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IS IT POSSIBLE TO GIVE A MORE PRECISE FORMULATION OF THE CRITERION OF MAXIMAL ACCRETIVITY FOR ONE EXTENSION OF NONNEGATIVE OPERATOR?

O. G. Storozh. *Is it possible to give a more precise formulation of the criterion of maximal accretivity for one extension of nonnegative operator?*, Mat. Stud. **54** (2020), 107–108.

The conditions being necessary and sufficient for maximal accretivity and maximal nonnegativity of some closed linear operators in Hilbert space are announced. The following problem is proposed: write down these conditions in more convenient form (one of the admissible variants is indicated).

Let H be a complex Hilbert space equipped with inner product $(\cdot|\cdot)$. The role of the initial object in this communication is played by a closed linear nonnegative operator $L_0\colon H \to H$ having the domain $D(L_0)$ dense in H. Under L, L_F, L_K we understand its adjoint, hard (Friedrichs), and soft (Neumann–Krein) extensions, respectively. Suppose that a fixed boundary value space (G, Γ_1, Γ_2) of L_0 and the corresponding Weyl function $M(\lambda)$ are given (we refer a reader to [1, p. 256–264] for the details). We keep the following notations: $D(T), \ker T$ are the domain and the kernel of (a linear) operator T, respectively; $\mathcal{B}(T)$ is the set of all linear bounded operators $A\colon G \to G$ such that $D(A) = G$; for each $A \in \mathcal{B}(T)$ where G is a Hilbert space, A^* means the adjont of A. The main object of our investigation is the operator $L_1 \subset L$ such that

$$D(L_1) = \{y \in D(L)\colon A_1\Gamma_1 y + A_2\Gamma_2 y = 0\},$$

where $A_1, A_2 \in \mathcal{B}(G)$. We assume below that L_0 is not positively definite operator, nevertheless

$$D(L_F) + D(L_K) = D(L), \quad D(L_F) \cap D(L_K) = D(L_0);$$

sequently there exists the strong limit $s\text{-}\lim_{\lambda\to -0} M(\lambda) := M_0 (\in \mathcal{B}(G))$. Moreover, we suppose that $D(L_F) = \ker \Gamma_2$ (the latter suggestion does not lead to the essential loss of generality).

Remind that a linear operator $T\colon H \to H$ is said to be an *accretive* if

$$\forall y \in D(T) \quad \operatorname{Re}(Ty|y) \geq 0,$$

and *maximal accretive* if, besides, it has no accretive extensions in H.

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Theorem 1. *L_1 is a maximal accretive (maximal nonnegative) operator if and only if*

i) $A_1M_0A_1^* + \mathrm{Re}(A_1A_2^*) \le 0 \ \ (A_1M_0A_1^* + A_1A_2^* \le 0)$;

ii) for some (sequently for each) $\lambda < 0 \quad \ker(A_1 - A_2 - A_1M(\lambda)) = \{0\}$

(compare with [1, p.373–374]).

Problem 1. *Is it correct (under the expressed above assumptions) to replace ii) by*

ii)′ $\ker(A_1 - A_2 - A_1M_0) = \{0\}$*?*

In the case when L_F is a positively definite operator, it is true.

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