}^nX)$. Thus, the radius function of δ_x on $H_{b\mathbb{P}}(X)$ is less or equal than the radius function of δ_x considered as a functional on $H_b(X)$. But due to [2] we know that the radius function of δ_x on $H_b(X)$ is equal to $\|x\|$. □

Proposition 2. *Let Z be a locally convex topological vector space and $H_0(Z)$ a subalgebra of $H(Z)$ which separates points of Z. Suppose that the both spectra of $H(Z)$ and $H_0(Z)$ consist of point evaluation functionals δ_z, $z \in Z$. Let $A\colon H(Z) \to H_0(Z)$ be a surjective continuous homomorphism. Then there exists an analytic injective mapping $\Phi\colon Z \to Z$ such that $A(f)(z) = f \circ \Phi(z)$ for every $f \in H(Z)$ and $z \in Z$.*

Proof. Let A' be the adjoint operator

$$A'\colon H_0'(Z) \to H'(Z), \quad A'(\varphi)(f) = \varphi(A(f)),$$

where $\varphi \in H_0'(Z)$, $f \in H(Z)$. Denote by $\widetilde{A}'$ the restriction of A' onto the subset of multiplicative functionals in $H'(Z)$ of the form δ_z, $z \in Z$, that is, onto the spectrum of $H_0(Z)$. Since A is an algebra homomorphism, $\widetilde{A}'$ maps the spectrum of $H_0(Z)$ to the spectrum of $H(Z)$.

Set $\Phi(z) = y$, where $\delta_y = \widetilde{A}'(\delta_z)$. Then for every $f \in H(Z)$, $f \circ \Phi = A(f) \in H_0(Z)$, that is, Φ is an analytic map by definition. To show that Φ is injective, let us suppose that $\Phi(z_1) = \Phi(z_2)$. Then $A(f)(z_1) = A(f)(z_2)$ for every $f \in H(Z)$. Since A is surjective, $g(z_1) = g(z_2)$ for every $g \in H_0$. Hence $z_1 = z_2$. □

Corollary 1. *Let $H(Z)$ and $H_0(Z)$ be as in Proposition 2 and A be a topological isomorphism of algebras. Then Φ is an analytic automorphism.*

Let $\mathbb{A} = \{A_1, A_2, \ldots, A_n, \ldots\}$ and $\mathbb{P} = \{P_1, P_2, \ldots, P_n, \ldots\}$ be sequences of algebraically independent polynomials on Banach spaces X and Y respectively, $\|A_n\|_1 = \|P_n\|_1 = 1$, $\deg A_n = \deg P_n = n$, $n \in \mathbb{N}$. Let us consider the following algebraic isomorphism of the algebras of polynomials $\Theta\colon \mathcal{P}_{\mathbb{A}}(X) \to \mathcal{P}_{\mathbb{P}}(Y)$ defined on the algebraic basis of $\mathcal{P}_{\mathbb{A}}(X)$ by $\Theta\colon A_n \mapsto P_n$. Then the algebraically adjoint operator

$$\Theta^*\colon \mathcal{P}^*_{\mathbb{P}}(Y) \to \mathcal{P}^*_{\mathbb{A}}(X)$$

is defined by

$$\Theta^*(\psi)(P) = (\psi \circ \Theta)(P), \quad \psi \in \mathcal{P}^*_{\mathbb{P}}(Y), \quad P \in \mathcal{P}_{\mathbb{A}}(X).$$

Here $\mathcal{P}^*_{\mathbb{A}}(X)$ is the space of all (not necessary continuous) linear functionals on $\mathcal{P}_{\mathbb{A}}(X)$. Let us denote by $\widetilde{\Theta^*}$ the restriction of Θ^* to the spectrum $M_{b\mathbb{P}}$ of $H_{b\mathbb{P}}(Y)$.

Theorem 1. *Suppose that $\widetilde{\Theta^*}$ maps $M_{b\mathbb{P}}$ to $M_{b\mathbb{A}}$ and there is a function $K\colon [0, +\infty) \to [0, +\infty)$, bounded on every segment in $[0, +\infty)$ such that*

$$R(\widetilde{\Theta^*}(\psi)) \le K\big(R(\psi)\big), \quad \psi \in M_{b\mathbb{P}}.$$

Then Θ is a continuous homomorphism which can be extended to a continuous homomorphism (which we denote by the same symbol Θ) from $H_{b\mathbb{A}}(X)$ to $H_{b\mathbb{P}}(Y)$.

