O. G. STOROZH

IS IT POSSIBLE TO GIVE A MORE PRECISE FORMULATION OF THE CRITERION OF MAXIMAL ACCRETMITY FOR ONE EXTENSION OF NONNEGATIVE OPERATOR?


The conditions being necessary and sufficient for maximal accretivity and maximal nonnegativity of some closed linear operators in Hilbert space are announced. The following problem is proposed: write down these conditions in more convenient form (one of the admissible variants is indicated).

Let $H$ be a complex Hilbert space equipped with inner product $(\cdot|\cdot)$. The role of the initial object in this communication is played by a closed linear nonnegative operator $L_0: H \to H$ having the domain $D(L_0)$ dense in $H$. Under $L, L_F, L_K$ we understand its adjoint, hard (Friedrichs), and soft (Neumann–Krein) extensions, respectively. Suppose that a fixed boundary value space $(G, \Gamma_1, \Gamma_2)$ of $L_0$ and the corresponding Weyl function $M(\lambda)$ are given (we refer a reader to [1, p. 256–264] for the details). We keep the following notations: $D(T), \ker T$ are the domain and the kernel of (a linear) operator $T$, respectively; $B(T)$ is the set of all linear bounded operators $A: G \to G$ such that $D(A) = G$; for each $A \in B(T)$ where $G$ is a Hilbert space, $A^*$ means the adjoint of $A$. The main object of our investigation is the operator $L_1 \subset L$ such that

$$D(L_1) = \{y \in D(L): A_1 \Gamma_1 y + A_2 \Gamma_2 y = 0\},$$

where $A_1, A_2 \in B(G)$. We assume below that $L_0$ is not positively definite operator, nevertheless

$$D(L_F) + D(L_K) = D(L), \quad D(L_F) \cap D(L_K) = D(L_0);$$

sequently there exists the strong limit $s\text{-}\lim_{\lambda \to 0} M(\lambda) := M_0(\in B(G))$. Moreover, we suppose that $D(L_F) = \ker \Gamma_2$ (the latter suggestion does not lead to the essential loss of generality).

Remind that a linear operator $T: H \to H$ is said to be an accretive if

$$\forall y \in D(T) \quad \text{Re}(Ty|y) \geq 0,$$

and maximal accretive if, besides, it has no accretive extensions in $H$.

2010 Mathematics Subject Classification: 47A55, 47Gxx.

Keywords: Hilbert space; operator; accretivity.

doi:10.30970/ms.54.1.107-108

© O. G. Storozh, 2020
Theorem 1. \(L_1\) is a maximal accretive (maximal nonnegative) operator if and only if
i) \(A_1 M_0 A_1^* + \text{Re}(A_1 A_2^*) \leq 0\) (\(A_1 M_0 A_1^* + A_1 A_2^* \leq 0\));
ii) for some (sequently for each) \(\lambda < 0\) \(\ker(A_1 - A_2 - A_1 M(\lambda)) = \{0\}\)
(compare with [1, p.373–374]).

Problem 1. Is it correct (under the expressed above assumptions) to replace ii) by
\[\text{ii}' \quad \ker(A_1 - A_2 - A_1 M_0) = \{0\}\?\]

In the case when \(L_F\) is a positively definite operator, it is true.

REFERENCES


Faculty of Mechanics and Mathematics
Lviv Ivan Franko National University
storog@ukr.net

Received 31.05.2020
Revised 31.08.2020