Proof. For every $y \in Y$ let $\psi_y = \widetilde{\Theta^*}(\delta_y) \in M_{b\mathbb{A}}$. If $a \in H_{b\mathbb{A}}(X)$, then $a(x)$ can be written as

$$a(x) = \sum_{n=0}^{\infty} \sum_{k_1 + 2k_2 + \ldots + nk_n = n} \alpha_{k_1 \ldots k_n} A_1^{k_1}(x) A_2^{k_2}(x) \ldots A_n^{k_n}(x),$$

where $k_1, \ldots, k_n \in \mathbb{N} \bigcup \{0\}$ and $\alpha_{k_1 \ldots k_n} \in \mathbb{C}$. So we can formally extend Θ to $H_{b\mathbb{A}}(X)$ by

$$\Theta(a) = \Theta\Big(\sum_{n=0}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1\ldots k_n} A_1^{k_1} A_2^{k_2} \ldots A_n^{k_n}\Big) =$$

$$= \sum_{n=0}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1\ldots k_n} (\Theta(A_1))^{k_1} (\Theta(A_2))^{k_2} \ldots (\Theta(A_n))^{k_n} =$$

$$= \sum_{n=0}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1\ldots k_n} P_1^{k_1} P_2^{k_2} \ldots P_n^{k_n}.$$

Let us show that $\Theta(a) \in H_{b\mathbb{P}}(Y)$. For every $y \in Y$

$$\Theta(a)(y) = \sum_{n=0}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1\ldots k_n} P_1^{k_1}(y) \ldots P_n^{k_n}(y) =$$

$$= \sum_{n=0}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1\ldots k_n} (\psi_y(A_1))^{k_1} \ldots (\psi_y A_n)^{k_n} = \psi_y(a).$$

Since the function $a \in H_{b\mathbb{A}}(X)$ and $\psi_y \in M_{b\mathbb{A}}$, $\psi_y(a)$ is well defined and so the function $\Theta(a)(y)$ is also well defined for all $y \in Y$.

If $\|y\| \leq r$, then

$$|\Theta(a)(y)| = \Big|\sum_{n=0}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1\ldots k_n} P_1^{k_1}(y) \ldots P_n^{k_n}(y)\Big| \leq$$

$$= |\psi_y(a)| \leq \sup_{\{\psi\in M_{b\mathbb{A}}:\ R(\psi)\leq K(\|y\|)\}} |\psi(a)| \leq \sup_{\{\psi\in M_{A}:\ R(\psi)\leq \sup_{[0,r]} K(\gamma)\}} |\psi(a)| \leq$$

$$\leq \|\psi\|_{\sup_{[0,r]} K(\gamma)} \|a\|_{\sup_{[0,r]} K(\gamma)} = \|a\|_{\sup_{[0,r]} K(\gamma)} < \infty,$$

since a is of bounded type. Thus $\Theta(a)$ is bounded on bounded subsets. Hence, if

$$a_{(m)} = a - \sum_{n=m}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1\ldots k_n} A_1^{k_1} A_2^{k_2} \ldots A_n^{k_n},$$

then $\big|\Theta\big(a_{(m)}\big)(y)\big| \leq \big\|a_{(m)}\big\|_{\sup_{[0,r]} K(\gamma)} \to 0 \quad (m \to \infty)$.

According to the definition, Θ maps homogeneous polynomials into homogeneous polynomials. So, $\Theta(a)$ can be approximated by polynomials $\Theta(a) - \Theta\big(a_{(m)}\big)$ uniformly on bounded subsets of Y. Therefore, we have that for every $a \in H_{b\mathbb{A}}(X)$, $\Theta(a)$ is a well-defined function on Y which belongs to the closure of $\mathcal{P}_{\mathbb{P}}(Y)$ in $H_{b\mathbb{P}}(Y)$. So $\Theta(a) \in H_{b\mathbb{P}}(Y)$.

Next let us show that Θ is continuous. It is sufficient to prove that for all $r_1 > 0$ and $a \in H_{b\mathbb{A}}(X)$ there exist $C > 0$ and $r_2 > 0$ such that the inequality $\|\Theta(a)\|_{r_1} \leq C\|a\|_{r_2}$ holds. We have

$$\|\Theta(a)\|_{r_1} = \sup_{\{y\in Y:\ \|y\|\leq r_1\}} |\Theta(a)(y)| \leq \sup_{\{\psi\in M_{b\mathbb{A}}:\ R(\psi)\leq \max\{\sup_{[0,r_1]} K(\gamma),1\}\}} |\psi(a)| \leq$$

$$\leq \|\psi\|_{\max\{\sup_{[0,r_1]} K(\gamma),1\}} \|a\|_{\max\{\sup_{[0,r_1]} K(\gamma),1\}} = \|a\|_{\max\{\sup_{[0,r_1]} K(\gamma),1\}}.$$

Thus $C = 1$ and $r_2 = \max\{\sup_{[0,r_1]} K(\gamma), 1\}$ and so Θ is continuous. $\square$

Since both $\mathbb{A} = \{A_1, A_2, \ldots, A_n, \ldots\}$ and $\mathbb{P} = \{P_1, P_2, \ldots, P_n, \ldots\}$ are sequences of algebraically independent polynomials, the homomorphism Θ is injective. Thus we have the following corollary.

Corollary 2. *If the mapping Θ is surjective, then under conditions of Theorem 1, Θ is a topological isomorphism.*

Proof. It is enough to observe that Θ^{-1} is continuous because of the Inverse Map Theorem for Fréchet spaces (see e. g. [19, Corollaries 2.12]). $\square$

3. Applications for algebras of symmetric analytic functions. Let S be a group of isometries on a Banach space X. A function $f\colon X \to \mathbb{C}$ is said to be S-*symmetric* (or just *symmetric*) if $f(\sigma(x)) = f(x)$ for all $\sigma \in S$ and $x \in X$. We denote by $\mathcal{P}_s(X)$ the algebra of all symmetric polynomials on X and by $H_{bs}(X)$ its completion in $H_b(X)$. For many cases $\mathcal{P}_s(X)$ has an algebraic basis $\mathbb{P}$ and so $H_{bs}(X) = H_{b\mathbb{P}}(X)$. In [14] it is proved that if S is the group of all measurable automorphisms of $[0;1]$ which preserve the Lebesgue measure, then polynomials

$$R_n(x) = \int_{[0;1]} (x(t))^n dt, \quad x \in L_\infty[0;1]$$

form an algebraic basis in the algebra of symmetric polynomials $\mathcal{P}_s(L_\infty[0;1])$. The spectrum $M_{bs}(L_\infty[0;1])$ of $H_{bs}(L_\infty[0;1])$ coincides with the set of point evaluation functionals and can be described as the set of sequences

$$\Lambda^\times = \{\xi_n \colon \xi_n = R_n(x),\, x \in L_\infty[0;1],\, n \in \mathbb{N}\} = \{\xi_n \in \mathbb{C}\colon \sup_n |\xi_n|^{1/n} < \infty\}.$$

The set $\Lambda^\times$ can be naturally identified with the (DF)-space $H'(\mathbb{C})_\beta$, the strong dual to the Fréchet space $H(\mathbb{C})$ of entire functions on $\mathbb{C}$. According to [13], algebra $H_{bs}(L_\infty[0;1])$ is isomorphic to the algebra $H(H'(\mathbb{C})_\beta)$ of all entire functions on $H'(\mathbb{C})_\beta$. Similar results can be obtained for some other algebras of symmetric analytic functions of bounded type [20].

In [15] (see also [11]) was considered algebra $H_{b\mathbb{I}}(\ell_\infty)$ generated by polynomials $I_n(y) = y_n^n$, $y = (y_1, y_2, \dots) \in \ell_\infty$ and proved that the set of sequences $\{\xi_n \colon \xi_n = I_n(y),\, y \in \ell_\infty,\, n \in \mathbb{N}\}$ coincides with the set $\Lambda^\times$ defined above. So the spectrum of $H_{bs}(L_\infty[0;1])$ coincides with the spectrum of $H_{b\mathbb{I}}(\ell_\infty)$ as a point set and if $\Theta\colon R_n \mapsto I_n$, then $\widetilde{\Theta^*}$ is a bijection from $M_{b\mathbb{I}}$ onto $M_{bs}(L_\infty[0;1])$. Thus we have the following result.

Theorem 2. *There exists a topological isomorphism* $\Theta\colon H_{bs}(L_\infty[0;1]) \to H_{b\mathbb{I}}(\ell_\infty)$ *such that* $\Theta\colon R_n \mapsto I_n$.

Proof. Note first that according to [15, 14], both $M_{b\mathbb{I}}$ and $M_{bs}(L_\infty[0;1])$ consists of point evaluation functionals. Also, for every $\delta_y \in M_{b\mathbb{I}}$, $y \in \ell_\infty$ we have $R(\delta_y) = \|y\|$. Indeed, let y_n be such that $\|y\| - |y_n| \le \varepsilon$. Then

$$R(\delta_y) = \sup_{Q \in \mathcal{P}_{\mathbb{I}}(^n\ell_\infty),\, \|Q\| \le 1} |Q(y)|^{1/n} \ge |I_n(y)|^{1/n} = |y_n| \ge \|y\| - \varepsilon.$$

Since it is true for every $\varepsilon > 0$, $R(\delta_y) \ge \|y\|$. But from Proposition 1 we have the inverse inequality.

Let $y = (y_1, y_2, \dots, y_n, \dots) \in \ell_\infty$ be an arbitrary vector. Then the sequence of complex numbers $\xi = (\xi_1, \xi_2^2, \dots, \xi_n^n, \dots) = (y_1, y_2^2, \dots, y_n^n, \dots)$ satisfies the condition $\sup_n \sqrt[n]{|\xi_n|} < \infty$. According to [14] there exists $x_\xi \in L_\infty[0,1]$ such that $R_n(x_\xi) = \xi_n$ for every $n \in \mathbb{N}$ and $\|x_\xi\| \le \frac{2}{M} \sup_{n \in \mathbb{N}} \sqrt[n]{|\xi_n|}$, where $M = \prod_{n=1}^\infty \cos\left(\frac{\pi}{2}\frac{1}{n+1}\right)$. Note that $0 < M < 1$. Thus $\widetilde{\Theta^*}(\delta_y) = x_\xi$ and using Proposition 1 we have $R(\delta_{x_\xi}) \le \|x_\xi\| \le \frac{2}{M} \sup_{n \in \mathbb{N}} \sqrt[n]{|y_n|^n} = \frac{2\|y\|}{M} = K(\|y\|) = K(R(\delta_y))$, where $K(t) = 2t/M$. Thus we can apply Corollary 2. $\square$

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